On t-Best Coapproximation in fuzzy anti-2-normed linear spaces

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Abstract
In this paper, we study the concept of t-best coapproximation in fuzzy anti-2-normed linear spaces. We introduce the notion of t-best coapproximation, t-coproximinal sets, t-coChebyshev sets and t-orthogonality and prove some interesting theorems to characterization of t-best coapproximation elements.

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1. Introduction
The concept of best coapproximation was introduced by Franchetti and Furi [3], in order to study some characteristic properties of real Hilbert spaces, and such problems were considered further by Papini and Singer [9] and Rao and Saravanan [10]. The concept of 2-norm on a linear space has been introduced and developed by Gähler in [4, 5] and Gunawan and Mashadi [5]. The idea of fuzzy norm was initiated by Katsaras in [8]. In [7] Iqbal H. Jebril and Samanta introduced fuzzy anti-norm on a linear space depending on the idea of fuzzy anti-norm was introduced by Bag and Samanta [2] and investigated

In this paper, we consider the set of $t$-best coapproximation in fuzzy anti-2-normed linear spaces and then prove several theorems pertaining to this set.

### 2. Preliminaries

**Definition 2.1.** Let $X$ be a real linear space of dimension greater than one and let $\|\cdot, \cdot\|$ be a real valued function on $X \times X$ satisfying the following conditions

1. $2N_1$: $\|x, y\| = 0$ if and only if $x$ and $y$ are linearly dependent
2. $2N_2$: $\|x, y\| = \|y, x\|$ for all $x, y \in X$
3. $2N_3$: $\|\alpha x, y\| = |\alpha| \|x, y\|$, for every $\alpha \in R$
4. $2N_4$: $\|x, y + z\| \leq \|x, y\| + \|x, z\|

Then the function $\|\cdot, \cdot\|$ is called a 2-norm on $X$ and the pair $(X, \|\cdot, \cdot\|)$ is called a 2-normed linear space.

**Example 2.2.** Let $X = R^3$ be a real linear space. Define $\|\cdot, \cdot\| : X \times X \rightarrow R$ by $\|x, y\| = \max\{|x_1y_2 - x_2y_1|, |x_2y_3 - x_3y_2|, |x_3y_1 - x_1y_3|\}$, where $x = (x_1, x_2, x_3)$, $y = (y_1, y_2, y_3)$ are in $R^3$. Then $(X, \|\cdot, \cdot\|)$ is a 2-normed linear space.

**Definition 2.3.** Let $X$ be a linear space over a real field $F$. A fuzzy subset $N$ of $X \times X \times R$ is called a fuzzy 2-norm on $X$ if the following conditions are satisfied for all $x, y, z \in X$.

1. $2N_1$: For all $t \in R$ with $t \leq 0$, $N(x, y, t) = 0$,
2. $2N_2$: For all $t \in R$ with $t > 0$, $N(x, y, t) = 1$ if and only if $x, y$ are linearly dependent
3. $2N_3$: $N(x, y, t)$ is invariant under any permutation of $x, y$
4. $2N_4$: For all $t \in R$ with $t > 0$, $N(x, cy, t) = N\left(x, y, \frac{t}{|c|}\right)$ if $c \neq 0, c \in F$
5. $2N_5$: For all $s, t \in R$, $N(x, y + z, s + t) \geq \min\{N(x, y, s), N(x, z, t)\}$
6. $2N_6$: $N(x, y, t)$ is a non-decreasing function of $t \in R$ and $\lim_{t \to \infty} N(x, y, t) = 1$.

Then the pair $(X, N)$ is called a fuzzy 2-normed linear space (briefly F-2-NLS).

**Example 2.4.** Let $(X, \|\cdot, \cdot\|)$ be a 2-normed linear space. Define

$$N(x, y, t) = \begin{cases} \frac{t}{t + \|x, y\|}, & \text{if } t > 0, \ t \in R, \ x, y \in X \\ 0, & \text{if } t \leq 0, \ t \in R, \ x, y \in X. \end{cases}$$

Then $(X, N)$ is a fuzzy 2-normed linear space.
Definition 2.5. Let $X$ be a linear space over a real field $F$. A fuzzy subset $N$ of $X \times X \times R$ is called a fuzzy anti-2-norm on $X$ if the following conditions are satisfied for all $x, y, z \in X$:

1. $(a - 2 - N_1)$: For all $t \in R$ with $t \leq 0$, $N(x, y, t) = 1$,
2. $(a - 2 - N_2)$: For all $t \in R$ with $t > 0$, $N(x, y, t) = 0$ if and only if $x, y$ are linearly dependent,
3. $(a - 2 - N_3)$: $N(x, y, t)$ is invariant under any permutation of $x, y$,
4. $(a - 2 - N_4)$: For all $t \in R$ with $t > 0$, $N(cx, y, t) = N(x, y, \frac{t}{|c|})$ if $c \neq 0, c \in F$,
5. $(a - 2 - N_5)$: For all $s, t \in R$, $N(x, y + z, s + t) \leq \max\{N(x, y, s), N(x, z, t)\}$,
6. $(a - 2 - N_6)$: $N(x, y, t)$ is a non-increasing function of $t \in R$ and $\lim_{t \to \infty} N(x, y, t) = 0$.

Then the pair $(X, N)$ is called a fuzzy anti-2-normed linear space (briefly Fa-2-NLS).

Remark 2.6. From $(a - 2 - N_3)$, it follows that in Fa-2-NLS,

1. $(a - 2 - N_4)$: For all $t \in R$ with $t > 0$, $N(cx, y, t) = N(x, y, \frac{t}{|c|})$ if $c \neq 0, c \in F$,
2. $(a - 2 - N_5)$: For all $s, t \in R$, $N(x + z, y, s + t) \leq \max\{N(x, y, s), N(z, y, t)\}$.

Example 2.7. Let $(X, \|\cdot, \cdot\|)$ be a 2-normed linear space. Define

$$N(x, y, t) = \left\{ \begin{array}{ll} \frac{\|x, y\|}{t + \|x, y\|}, & \text{if } t > 0, \ t \in R, \ x, y \in X \\
1, & \text{if } t \leq 0, \ t \in R, \ x, y \in X. \end{array} \right.$$

Then $(X, N)$ is a Fuzzy anti-2-normed linear space.

Definition 2.8. A sequence $\{x_k\}$ in a fuzzy anti-2-normed linear space $(X, N)$ is said to be $t$-convergent to $x \in X$ if given $t > 0, 0 < r < 1$, there exists an integer $n_0 \in N$ such that $N(x_1, x_k - x, t) < r$, for all $k \geq n_0$.

Theorem 2.9. In a fuzzy anti-2-normed linear space $(X, N)$, a sequence $\{x_k\}$ is $t$-convergent to $x \in X$ if and only $\lim_{k \to \infty} N(x_1, x_k - x, t) = 0, \forall t > 0$.

Definition 2.10. Let $(X, N)$ be a fuzzy anti-2-normed linear space. Let $\{x_k\}$ be a sequence in $X$ then $\{x_k\}$ is said to be $t$-Cauchy sequence if $\lim_{k \to \infty} N(x_1, x_{k+p} - x_k, t) = 0, \forall t > 0$ and $p = 1, 2, 3, \ldots$.

A fuzzy anti-2-normed linear space $(X, N)$ is said to be complete if every Cauchy sequence in $X$ is convergent. A complete fuzzy anti-2-normed linear space $(X, N)$ is called a fuzzy anti-2-Banach space. The open ball $B(x, r, t)$ and the closed ball $B[x, r, t]$ with the center $x \in X$ and radius $0 < r < 1, t > 0$ are defined as follows:

$$B(x, r, t) = \{y \in X : N(x_1, x - y, t) < r\}$$

$$B[x, r, t] = \{y \in X : N(x_1, x - y, t) \leq r\}.$$
A subset $A$ of $X$ is said to be $t$-open if there exists $r \in (0, 1)$ such that $B(x, r, t) \subset A$ for all $x \in A$ and $t > 0$. A subset $A$ of $X$ is said to be $t$-closed if for any sequence $\{x_k\}$ in $A$ converges to $x \in A$. i.e., $\lim_{k \to \infty} N(x_1, x_k - x, t) = 0$, for all $t > 0$ implies that $x \in A$. A subset $A$ of $X$ is said to be $t$-compact if for every sequence $\{x_k\}$ in $A$ has a subsequence $\{x_{n_k}\}$ which $t$-converges to an element $x_0 \in A$.

### 3. $t$-Best Coapproximation

**Definition 3.1.** Let $A$ be a nonempty subset of fuzzy anti-2-normed linear space $(X, N)$ and $t > 0$. For $x \in X$, an element $y_0 \in A$ is said to be a $t$-best coapproximation of $x$ from $A$ if $N(x, y_0 - y, t) \leq N(x, x - y, t)$, for all $y \in A$.

The set of all elements of $t$-best coapproximation of $x$ from $A$ is denoted by $R^t_A(x)$ and is defined as

$$R^t_A(x) = \{y_0 \in A : N(x, y_0 - y, t) \leq N(x, x - y, t), \forall y \in A\}$$

For $t > 0$ putting

$$\hat{A}_t^x = \{x \in X : N(x, y, t) \leq N(x, x - y, t), \forall y \in A\} = (R^t_A)^{-1}(0).$$

It is clear $y_0 \in R^t_A(x)$ if and only if $x - y_0 \in \hat{A}_t^x$.

**Definition 3.2.** Let $A$ be a non empty subset of a fuzzy anti-2-normed linear space $(X, N)$. If for $t > 0$ and each $x \in X$ has at least (respectively exactly) one $t$-best coapproximation in $A$, then $A$ is called a $t$-coproximinal (respectively $t$-coChebyshev) set. Also $A$ is called $t$-quasi-coChebyshev set if $R^t_A(x)$ is a $t$-compact set.

**Theorem 3.3.** Let $(X, N)$ be a fuzzy anti-2-normed linear space, and $A$ be a subspace of $X$ and $t > 0$. Then for each $x \in X$

(i) $A$ is a $t$-coproximinal if and only if $X = A + \hat{A}_t^x$.

(ii) $A$ is a $t$-coChebyshev subspace if and only if $X = A \oplus \hat{A}_t^x$.

**Proof.** (i) ($\Rightarrow$) Assume that $A$ is $t$-coproximinal, $x \in X$ and $y_0 \in R^t_A(x)$. Then $x - y_0 \in \hat{A}_t^x$. Now, $x = y_0 + (x - y_0) \in A + \hat{A}_t^x$. Hence $X = A + \hat{A}_t^x$.

($\Leftarrow$) Let $x \in X = A + \hat{A}_t^x$. Then $x = y_0 + \tilde{y}$, $y_0 \in A$, $\tilde{y} \in \hat{A}_t^x$ and so $0 \in R^t_A(\tilde{y}) = R^t_A(x - y_0)$. Since, $N(x_1, 0 - (x - y_0), t) \leq N(x_1, y - (x - y_0), t)$, so $N(x_1, y_0 - x, t) \leq N(x_1, (y + y_0) - x, t)$, where $y + y_0 \in A$; hence $y_0 \in R^t_A(x)$. Therefore $A$ is $t$-coproximinal.

(ii) ($\Rightarrow$) Suppose that $A$ is $t$-coChebyshev subspace, $x \in X$, and $x = y_1 + \tilde{y}_1 = y_2 + \tilde{y}_2$, where $y_1, y_2 \in A$ and $\tilde{y}_1, \tilde{y}_2 \in \hat{A}_t^x$. We show that $y_1 = y_2$ and $\tilde{y}_1 = \tilde{y}_2$. Since $x = y_1 + \tilde{y}_1 = y_2 + \tilde{y}_2$, then $x - y_1 = y_2, x - y_2 = \tilde{y}_2$, this implies that $y_1, y_2 \in R^t_A(x)$. Therefore $y_1 = y_2$, it follows that $\tilde{y}_1 = \tilde{y}_2$. Thus $X = A \oplus \hat{A}_t^x$.

($\Leftarrow$) Let $X = A \oplus \hat{A}_t^x$, and suppose for $x \in X$, there exists $y_1, y_2 \in R^t_A(x)$. Then $x - y_1,$
Theorem 3.4. Let \( A \) be a non empty subset of a fuzzy anti-2-normed linear space \((X, N)\). Then for \( t > 0 \) and each \( x \in X \),

(i) \( R^t_{A+y}(x + y) = R^t_A(x) + y \), for every \( x, y \in X \).

(ii) \( R^t_{\alpha A}(\alpha x) = \alpha R^t_A(x) \), for every \( x \in X \) and \( \alpha \in R \setminus \{0\} \).

(iii) \( A \) is \( t \)-coproximinal (respectively \( t \)-coChebyshev) if and only if \( A + y \) is \( t \)-coproximinal (respectively \( t \)-coChebyshev), for any \( y \in X \).

(iv) \( A \) is \( t \)-coproximinal (respectively \( t \)-coChebyshev) if and only if \( \alpha A \) is \( |\alpha|t \)-coproximinal (respectively \( |\alpha|t \)-coChebyshev), for any given \( \alpha \in R \setminus \{0\} \).

Proof. (i) For any \( x, y \in X, t > 0, y_0 \in R^t_{A+y}(x+y) \) if and only if \( N(x_1, y_0-(a+y), t) \leq N(x_1, x+y-(a+y), t) \) for all \( (a+y) \in A+y \) if and only if \( N(x_1, (y_0-y)-a, t) \leq N(x_1, x-a, t) \) for all \( a \in A \), if and only if, \((y_0-y) \in R^t_A(x), \) i.e., \( y_0 \in R^t_A(x) + y \).

(ii) For any \( x \in X, \alpha \in R \setminus \{0\} \) and \( t > 0, y_0 \in R^t_{\alpha A}(\alpha x) \) if and only if, \( N(x_1, (y_0-\alpha a), |\alpha|t) \leq N(x_1, (\alpha x-\alpha a), |\alpha|t) \) for all \( a \in A \) if and only if

\[
N \left( x_1, \frac{1}{|\alpha|} y_0 - a \right), |\alpha|t \right) \leq N(x_1, (x-a), t)
\]

for all \( a \in A \) if and only if \( \frac{1}{|\alpha|} y_0 \in R^t_A(x) \) if and only if \( y_0 \in \alpha R^t_A(x) \). Therefore

\( R^t_{\alpha A}(\alpha x) = \alpha R^t_A(x) \)

(iii) The proof is an immediate consequence of (i).

(iv) The proof is an immediate consequence of (ii).

Corollary 3.5. Let \( M \) be a nonempty subspace of a fuzzy anti-2-normed linear space \((X, N)\). Then for \( t > 0 \) and each \( x \in X \),

(i) \( R^t_M(x + y) = R^t_M(x) + y \), for every \( x, y \in X \),

(ii) \( R^t_M(\alpha x) = \alpha R^t_M(x) \), for every \( x \in X \) and \( \alpha \in R \setminus \{0\} \).

Proof. The proof is an immediate consequence of theorem 3.4 and this fact that \( M + y = M \) and \( \alpha M = M \) for all \( y \in M \) and \( \alpha \in R \setminus \{0\} \).

Definition 3.6. For \( x \in X, a \in A, 0 < r < 1, \) and \( t > 0, \) define \( e^r_t(a) = N(x_1, x-a, t) \).

Theorem 3.7. Let \((X, N)\) be a fuzzy anti-2-normed linear space, \( A \) be a subset of \( X \), \( x \in X \setminus \overline{A} \) and \( t > 0 \). Then we have

\[
R^t_A(x) = \bigcap_{a \in A} B[a, e^r_t(a), t] \bigcap A.
\]
Theorem 3.10. For each \( a \in A \) we have \( R^1_A(x) \subseteq \{ B[a, e^t_a(x), t] \} \bigcap A \). Therefore \( R^1_A(x) \subseteq \bigcap_{a \in A} B[a, e^t_a(x), t] \bigcap A \). Conversely, let \( y \in \bigcap_{a \in A} B[a, e^t_a(x), t] \bigcap A \), then we have \( y \in A \), and for each \( a \in A \), \( N(x_1, a - y, t) \leq e^t_a(x) = N(x_1, x - a, t) \), which implies that \( y \in R^1_A(x) \). So \( \bigcap_{a \in A} B[a, e^t_a(x), t] \bigcap A \subseteq R^1_A(x) \), which completes the proof.

Corollary 3.8. Let \((X, N)\) be a fuzzy anti-2-normed linear space, \(A\) be a subset of \(X\), \(x \in X \setminus A\) and \(t > 0\). Then

(i) The set \( R^t_A(x) \) is \( t \)-bounded.

(ii) If \( A \) is \( t \)-closed then \( R^t_A(x) \) is \( t \)-closed.

Theorem 3.9. Let \((X, N)\) be fuzzy anti-2-normed linear space. For each \( x \in X \) and \( t > 0 \), if \( A \) is a convex subset of \( X \), then \( R^t_A(x) \) is a convex subset of \( A \) (for \( R^t_A(x) \neq \emptyset \)).

Proof. Let \( z_1, z_2 \in R^t_A \), then for \( t > 0 \) and each \( x \in X \), \( N(x_1, y - z_1, t) \leq N(x_1, x - y, t) \) and \( N(x_1, y - z_2, t) \leq N(x_1, x - y, t) \) for all \( y \in A \). Now for each \( \lambda \in (0, 1) \) we have

\[
N(x_1, y - (\lambda z_1 + (1 - \lambda)z_2), t) = N(x_1, \lambda y - \lambda z_1 + y - \lambda y - z_2 + \lambda z_2, t) \\
= N(x_1, \lambda(y - z_1) + (1 - \lambda)(y - z_2), \lambda t + (1 - \lambda)t) \\
\leq \max \left\{ N\left(x_1, y - z_1, \frac{\lambda t}{\lambda}\right), N\left(x_1, y - z_2, \frac{(1 - \lambda)t}{1 - \lambda}\right) \right\} \\
\leq \max \left\{ N\left(x_1, x - y, \frac{\lambda t}{\lambda}\right), N\left(x_1, x - y, \frac{(1 - \lambda)t}{1 - \lambda}\right) \right\} \\
\leq N(x_1, x - y, t).
\]

So \( \lambda z_1 + (1 - \lambda)z_2 \in R^t_A(x) \) and \( R^t_A(x) \) is convex.

Theorem 3.10. For \( t > 0 \) and each \( x \in X \). Let \( A \) be a \( t \)-coproximinal subspace of fuzzy anti-2-normed linear space \((X, N)\). Then

(i) If \( \tilde{A}^t_x \) is a \( t \)-compact set then \( A \) is \( t \)-quasi-CoChebyshev.

(ii) If \( \tilde{A}^t_x \) is a \( t \)-closed set then \( R^t_A(x) \) is \( t \)-closed, for every \( x \in X \).

Proof. (i) Suppose \( x \in X \) and \( \{y_n\} \) is a sequence in \( R^t_A(x) \). Since \( x - y_n \in \tilde{A}^t_x \) and \( \tilde{A}^t_x \) is a \( t \)-compact set, there exists a subsequence \( \{x - y_{n_k}\} \) that \( t \)-convergent to \( x - y_0 \in \tilde{A}^t_x \). Consequently, \( \{y_n\} \) has a subsequence \( y_{n_k} \rightarrow y_0 \in R^t_A(x) \) and hence \( A \) is \( t \)-quasi-CoChebyshev.

(ii) The proof is similar to (i).
**Definition 3.11.** A subset $A$ of a fuzzy anti-$2$-normed linear space $(X, N)$ is said to be $t$-boundedly compact if every $t$-bounded sequence in $A$ has a subsequence $t$-converging to an element of $X$.

**Theorem 3.12.** Suppose for some $t > 0$ and each $x \in X$, $A$ is a $t$-boundedly compact and $t$-closed subset of a fuzzy anti-$2$-normed linear space $(X, N)$ then $A$ is $t$-quasi-coChebyshev.

**Proof.** Let $\{y_n\}$ be any sequence in $R^t_A(x)$. Then $N(x_1, y_n - y, t) \leq N(x_1, x - y, t)$ for every $y \in A$. Since $R^t_A(x)$ is $t$-bounded, $\{y_n\}$ is a $t$-bounded sequence in $A$, and so $\{y_n\}$ has a $t$-convergent subsequence $\{y_{n_k}\}$, let $y_{n_k} \rightarrow y_0 \in A$, as $A$ is $t$-closed. Consider $N(x_1, y_0 - y, t) = \lim_{k} N(x_1, y_{n_k} - y, t) \leq N(x_1, x - y, t)$, for every $y \in A$.

So $y_0 \in R^t_A(x)$, which implies that $A$ is $t$-quasi-coChebyshev. \[\blacksquare\]

**Definition 3.13.** Let $(X, N)$ be a fuzzy anti-$2$-normed linear space and $A$ be a subset of $X$. For $t > 0$ and an element $x \in X$ is said to be $t$-orthogonal to an element $y \in X$ and we denote it by $x \perp^t_y$, if $N(x_1, x + \lambda y, t) \geq N(x_1, x, t)$ for all scalar $\lambda \in R, \lambda \neq 0$. We say $A \perp^t_y$ if $x \perp^t_y$ for every $x \in A$.

**Theorem 3.14.** For $t > 0$ and each $x \in X$ and $y_0 \in A$, let $(X, N)$ be a fuzzy anti-$2$-normed linear space and $A$ be a subspace of $X$. If $A \perp^t_x x - y_0$ then $y_0 \in R^t_A(x)$.

**Proof.** Suppose $t > 0, x \in X$ and $A \perp^t_x x - y_0$. Then $N(x_1, a + \lambda(x - y_0), t) \geq N(x_1, a, t)$ for all $a \in A$ and all scalar $\lambda \in R, \lambda \neq 0$. Then $N \left(x_1, x - y_0 + \frac{t}{|\lambda|} a', \frac{t}{|\lambda|} \right) \geq N \left(x_1, \lambda^{-1} a, \frac{t}{|\lambda|} \right)$. Hence $N \left(x_1, x - a', \frac{t}{|\lambda|} \right) \geq N \left(x_1, y_0 - a', \frac{t}{|\lambda|} \right)$, where $a' = y_0 - \lambda^{-1} a$. Now if $\lambda = 1$ then, $N(x_1, y - y_0, t) \geq N(x_1, x - y, t)$ for all $y \in A$ and so $y_0 \in R^t_A(x)$. \[\blacksquare\]

4. **$F$-Best coapproximation**

**Definition 4.1.** Let $A$ be a nonempty subset of a fuzzy anti-$2$-normed linear space $(X, N)$. An element $y_0 \in A$ is said to be an $F$-best coapproximation of $x$ from $A$ if it is a $t$-best coapproximation of $x$ from $A$, for every $t > 0$, i.e., $y_0 \in \bigcap_{t \in (0, \infty)} R^t_A(x)$.

The set of all elements of $F$-best coapproximation of $x$ from $A$ is denoted by $FR^t_A(x)$, i.e., $FR^t_A(x) = \bigcap_{t \in (0, \infty)} R^t_A(x)$.

If each $x \in X$ has at least (respectively exactly) one $F$-best coapproximation in $A$, then $A$ is called $F$-coproximinal (respectively $F$-coChebyshev) set.
Example 4.2. Let $X = R^3$. Define $N : X \times X \times X \times [0, \infty) \rightarrow [0, 1]$ by

\[
N(x_1, x_2, x_3, t) = \frac{\|x_1, x_2, x_3\|}{t} \quad \text{if } t > 0, \quad t \in R, \quad x_1, x_2, x_3 \in X
\]

\[
= 1 \quad \text{if } t \leq 0, \quad t \in R, \quad x_1, x_2, x_3 \in X,
\]

where $\|x_1, x_2, x_3\| = \min_{1 \leq i \leq 3} \sum_{j=1}^{3} |x_{ij}|$. Then $(X, N)$ is a fuzzy anti-3-normed linear space. Let

\[
A = \left\{(a, b, c) \in R^3 : a^2 + b^2 \leq 1, \quad 0 \leq c \leq a^2 + b^2\right\}
\]

and $x_1 = (1, 0, 0), x_2 = (0, 1, 0), x = (0, 0, 4)$ are in $X$. Let $a_0 = (0, -1, 1)$ and $a_1 = (0, 1, 1)$ are in $A$. Hence $a_0 = (0, -1, 1)$ and $a_1 = (0, 1, 1)$ are $F$-best coapproximations of $x = (0, 0, 4)$ from $A$. Then $(0, -1, 1), (0, 1, 1) \in FR^*_A(0, 0, 4)$. So, $A$ is not a $F$-coChebyshev set.

Theorem 4.3. Let $\{\|\cdot, \cdot\|_\alpha^* : \alpha \in (0, 1]\}$ be a descending family of $\alpha$-2-norm on $X$ corresponding to the fuzzy anti-2-norm on $X$. Then $y_0 \in A$ is a best coapproximation to $x \in X$ in the descending family of $\alpha$-2-norm on $X$ corresponding to the fuzzy anti-2-norm on $X$ if and only if $y_0$ is a $F$-best coapproximation to $x$ in the fuzzy anti-2-normed linear space $(X, N)$.

Proof. For each $x \in X$, $y_0$ is a best coapproximation to $x \in X$ in the descending family of $\alpha$-2-norm on $X$ corresponding to the fuzzy anti-2-norm on $X$ if and only if $\|x_1, y - y_0\|_\alpha^* \leq \|x_1, x - y\|_\alpha^*$, for every $y \in A$, if and only if

\[
t \geq \frac{t + \|x_1, y - y_0\|_\alpha^*}{t + \|x_1, x - y\|_\alpha^*}
\]

for every $y \in A$ and $t \in (0, \infty)$, if and only if $N(x_1, y - y_0, t) \leq N(x_1, x - y, t)$ for every $y \in A$ and $t \in (0, \infty)$ if and only if $y_0 \in FR^*_A(x)$. $lacksquare$

Definition 4.4. Let $(X, N)$ be a fuzzy anti-2-normed linear space and $A$ be a subset of $X$. For each element $x \in X$ is said to be $F$-orthogonal to an element $y \in X$ and we denote it by $x \perp^F y$, if for every $t > 0, x \perp^F_t y$. We say $A \perp^F y$ if $x \perp^F_t y$ for every $x \in A$.

Theorem 4.5. Let $\{\|\cdot, \cdot\|_\alpha^* : \alpha \in (0, 1]\}$ be a descending family of $\alpha$-2-norm on $X$ corresponding to the fuzzy anti-2-norm on $X$. Then $x \in X$ is Brikhoff orthogonal to $y \in X$ in the descending family of $\alpha$-2-norm on $X$ corresponding to the fuzzy anti-2-norm on $X$ if and only if $x$ is a $F$-orthogonal to $y$ in the fuzzy anti-2-normed linear space $(X, N)$.

Proof. For each $x \in X$, $x$ is a Brikhoff orthogonal to $y \in X$ in the descending family of $\alpha$-2-norm on $X$ corresponding to the fuzzy anti-2-norm on $X$ if and only if $\|x_1, x\|_\alpha^* \leq \|x_1, x + \lambda y\|_\alpha^*$, for every $\lambda \in R \setminus \{0\}$, if and only if

\[
t \geq \frac{t}{t + \|x_1, x\|_\alpha^*} \geq \frac{t}{t + \|x_1, x + \lambda y\|_\alpha^*}
\]
for every $\lambda \in R\backslash\{0\}$ and $t > 0$, if and only if $N(x_1, x + \lambda y, t) \leq N(x_1, x, t)$ for every $\lambda \in R\backslash\{0\}$ and $t > 0$ if and only if $x \perp_{F} y$.

**Remark 4.6.** The converse of theorem 3.14 is true, if we replace $t$-orthogonality with $F$-orthogonality.

## 5. Conclusion

In this paper, we introduced the concept of $t$-best coapproximation and $F$-best coapproximation in fuzzy anti-2-normed linear spaces and then prove several theorems pertaining to this sets.

## References


