Edge Magic Combinations of Spiders

Dr. A. Solairaju¹ and C. Ragavan²

¹Associate Professor of Mathematics, Jamal Mohamed College, Tiruchirappalli, Tamilnadu, India
E-mail: solaijmc@yahoo.in
²Lecturer in Mathematics, Sri Vidya Mandir Arts & Science College, Uthangarai, Krishnagiri, India
E-mail: ragavanrahul@rediffmail.com

Abstract

A total edge magic labeling of a graph with n vertices and m edges is a bijection f from VUE to the integers 1, 2, …, m + n such that there exists a constant s for any edge (u, v) in E satisfying f(u) + f(v) + f(u, v) = s. A graph is called a total edge magic graph if it has a total edge magic labeling. In this paper, we combine the spider graphs S_m,2 (m > 0) with star graphs S_n (n > 0) forming new graphs and show them to be total edge magic.

Keywords: Graph labeling; Antimagic; Spiders; AMS Classification code: 05C78

Introduction

All graphs considered here are finite simple and undirected. The vertex set and edge set are denoted by V(G) and E(G) respectively. Our notations and terminology are as in [2]. We refer to [1] for some basic concepts. Labeled graphs form useful models for a wide range of applications such as coding theory, X-ray crystallography, radar, astronomy, circuit design, communication network addressing and database management [1].

A total edge magic labeling of a graph with n vertices and m edges is a bijection f from VUE to the integers 1, 2, …, m + n such that there exists a constant s for any edge (u, v) in E satisfying f(u) + f(v) + f(u, v) = s. A graph is called a total edge magic graph if it has a total edge magic labeling [3, 4].
Main Results
Theorem 2.1: The graph $S_{3,2} \ast S_2 \ast S_2 \ast S_n$ is total edge magic, where $S_{3,2}$ is a spider graph on seven vertices and $S_n$ is a star graph with $n+1$ vertices, for $n \geq 1$.

Proof: Let $v_1, v_2, v_3, \ldots, v_7$ be the vertices of the spider graph $S_{3,2}$ with the label $v_1$ denoting the center and $v_5, v_6, v_7$ referring the outer vertices and $v_2, v_3, v_4$ representing the inner vertices. Connect the center of the star $S_2$ with the vertex $v_7$ and name it as $v_7$. Mark the other 2 vertices of $S_2$ by giving the labels $v_8, v_9$. Connect the center of the star $S_2$ with the vertex $v_6$ and name it as $v_6$. Mark the other 2 vertices of $S_2$ by giving the labels $v_{10}, v_{11}$. Connect the center of the star $S_n$ with the vertex $v_5$ and name it as $v_5$. Mark the other $n$ vertices of $S_n$ by giving the labels $v_{12}, v_{13}, \ldots, v_{n+11}$ continuously starting from one pendant vertex till the last pendant vertex.

Label the edges incident on $v_1$ and respectively with the vertices $v_2, v_3, v_4$ as $e_1, e_2, e_3$. Consider the other edge incident with $v_7$ and with the inner vertex $v_4$ and name it as $e_4$. Label the edges incident on $v_7$ and respectively with the vertex $v_8, v_9$ as $e_7, e_8$. Label the edges incident on $v_6$ and respectively with the vertices $v_{10}, v_{11}$ as $e_9, e_{10}$. Label the edges incident on $v_3$ and respectively with the vertices $v_{12}, v_{13}, \ldots, v_{n+11}$ as $e_{11}, e_{12}, \ldots, e_{n+10}$.

The new graph formed is denoted by $S_{3,2} \ast S_2 \ast S_2 \ast S_n$ and we can see that it is a tree on $n+11$ vertices and $n+10$ edges. The labeling $f$ on the vertices of $S_{3,2} \ast S_2 \ast S_2 \ast S_n$ defined by

\[
\begin{align*}
    f(v_i) &= n+10 + i \text{ for } i = 1; \\
    f(v_i) &= n+4 + i \text{ for } i = 2; \\
    f(v_i) &= n+7 + i \text{ for } i = 3; \\
    f(v_i) &= n+3 + i \text{ for } i = 7; \\
    f(v_i) &= n+10 + i \text{ for } i = 8; \\
    f(v_i) &= n+3 + i \text{ for } i = 9; \\
    f(v_i) &= n+8 + i \text{ for } i = 4; \\
    f(v_i) &= n+11 + i \text{ for } i = 5; \\
    f(v_i) &= n+12 + i \text{ for } 8 \leq i \leq n+11.
\end{align*}
\]

is a total edge magic labeling for the graph $S_{3,2} \ast S_2 \ast S_2 \ast S_n$. Here the magic constant $s=3n+34$.

The labeling $f$ on the edges of $S_{3,2} \ast S_2 \ast S_2 \ast S_n$ is defined by

\[
\begin{align*}
    f(e_i) &= n+5 + i \text{ for } i = 1; \\
    f(e_i) &= n+9 + i \text{ for } i = 2; \\
    f(e_i) &= n+7 + i \text{ for } i = 3; \\
    f(e_i) &= n+3 + i \text{ for } i = 4; \\
    f(e_i) &= n+5 + i \text{ for } i = 5; \\
    f(e_i) &= n+13 + i \text{ for } 7 \leq i \leq 8; \\
    f(e_i) &= n+11 + i \text{ for } 8 \leq i \leq 10; \\
    f(e_i) &= n+12 + i \text{ for } 11 \leq i \leq n+10.
\end{align*}
\]

Theorem 2.2 The graph $S_{3,2} \ast S_2 \ast S_3 \ast S_n$ is total edge magic, where $S_{3,2}$ is a spider graph on seven vertices and $S_n$ is a star graph with $n+1$ vertices, for $n \geq 1$.

Proof: Let $v_1, v_2, v_3, \ldots, v_7$ be the vertices of the spider graph $S_{3,2}$ with the label $v_1$ denoting the center and $v_5, v_6, v_7$ referring the outer vertices and $v_2, v_3, v_4$ representing the inner vertices. Connect the center of the star $S_2$ with the vertex $v_7$ and name it as $v_7$. Mark the other 2 vertices of $S_2$ by giving the labels $v_8, v_9$. Connect the center of the star $S_3$ with the vertex $v_6$ and name it as $v_6$. Mark the other 3 vertices of $S_3$ by giving
the labels \(v_{10}, v_{11}, v_{12}\). Connect the center of the star \(S_{n-1}\) with the vertex \(v_5\) and name it as \(v_5\). Mark the other \(n-1\) vertices of \(S_n\) by giving the labels \(v_{13}, \ldots, v_{n+11}\) continuously starting from one pendant vertex till the last pendant vertex.

Label the edges incident on \(v_1\) and respectively with the vertices \(v_2, v_3, v_4\) as \(e_1, e_2, e_3\). Consider the edge incident with \(v_5\) and with the inner vertex \(v_2\) and name it as \(e_4\). Attach the label \(e_5\) to the edge incident on \(v_6\) and with the inner vertex \(v_3\). Consider the other edge incident with \(v_7\) and with the inner vertex \(v_4\) and name it as \(e_6\). Label the edges incident on \(v_7\) and respectively with the vertices \(v_8, v_9\) as \(e_7, e_8\). Label the edges incident on \(v_8\) and respectively with the vertices \(v_{10}, v_{11}, v_{12}\) as \(e_9, e_{10}, e_{11}\). Label the edges incident on \(v_5\) and respectively with the vertices \(v_{13}, v_{14}, \ldots, v_{n+11}\) as \(e_{12}, e_{13}, \ldots, e_{n+10}\).

The new graph formed is denoted by \(S_{3,2}*S_3*S_3*Sn\) and we can see that it is a tree on \(n+13\) vertices and \(n+12\) edges. The total edge magic labeling \(f\) on the vertices of \(S_{3,2}*S_2*S_2*S_3*Sn\) is the same as that defined for \(S_{3,2}*S_2*S_2*S_n\) in theorem 2.1 except for the following: \(f(v_i) = n + 2 + i\) for \(i = 7\).

Here also the magic constant \(s = 3n+34\).

The labeling \(f\) on the edges of \(S_{3,2}*S_2*S_3*S_n\) is defined as in theorem 2.1 except for \(f(e_i) = n + 4 + i\) for \(i = 1; f(e_i) = n + 14 + i\) for \(i = 7, 8; f(e_i) = n + 9 + i\) for \(i = 11\).

**Theorem 2.3:** The graph \(S_{3,2}*S_3*S_3*S_n\) is total edge magic, where \(S_{3,2}\) is a spider graph on seven vertices and \(S_n\) is a star graph with \(n+1\) vertices, for \(n \geq 1\).

**Proof:** Let \(v_1, v_2, v_3, \ldots, v_7\) be the vertices of the spider graph \(S_{3,2}\) with the label \(v_1\) denoting the center and \(v_5, v_6, v_7\) referring the outer vertices and \(v_2, v_3, v_4\) representing the inner vertices. Connect the center of the star \(S_3\) with the vertex \(v_7\) and name it as \(v_7\). Mark the other \(3\) vertices of \(S_3\) by giving the labels \(v_6, v_9, v_{10}\). Connect the center of the star \(S_3\) with the vertex \(v_6\) and name it as \(v_6\). Mark the other \(3\) vertices of \(S_3\) by giving the labels \(v_{11}, v_{12}, v_{13}\). Connect the center of the star \(S_n\) with the vertex \(v_5\) and name it as \(v_5\). Mark the other \(n\) vertices of \(S_n\) by giving the labels \(v_{14}, v_{15}, \ldots, v_{n+13}\) continuously starting from one pendant vertex till the last pendant vertex.

Label the edges incident on \(v_1\) and respectively with the vertices \(v_2, v_3, v_4\) as \(e_1, e_2, e_3\). Consider the edge incident with \(v_5\) and with the inner vertex \(v_2\) and name it as \(e_4\). Attach the label \(e_5\) to the edge incident on \(v_6\) and with the inner vertex \(v_3\). Consider the other edge incident with \(v_7\) and with the inner vertex \(v_4\) and name it as \(e_6\). Label the edges incident on \(v_7\) and respectively with the vertices \(v_8, v_9, v_{10}\) as \(e_7, e_8, e_9\). Label the edges incident on \(v_8\) and respectively with the vertices \(v_{11}, v_{12}, v_{13}\) as \(e_{10}, e_{11}, e_{12}\). Label the edges incident on \(v_5\) and respectively with the vertices \(v_{14}, v_{15}, \ldots, v_{n+13}\) as \(e_{13}, e_{14}, \ldots, e_{n+12}\).

The new graph formed is denoted by \(S_{3,2}*S_3*S_3*S_n\) and we can see that it is a tree on \(n+13\) vertices and \(n+12\) edges. The labeling \(f\) on the vertices of \(S_{3,2}*S_3*S_3*S_n\) defined by

\[
\begin{align*}
f(v_1) &= n + 8 + i \text{ for } i = 1; \\
f(v_2) &= n + 12 + i \text{ for } i = 2; \\
f(v_3) &= n + 13 + i \text{ for } i = 3; \\
f(v_4) &= n + 14 + i \text{ for } i = 4; \\
f(v_5) &= n + 9 + i \text{ for } i = 5; \\
f(v_6) &= n + 11 + i \text{ for } i = 6; \\
f(v_7) &= n + 4 + i \text{ for } i = 7; \\
f(e_i) &= n + 14 - i \text{ for } 8 \leq i \leq (n + 13).
\end{align*}
\]
is a total edge magic labeling for the graph $S_{3,2}^* S_3^* S_3^* S_n$. Here the magic constant $s = 3n + 40$.

The labeling $f$ on the edges of $S_{3,2}^* S_3^* S_3^* S_n$ is defined by

- $f(e_i) = n + 12 + i$ for $i = 1$;
- $f(e_i) = n + 9 + i$ for $i = 3$;
- $f(e_i) = n + 2 + i$ for $i = 5$;
- $f(e_i) = n + 16 + i$ for $i = 7,8,9$;
- $f(e_i) = n + 10 + i$ for $i = 10,11,12$;
- $f(e_i) = n + 13 + i$ for $13 \leq i \leq n + 12$.

**Theorem 2.4:** The graph $S_{3,2}^* S_3^* S_3^* S_n$ is total edge magic, where $S_{3,2}$ is a spider graph on seven vertices and $S_n$ is a star graph with $n + 1$ vertices, for $n \geq 1$.

**Proof:** Let $v_1, v_2, v_3, \ldots, v_7$ be the vertices of the spider graph $S_{3,2}$ with the label $v_1$ denoting the center and $v_3, v_6, v_7$ referring the outer vertices and $v_2, v_3, v_4$ representing the inner vertices. Connect the center of the star $S_3$ with the vertex $v_7$ and name it as $v_7$. Mark the other 3 vertices of $S_3$ by giving the labels $v_8, v_9, v_{10}$. Connect the center of the star $S_4$ with the vertex $v_6$ and name it as $v_6$. Mark the other 4 vertices of $S_4$ by giving the labels $v_{11}, v_{12}, v_{13}, v_{14}$. Connect the center of the star $S_{n-1}$ with the vertex $v_5$ and name it as $v_5$. Mark the other $n-1$ vertices of $S_{n-1}$ by giving the labels $v_{15}, v_{16}, \ldots, v_{n+13}$ continuously starting from one pendant vertex till the last pendant vertex.

Label the edges incident on $v_7$ and respectively with the vertices $v_1, v_2, v_3, v_4$ as $e_1, e_2, e_3$. Consider the edge incident with $v_7$ and with the inner vertex $v_2$ and name it as $e_4$. Attach the label $e_5$ to the edge incident on $v_6$ and with the inner vertex $v_3$. Consider the other edge incident with $v_7$ and with the inner vertex $v_4$ and name it as $e_6$. Label the edges incident on $v_7$ and respectively with the vertices $v_8, v_9, v_{10}$ as $e_7, e_8, e_9$. Label the edges incident on $v_9$ and respectively with the vertices $v_{11}, v_{12}, v_{13}, v_{14}$ as $e_{10}, e_{11}, e_{12}, e_{13}$. Label the edges incident on $v_5$ and respectively with the vertices $v_{15}, v_{16}, \ldots, v_{n+13}$ as $e_{14}, e_{15}, \ldots, e_{n+12}$.

The new graph formed is denoted by $S_{3,2}^* S_3^* S_3^* S_{n-1}$ and we can see that it is a tree on $n+13$ vertices and $n+12$ edges. The total edge magic labeling $f$ on the vertices of $S_{3,2}^* S_3^* S_3^* S_{n-1}$ is the same as that defined for $S_{3,2}^* S_3^* S_3^* S_n$ in theorem 2.3 except for the following:

- $f(v_i) = n + 3 + i$ for $i = 7$
- Here also the magic constant $s = 3n + 40$.

The labeling $f$ on the edges of $S_{3,2}^* S_3^* S_3^* S_{n-1}$ is defined as in theorem 2.3 except for $f(e_i) = n + 5 + i$ for $i = 6$; $f(e_i) = n + 17 + i$ for $i = 7,8,9$.

**Concluding Remarks**

We are further investigating on the total edge magic labelings of the subsequent graphs and whether it is possible to find a total edge magic labeling for a general graph $S_{m,r}^* S_p^* S_q^* S_n$ for $m \geq 3$, $p \geq 1$, $q \geq 1$, $n \geq 1$, $r \geq 2$.  


References


