

## **Multi attribute decision making using decision makers attitude in intuitionistic fuzzy context**

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### **Abstract**

Zhiping Chen and Wei Yang, introduced an algorithm based on decision makers risk attitude, to rank alternatives in decision making problems for interval valued intuitionistic fuzzy sets (IVIFSs for short). They applied this method in situations having exactly known and partly known criteria weight. If the weights of the criteria are only partially known, they introduced an optimization model to find these weights. By rectifying the drawbacks in the accuracy function and decision function given by them, we propose an accuracy function and a new multiple criteria decision making function to solve decision making problems.

**AMS subject classification:**

**Keywords:** Intuitionistic Fuzzy Sets(IFSSs), score function, accuracy function.

### **1. Introduction**

Following the introduction of Fuzzy set(FS for short) by L. A. Zadeh in 1965, Krassimir Atanassov introduced the notion of IFS which has been found a better tool to model decision problems[1]. Multicriteria decision making methods based on IFS theoretical tools were introduced in the decision theory in 2007 by Z. S. Xu. This was extended to IVIFS [2]. Later many researchers studied the problem of ranking IFSSs. Lakshmana Gomathi Nayagam, S. Muralikrishnan and Geetha Sivaraman studied this problem [3]

and detected an error in the accuracy function given by Xu. Zhiping Chen and Wei Yang [4] studied multiple criteria decision making problems based on IVIFS, to rank the alternatives. They used the accuracy function which was detected wrong by [3]. In this paper we propose another simple accuracy function to rectify the error. Section 2 contains basic definitions and results. In section 3, contains an accuracy function, its properties and a decision function. An illustration is given in section 4.

## 2. Preliminaries

**Definition 2.1. [1]** Let  $X$  be a given set. An Intuitionistic fuzzy set  $A$  in  $X$  is given by,

$$A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$$

where  $\mu_A, \nu_A : X \rightarrow [0, 1]$ ,  $\mu_A(x)$  is the degree of membership of the element  $x$  in  $A$  and  $\nu_A(x)$  is the degree of non membership of  $x$  in  $A$ , and  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ . For each  $x \in X$ ,  $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$  is the degree of hesitation.

**Definition 2.2. [5]** Let  $D[0, 1]$  be the set of all closed subintervals of the interval  $[0, 1]$ . Let  $X \neq \emptyset$  be a given set. An interval valued intuitionistic fuzzy set  $A$  in  $X$  is given by

$$A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\},$$

where  $\mu_A : X \rightarrow D[0, 1]$ ,  $\nu_A : X \rightarrow D[0, 1]$  with the condition

$$0 \leq \sup_x \mu_A(x) + \sup_x \nu_A(x) \leq 1.$$

The intervals  $\mu_A(x)$  and  $\nu_A(x)$  denote, respectively, the degree of belongingness and the degree of non belongingness of the element  $x$  to the set  $A$ . Thus, for each  $x \in X$ ,  $\mu_A(x)$  and  $\nu_A(x)$  are closed intervals whose lower and upper end points are respectively, denoted by  $\mu_{AL}(x)$ ,  $\mu_{AU}(x)$  and  $\nu_{AL}(x)$ ,  $\nu_{AU}(x)$ .  $A$  can be denoted by,  $A = \{(x, [\mu_{AL}(x), \mu_{AU}(x)], [\nu_{AL}(x), \nu_{AU}(x)]) : x \in X\}$ , where  $0 \leq \mu_{AU}(x) + \nu_{AU}(x) \leq 1$ ,  $\mu_{AL}(x) \geq 0$  and  $\nu_{AL}(x) \geq 0$ . We will denote the set of all the IVIFS in  $X$  by IVIFS( $X$ ).

**Definition 2.3. [5]** Let  $A, B \in \text{IVIFS}(X)$ . A subset relation is defined by

$$A \subseteq B \Leftrightarrow \mu_{AL}(x) \leq \mu_{BL}(x), \mu_{AU}(x) \leq \mu_{BU}(x)$$

and

$$\nu_{AL}(x) \geq \nu_{BL}(x), \nu_{AU}(x) \geq \nu_{BU}(x), \forall x \in X.$$

**Definition 2.4. [5]** Two IVIFSs  $A$  and  $B$  are equal iff  $A \subset B$  and  $B \subset A$  Following the introduction of operations of IFSs by Atanassov, Xu and Yager [2], Xu [7, 9] introduced the notion of aggregation operator of intuitionistic fuzzy numbers (IFNs for short).

**Definition 2.5. [6]** A fuzzy set  $A$  on  $R$ , the set of real numbers, is said to be a fuzzy number if  $A$  satisfies the following properties

- (i) A must be a normal fuzzy subset of R;
- (ii) each  $\alpha$  cut of A must be closed interval for every  $\alpha \in (0, 1]$ ;
- (iii) the support of A,  ${}^{0+}A$ , must be bounded.

**Definition 2.6.** [1] An intuitionistic fuzzy set (IFS)  $A = (\mu_A, \nu_A)$  of R is said to be an intuitionistic fuzzy number if  $\mu_A$  and  $\nu_A$  are fuzzy numbers with  $\mu_A, \nu_A \in [0, 1]$  and  $\mu_A + \nu_A \leq 1$ . Final ranking of the alternatives in MADM problems is determined by the ranking of the corresponding IFNs [7].

**Definition 2.7.** [7,8] Assume  $a_i = (\mu_{a_i}, \nu_{a_i})$  are IFNs, and  $b_i = (\mu_{b_i}, \nu_{b_i})$  are ordered IFNs of  $a_i = (\mu_{a_i}, \nu_{a_i})$  from large to small for  $i = 1, \dots, n$

- (i) If  $\omega = (\omega_1, \dots, \omega_n)$  is the weight vector of  $(a_1, \dots, a_n)$ , then the aggregation operator of intuitionistic fuzzy weighted average is defined by

$$IFWA_\omega(a_1, \dots, a_n) = \omega_1 a_1 + \dots + \omega_n a_n$$

- (ii) If  $\omega = (\omega_1, \dots, \omega_n)$  is the exponential weight vector of  $(a_1, \dots, a_n)$ , then the aggregation operator of intuitionistic fuzzy weighted geometric is defined by

$$IFWG_\omega(a_1, \dots, a_n) = a_1^{\omega_1} \cdots a_n^{\omega_n}$$

In 2007 Xu [7] applied aggregation operators as a better tool to obtain a single IFN for each alternative, and then compared the aggregated IFNs for comparing the alternatives. Later, besides Xu, many researchers studied the problem of ranking of IFNs. Noted among them are Lakshmana Gomathi Nayagam, V., Muralikrishnan S. and Geetha Sivaraman [3].

**Definition 2.8.** [9] Let  $\alpha = ([a, b], [c, d])$  be an IVIFN, then the score function  $S'(\alpha)$  and accuracy function  $H'(\alpha)$  of  $\alpha$  are defined as

$$S'(\alpha) = \frac{1}{2}(a - c + b - d) \quad (1)$$

and

$$H'(\alpha) = \frac{1}{2}(a + b + c + d) \quad (2)$$

greater the value of  $S'(\alpha)$  and  $H'(\alpha)$ , higher will be the rank for  $\alpha$ .

**Definition 2.9.** [4] Let

$$\begin{aligned} \alpha_1 &= ([a_1, b_1], [c_1, d_1]), \\ \alpha_2 &= ([a_2, b_2], [c_2, d_2]), \dots, \\ \alpha_n &= ([a_n, b_n], [c_n, d_n]) \end{aligned}$$

be IVIFNs and  $w = (w_1, w_2, \dots, w_n)$  be the weight vector of these IVIFNs, then the weighted score function and weighted accuracy function are defined by

$$S'_w(\alpha_1, \alpha_2, \dots, \alpha_n) = w_1 S'(\alpha_1) + w_2 S'(\alpha_2) + \dots + w_n S'(\alpha_n) \quad (3)$$

and

$$H'_w(\alpha_1, \alpha_2, \dots, \alpha_n) = w_1 H'(\alpha_1) + w_2 H'(\alpha_2) + \dots + w_n H'(\alpha_n) \quad (4)$$

**Definition 2.10. [4]** For IVIFNs

$$\begin{aligned} \alpha_1 &= ([a_1, b_1], [c_1, d_1]), \\ \alpha_2 &= ([a_2, b_2], [c_2, d_2]), \\ \alpha_n &= ([a_n, b_n], [c_n, d_n]) \end{aligned}$$

with weights  $w = (w_1, w_2, \dots, w_n)$ , and if weighted score function and weighted accuracy function are as given in (3) and (4) then for any  $\lambda \in R$ , the set of real numbers, the decision function  $D(\alpha_1, \alpha_2, \dots, \alpha_n)$  is defined as

$$D(\alpha_1, \alpha_2, \dots, \alpha_n) = S'_w(\alpha_1, \alpha_2, \dots, \alpha_n) + \lambda H'_w(\alpha_1, \alpha_2, \dots, \alpha_n) \quad (5)$$

Here different  $\lambda \in R$  can be used to indicate different risk attitude of the decision maker. If  $\lambda < 0$ , the decision maker is risk averse, and larger the  $|\lambda|$  is, the more risk averse the decision maker is. If  $\lambda = 0$ , the decision maker considers only the weighted score function, and is thus risk neutral. If  $\lambda > 0$ , the decision maker is risk seeking. Larger the value of  $\lambda$  is, the more risk seeking the decision maker is. During the decision making process, the decision makers can choose suitable  $\lambda$  according to their risk attitude.

### 3. New accuracy function

The accuracy function given in definition 2.8 is found to be wrong by [3], so we define a new accuracy function in this section.

**Definition 3.1.** Let  $A = ([a, b], [c, d])$  be an interval valued intuitionistic fuzzy number (IVIFN for short), then the new accuracy function is defined as  $S(A) = \frac{(a + b - cd)}{2}$ .

**Definition 3.2.** Let

$$\begin{aligned} A_1 &= ([a_1, b_1], [c_1, d_1]), A_2 = ([a_2, b_2], [c_2, d_2]), \dots, \\ A_n &= ([a_n, b_n], [c_n, d_n]) \end{aligned}$$

be n IVIFNs with weights  $w = (w_1, w_2, \dots, w_n)$ , then the weighted accuracy function is defined as

$$T_w(A_1, A_2, \dots, A_n) = w_1 S(A_1) + w_2 S(A_2) + \dots + w_n S(A_n).$$

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**Proposition 3.3.** For any interval valued intuitionistic fuzzy set  $A = ([a, b], [c, d])$ , the new proposed score  $S(A) \in [-1, 1]$ .

**Theorem 3.4.** For any two comparable IVIFS A and B, if  $A \subset B$ , then  $S(A) < S(B)$ .

*Proof.* Let  $A = ([a_1, b_1], [c_1, d_1])$  and  $B = ([a_2, b_2], [c_2, d_2])$  be two comparable IVIFS such that  $A \subset B$ , then by definition 3.1.

$$S(B) - S(A) = \frac{(a_2 - a_1)}{2} + \frac{(b_2 - b_1)}{2} + \frac{(c_1 d_1 - c_2 d_2)}{2} > 0$$

■

**Definition 3.5.** Using the new weighted accuracy function given in definition 3.2, and score function given in (3) for alternatives

$$A_1 = ([a_1, b_1], [c_1, d_1]), A_2 = ([a_2, b_2], [c_2, d_2]), \dots, A_n = ([a_n, b_n], [c_n, d_n])$$

we define the decision function as

$$R(A_1, A_2, \dots, A_n) = S'_w(A_1, A_2, \dots, A_n) + \lambda T_w(A_1, A_2, \dots, A_n)$$

That is,

$$R(A_1, A_2, \dots, A_n) = \sum_{j=1}^n w_j S'(A_j) + \lambda \sum_{j=1}^n w_j S(A_j) \quad (6)$$

#### 4. Illustration

A decision maker has to select one among the five candidates say  $A_1, A_2, A_3, A_4, A_5$  for a teaching post, based on three criteria,  $C_1$  (Basic Qualification)  $C_2$  (Teaching Experience)  $C_3$  (Highly Qualified) with weight vector  $w = (.3, .4, .3)$ . The interval valued intuitionistic fuzzy decision matrix is given below.

	$C_1$	$C_2$	$C_3$
$A_1$	([.35, .65], [.2, .3])	([.5, .6], [.3, .4])	([.4, .5], [.1, .3])
$A_2$	([.7, .8], [.1, .2])	([.6, .7], [.1, .3])	([.2, .3], [.4, .5])
$A_3$	([.2, .4], [.4, .5])	([.4, .5], [.1, .2])	([.3, .4], [.1, .2])
$A_4$	([.6, .7], [.2, .3])	([.6, .7], [.2, .3])	([.4, .8], [.1, .2])
$A_5$	([.3, .5], [.2, .4])	([.1, .3], [.6, .7])	([.3, .4], [.5, .6])

Suppose the decision maker is risk neutral, then take  $\lambda = 0$ . Then by the decision function (6), we get

$$\begin{aligned} R(A_1) &= 0.23, R(A_2) = 0.3, R(A_3) = 0.135, R(A_4) = 0.415, \\ R(A_5) &= -0.21. \end{aligned}$$

The rank is

$$R(A_5) < R(A_3) < R(A_1) < R(A_2) < R(A_4).$$

Which means that  $A_5 \prec A_3 \prec A_1 \prec A_2 \prec A_4$ . The best alternative is thus  $A_4$ . If the decision maker is risk seeking, then take  $\lambda = 1$ , then

$$R(A_1) = 0.698, R(A_2) = 0.821, R(A_3) = 0.473, R(A_4) = 1.026, R(A_5) = -0.046.$$

Here also the rank is  $R(A_5) < R(A_3) < R(A_1) < R(A_2) < R(A_4)$ .

Which shows  $A_5 \prec A_3 \prec A_1 \prec A_2 \prec A_4$ . Thus the best alternative is again  $A_4$ . If the decision maker is risk averse, by taking  $\lambda = -1$ , the decision function (6) gives  $R(A_1) = -0.238, R(A_2) = -0.221, R(A_3) = -0.203, R(A_4) = -0.196, R(A_5) = -0.374$ .

Here the rank is  $A_5 \prec A_1 \prec A_2 \prec A_3 \prec A_4$ .

## 5. Conclusion

In this paper we introduced a multiple criteria decision making algorithm given by Zhiping Chen and Wei Yang, to rank the alternatives, and is based on the weighted score function and weighted accuracy function and a decision function which reflect the decision maker's attitude. By rectifying the drawbacks in the decision function given by them, we proposed another accuracy function and a decision function to rank the alternatives.

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