

Notes on Interval Valued Fuzzy Generalized Semipreclosed Sets

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Abstract:

In this paper, we study some of the properties of interval valued fuzzy generalized semipre closed sets. Also we have provided the relation between interval valued fuzzy generalized semipre closed sets with other interval valued fuzzy sets. Note: interval valued is denoted as IV.

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INTRODUCTION:

The concept of a fuzzy subset was introduced and studied by L.A.Zadeh [14] in the year 1965, the subsequent research activities in this area and related areas have found applications in many branches of science and engineering . The following papers have motivated us to work on this paper C.L.Chang [3] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces many researchers like, and many others have contributed to the development of fuzzy topological spaces. Andrijevic [1] has introduced semipreclosed sets and Dontchev [4] has introduced generalized semipreclosed sets in general topology. After that the set was generalized to fuzzy topological spaces by saraf and khanna [12]. Tapas kumar mondal and S.K.Samantha [8] have introduced the topology of interval valued fuzzy

sets. Now we have generalized the set to interval valued fuzzy topological spaces. Some interesting theorems and results on interval valued fuzzy generalized semipreclosed sets are provided in this paper.

1. PRELIMINARIES:

1.1 Definition: [8] Let X be any nonempty set. A mapping $\bar{A}: X \rightarrow D[0, 1]$ is called an interval valued fuzzy subset (briefly, IVFS) of X , where $D[0, 1]$ denotes the family of all closed subintervals of $[0, 1]$ and $\bar{A}(x) = [A^-(x), A^+(x)]$, for all $x \in X$, where A^- and A^+ are fuzzy subsets of X such that $A^-(x) \leq A^+(x)$, for all $x \in X$. Thus $\bar{A}(x)$ is an interval (a closed subset of $[0, 1]$) and not a number from the interval $[0, 1]$ as in the case of fuzzy subset. Note that $\bar{0} = [0, 0]$ and $\bar{1} = [1, 1]$.

1.2 Remark: Let D^X be the set of all interval valued fuzzy subset of X .

1.3 Definition: Let $\bar{A} = \{ \langle x, \bar{\mu}_A(x) \rangle / x \in X \}$, $\bar{B} = \{ \langle x, \bar{\mu}_B(x) \rangle / x \in X \} \in D^X$. We define the following relations and operations:

1. $\bar{A} \subseteq \bar{B}$ if and only if $\bar{\mu}_A(x) \leq \bar{\mu}_B(x)$, for all x in X .
2. $\bar{A} = \bar{B}$ if and only if $\bar{\mu}_A(x) = \bar{\mu}_B(x)$, for all x in X .
3. $(\bar{A})^c = \bar{1} - \bar{A} = \{ \langle x, \bar{1} - \bar{\mu}_A(x) \rangle / x \in X \}$.
4. $\bar{A} \cap \bar{B} = \{ \langle x, \min \{ \bar{\mu}_A(x), \bar{\mu}_B(x) \} \rangle / x \in X \}$.
5. $\bar{A} \cup \bar{B} = \{ \langle x, \max \{ \bar{\mu}_A(x), \bar{\mu}_B(x) \} \rangle / x \in X \}$.

1.4 Definition: [8] Let X be a set and \mathfrak{I} be a family of interval valued fuzzy sets (IVFSs) of X . The family \mathfrak{I} is called an interval valued fuzzy topology (IVFT) on X if and only if \mathfrak{I} satisfies the following axioms

1. $\bar{0}, \bar{1} \in \mathfrak{I}$,
2. If $\{ \bar{A}_i; i \in I \} \subseteq \mathfrak{I}$, then $\bigcup_{i \in I} \bar{A}_i \in \mathfrak{I}$,
3. If $\bar{A}_1, \bar{A}_2, \bar{A}_3, \dots, \bar{A}_n \in \mathfrak{I}$, then $\bigcap_{i=1}^n \bar{A}_i \in \mathfrak{I}$.

The pair (X, \mathfrak{I}) is called an interval valued fuzzy topological space (IVFTS). The members of \mathfrak{I} are called interval valued fuzzy open sets (IVFOS) in X . An interval valued fuzzy set \bar{A} in X is said to be interval valued fuzzy closed set (IVFCS) in X if and only if $(\bar{A})^c$ is an IVFOS in X .

1.5 Definition: Let (X, \mathfrak{I}) be an IVFTS and $\bar{A} = \langle x, \bar{\mu}_A(x) \rangle$ be an IVFS in X . Then the interval valued fuzzy interior and interval valued fuzzy closure are defined by $ivfint(\bar{A}) = \bigcup \{ \bar{G}: \bar{G} \text{ is an IVFOS in } X \text{ and } \bar{G} \subseteq \bar{A} \}$, $ivfcl(\bar{A}) = \bigcap \{ \bar{K}: \bar{K} \text{ is an IVFCS in } X \text{ and } \bar{A} \subseteq \bar{K} \}$. For any IVFS \bar{A} in (X, \mathfrak{I}) , we have $ivfcl(\bar{A}^c) = (ivfint(\bar{A}))^c$ and $ivfint(\bar{A}^c) = (ivfcl(\bar{A}))^c$.

1.6 Definition: An IVFS $\bar{A} = \langle x, \bar{\mu}_A(x) \rangle$ is an IVFT (X, \mathfrak{I}) is said to be an

1. interval valued fuzzy regular closed set (IVFRCS for short) if $\bar{A} = ivfcl(ivfint(\bar{A}))$.
2. interval valued fuzzy semiclosed set (IVFSCS for short) if $ivfint(ivfcl(\bar{A})) \subseteq \bar{A}$.
3. interval valued fuzzy preclosed set (IVFPCS for short) if $ivfcl(ivfint(\bar{A})) \subseteq \bar{A}$.
4. interval valued fuzzy α closed set (IVF α CS for short) if $ivfcl(ivfint(ivfcl(\bar{A}))) \subseteq \bar{A}$.
5. interval valued fuzzy β closed set (IVF β CS for short) if $ivfint(ivfcl(ivfint(\bar{A}))) \subseteq \bar{A}$.

1.7 Definition: An IVFS $\bar{A} = \langle x, \bar{\mu}_A(x) \rangle$ is an IVFT (X, \mathfrak{I}) is said to be an

1. interval valued fuzzy generalized closed set (IVFGCS for short) if $ivfcl(\bar{A}) = \bar{U}$, whenever $\bar{A} \subseteq \bar{U}$ and \bar{U} is an IVFOS.
2. interval valued fuzzy regular generalized closed set (IVFRGCS for short) if $ivfcl(\bar{A}) \subseteq \bar{U}$, whenever $\bar{A} \subseteq \bar{U}$ and \bar{U} is an IVFROS.

1.8 Definition: An IVFS $\bar{A} = \langle x, \bar{\mu}_A(x) \rangle$ is an IVFT (X, \mathfrak{I}) is said to be an

1. interval valued fuzzy semipreclosed set (IVFSPCS for short) if there exists an IVFPCS \bar{B} such that $ivfint(\bar{B}) \subseteq \bar{A} \subseteq \bar{B}$.
2. interval valued fuzzy semipreopen set (IVFSPOS for short) if there exists an IVFPOS \bar{B} such that $\bar{B} \subseteq \bar{A} \subseteq ivfcl(\bar{B})$.

1.9 Definition: Two IVFSs \bar{A} and \bar{B} are said to be not q -coincident ($\bar{A} \not q \bar{B}$ in short) if and only if $\bar{A} \subseteq \bar{B}^c$.

1.10 Definition: An IVFS \bar{A} in (X, \mathfrak{I}) is an IVFQ-set in X . if $ivfint(ivfcl(\bar{A})) = ivfcl(ivfint(\bar{A}))$.

1.11 Definition: Let \bar{A} be an IVFS in an IVFTS (X, \mathfrak{I}) . Then the interval valued fuzzy semipre interior of \bar{A} ($ivfspint(\bar{A})$ for short) and the interval valued fuzzy semipre closure of \bar{A} ($ivfspcl(\bar{A})$ for short) are defined by $ivfspint(\bar{A}) = \cup \{ \bar{G} : \bar{G} \text{ is an IVFSPOS in } X \text{ and } \bar{G} \subseteq \bar{A} \}$, $ivfspcl(\bar{A}) = \cap \{ \bar{K} : \bar{K} \text{ is an IVFSPCS in } X \text{ and } \bar{A} \subseteq \bar{K} \}$. For any IVFS \bar{A} in (X, \mathfrak{I}) , we have $ivfspcl(\bar{A}^c) = (ivfspint(\bar{A}))^c$ and $ivfspint(\bar{A}^c) = (ivfspcl(\bar{A}))^c$.

1.12 Definition: An IVFS \bar{A} in IVFTS (X, \mathfrak{I}) is said to be an interval valued fuzzy generalized semipreclosed set (IVFGSPCS for short) if $ivfspcl(\bar{A}) \subseteq \bar{U}$ whenever $\bar{A} \subseteq \bar{U}$ and \bar{U} is an IVFOS in (X, \mathfrak{I}) .

1.13 Example: Let $X = \{a, b\}$ and $\bar{G} = \{ \langle a, [0.3, 0.8] \rangle, \langle b, [0.1, 0.5] \rangle \}$. Then $\mathfrak{S} = \{ \bar{0}, \bar{G}, \bar{1} \}$ is an IVFT on X and the IVFS $\bar{A} = \{ \langle a, [0.3, 0.7] \rangle, \langle b, [0.1, 0.2] \rangle \}$ is an IVFGSPCS in (X, \mathfrak{S}) .

1.14 Definition: Let $\alpha \in D[0, 1]$. An interval valued fuzzy point (IVFP for short), written as \bar{p}_α is defined to be an IVFS of X is given by $\bar{p}_\alpha(x) = \begin{cases} \alpha & \text{if } x = p \\ [0, 0] & \text{otherwise.} \end{cases}$

2. Some properties of interval valued fuzzy generalized semipreclosed sets:

2.1 Theorem: Every IVFCS in (X, \mathfrak{S}) is an IVFGSPCS in (X, \mathfrak{S}) .

Proof: Let \bar{A} be an IVFCS in (X, \mathfrak{S}) . Assume that $\bar{A} \subseteq \bar{U}$ and \bar{U} is an IVFOS in (X, \mathfrak{S}) .

Then $ivfscpl(\bar{A}) \subseteq ivfcl(\bar{A}) = \bar{A} \subseteq \bar{U}$, by hypothesis. Hence \bar{A} is an IVFGSPCS in (X, \mathfrak{S}) .

2.2 Remark: The converse of above theorem need not be true as from the following example.

Proof: Consider the example, let $X = \{a, b\}$ and $\bar{G} = \{ \langle a, [0.3, 0.8] \rangle, \langle b, [0.1, 0.5] \rangle \}$. Then $\mathfrak{S} = \{ \bar{0}, \bar{G}, \bar{1} \}$ is an IVFT on X . Let $\bar{A} = \{ \langle a, [0.3, 0.7] \rangle, \langle b, [0.1, 0.2] \rangle \}$ be an IVFS in X . Then \bar{A} is an IVFGSPCS but not an IVFCS in X .

2.3 Theorem: Every IVFRCS in (X, \mathfrak{S}) is an IVFGSPCS in (X, \mathfrak{S}) .

Proof: Since every IVFRCS is an IVFCS, the proof is obvious from Theorem 2.1.

2.4 Remark: The converse of above theorem need not be true as from the following example.

Proof: Consider the example, let $X = \{a, b\}$ and $\bar{G} = \{ \langle a, [0.4, 0.8] \rangle, \langle b, [0.3, 0.6] \rangle \}$.

Then $\mathfrak{S} = \{ \bar{0}, \bar{G}, \bar{1} \}$ is an IVFT on X . Let $\bar{A} = \{ \langle a, [0.3, 0.6] \rangle, \langle b, [0.2, 0.4] \rangle \}$ be an IVFS in X . Then \bar{A} is an IVFGSPCS but not an IVFRCS in X .

2.5 Theorem: Every IVFGCS in (X, \mathfrak{S}) is an IVFGSPCS in (X, \mathfrak{S}) .

Proof: Let \bar{A} be an IVFGCS in (X, \mathfrak{S}) . Then assume that $\bar{A} \subseteq \bar{U}$ and \bar{U} is an IVFOS in (X, \mathfrak{S}) .

Since $ivf\text{spcl}(\bar{A}) \subseteq ivfcl(\bar{A})$ and $ivfcl(\bar{A}) \subseteq \bar{U}$, by hypothesis. Hence \bar{A} is an IVFGSPCS in X .

2.6 Remark: The converse of above theorem need not be true as from the following example.

Proof: Consider the example, let $X = \{a, b\}$ and $\bar{G} = \{\langle a, [0.3, 0.8] \rangle, \langle b, [0.1, 0.5] \rangle\}$. Then $\mathfrak{S} = \{\bar{0}, \bar{G}, \bar{1}\}$ is an IVFT on X . Let $\bar{A} = \{\langle a, [0.3, 0.7] \rangle, \langle b, [0.1, 0.2] \rangle\}$ be an IVFS in X . Then \bar{A} is an IVFGSPCS but not an IVFGCS in X .

2.7 Theorem: Every IVFSPCS in (X, \mathfrak{S}) is an IVFGSPCS in (X, \mathfrak{S}) .

Proof: Let \bar{A} be an IVFSPCS in X . Assume that $\bar{A} \subseteq \bar{U}$ and \bar{U} is an IVFOS in (X, \mathfrak{S}) . Then

Since $ivf\text{spcl}(\bar{A}) = \bar{A}$, we have $ivf\text{spcl}(\bar{A}) \subseteq \bar{U}$. Hence \bar{A} is an IVFGSPCS in (X, \mathfrak{S}) .

2.8 Remark: The converse of above theorem need not be true as from the following example.

Proof: Consider the example, let $X = \{a, b\}$ and $\bar{G} = \{\langle a, [0.3, 0.7] \rangle, \langle b, [0.3, 0.8] \rangle\}$. Then $\mathfrak{S} = \{\bar{0}, \bar{G}, \bar{1}\}$ is an IVFT on X . Let $\bar{A} = \{\langle a, [0.3, 0.7] \rangle, \langle b, [0.3, 0.9] \rangle\}$ be an IVFS in X . Then \bar{A} is an IVFGSPCS but not an IVFSPCS in X .

2.9 Theorem: Every IVFaCS in (X, \mathfrak{S}) is an IVFGSPCS in (X, \mathfrak{S}) .

Proof: Since every IVFaCS is an IVFSPCS, the proof is obvious from Theorem 2.7.

2.10 Remark: The converse of above theorem need not be true as from the following example.

Proof: Consider the example, let $X = \{a, b\}$ and $\bar{G} = \{\langle a, [0.5, 0.5] \rangle, \langle b, [0.6, 0.8] \rangle\}$. Then $\mathfrak{S} = \{\bar{0}, \bar{G}, \bar{1}\}$ is an IVFT on X . Let $\bar{A} = \{\langle a, [0.5, 0.5] \rangle, \langle b, [0.6, 0.9] \rangle\}$ be an IVFS in X . Then \bar{A} is an IVFGSPCS but not an IVFaCS in (X, \mathfrak{S}) .

2.11 Theorem: Every IVF β CS in (X, \mathfrak{S}) is an IVFGSPCS in (X, \mathfrak{S}) .

Proof: Let \bar{A} be an IVF β CS in X . Assume that $\bar{A} \subseteq \bar{U}$, \bar{U} is an IVFOS in (X, \mathfrak{S}) .

Then since $ivf\beta cl(\bar{A}) = \bar{A}$, we have $ivf\beta cl(\bar{A}) \subseteq \bar{U}$. Hence \bar{A} is an IVFGSPCS in (X, \mathfrak{S}) .

2.12 Remark: The converse of above theorem need not be true as from the following example.

Proof: Consider the example, let $X = \{a, b\}$ and $\bar{G} = \{ \langle a, [0.3, 0.7] \rangle, \langle b, [0.3, 0.8] \rangle \}$. Then $\mathfrak{I} = \{ \bar{0}, \bar{G}, \bar{1} \}$ is an IVFT on X. Let $\bar{A} = \{ \langle a, [0.3, 0.7] \rangle, \langle b, [0.3, 0.9] \rangle \}$ be an IVFS in X. Then \bar{A} is an IVFGSPCS but not an IVF β CS in X.

2.13 Theorem: Every IVFSCS in (X, \mathfrak{I}) is an IVFGSPCS in (X, \mathfrak{I}) .

Proof: Let \bar{A} be an IVFSCS in (X, \mathfrak{I}) . Since every IVFSCS is an IVFSPCS and by theorem 2.7, we have \bar{A} is an IVFGSPCS in (X, \mathfrak{I}) .

2.14 Remark: The converse of above theorem need not be true as from the following example.

Proof: Consider the example, let $X = \{a, b\}$ and $\bar{G} = \{ \langle a, [0.5, 0.5] \rangle, \langle b, [0.6, 0.8] \rangle \}$. Then $\mathfrak{I} = \{ \bar{0}, \bar{G}, \bar{1} \}$ is an IVFT on X. Let $\bar{A} = \{ \langle a, [0.5, 0.5] \rangle, \langle b, [0.6, 0.9] \rangle \}$ be an IVFS in X. Then \bar{A} is an IVFGSPCS but not an IVFSCS in X.

2.15 Theorem: Every IVFPCS in (X, \mathfrak{I}) is an IVFGSPCS in (X, \mathfrak{I}) .

Proof: Since every IVFPCS is an IVFSPCS, the proof is obvious from Theorem 2.7.

2.16 Remark: The converse of above theorem need not be true as from the following example.

Proof: Consider the example, let $X = \{a, b\}$ and $\bar{G} = \{ \langle a, [0.5, 0.5] \rangle, \langle b, [0.6, 0.8] \rangle \}$. Then $\mathfrak{I} = \{ \bar{0}, \bar{G}, \bar{1} \}$ is an IVFT on X. Let $\bar{A} = \{ \langle a, [0.5, 0.5] \rangle, \langle b, [0.6, 0.9] \rangle \}$ be an IVFS in X. Then \bar{A} is an IVFGSPCS but not an IVFPCS in X.

2.17 Remark: The union of any two IVFGSPCS in (X, \mathfrak{I}) is not an IVFGSPCS in (X, \mathfrak{I}) .

Proof: Consider the example, let $X = \{a, b\}$, and $\bar{G}_1 = \{ \langle a, [0.8, 0.8] \rangle, \langle b, [0.9, 0.9] \rangle \}$ and $\bar{G}_2 = \{ \langle a, [0.5, 0.5] \rangle, \langle b, [0.8, 0.8] \rangle \}$. Then $\mathfrak{I} = \{ \bar{0}, \bar{G}_1, \bar{G}_2, \bar{1} \}$ is an IVFT on X. Let $\bar{A} = \{ \langle a, [0.5, 0.5] \rangle, \langle b, [0.3, 0.3] \rangle \}$ and $\bar{B} = \{ \langle a, [0.3, 0.3] \rangle, \langle b, [0.9, 0.9] \rangle \}$ be two IVFSs in X. Then \bar{A} and \bar{B} are IVFGSPCS but $\bar{A} \cup \bar{B}$ is not an IVFGSPCS in X, since $\bar{A} \cup \bar{B} = \{ \langle a, [0.5, 0.5] \rangle, \langle b, [0.9, 0.9] \rangle \} \subseteq \bar{G}_1$ but $ivfspcl(\bar{A} \cup \bar{B}) = \bar{1} \notin \bar{G}_1$.

2.18 Remark: The intersection of two IVFGSPCS in (X, \mathfrak{I}) is not an IVFGSPCS in (X, \mathfrak{I}) .

Proof: Consider the example, let $X = \{a, b\}$ and $\bar{G} = \{ \langle a, [0.4, 0.4] \rangle, \langle b, [0.7, 0.7] \rangle \}$.

Then $\mathfrak{I} = \{ \bar{0}, \bar{G}, \bar{1} \}$ is an IVFT on X . Let $\bar{A} = \{ \langle a, [0.4, 0.4] \rangle, \langle b, [0.8, 0.8] \rangle \}$ and $\bar{B} = \{ \langle a, [0.9, 0.9] \rangle, \langle b, [0.7, 0.7] \rangle \}$, be IVFS in X . Then \bar{A} and \bar{B} are IVFGSPCS but $\bar{A} \cap \bar{B}$ is not an IVFGSPCS in X , since $\bar{A} \cap \bar{B} = \{ \langle a, [0.4, 0.4] \rangle, \langle b, [0.7, 0.7] \rangle \} \subseteq \bar{G}$ but $ivf\text{spcl}(\bar{A} \cap \bar{B}) = \bar{1} \notin \bar{G}$.

2.19 Theorem: Let (X, \mathfrak{I}) be an IVFTS. Then for every $\bar{A} \in \text{IVFGSPC}(X)$ and for every $\bar{B} \in \text{IVFS}(X)$, $\bar{A} \subseteq \bar{B} \subseteq ivf\text{spcl}(\bar{A})$ implies $\bar{B} \in \text{IVFGSPC}(X)$.

Proof: Let $\bar{B} \subseteq \bar{U}$ and \bar{U} is an IVFOS in (X, \mathfrak{I}) . Then since $\bar{A} \subseteq \bar{B}$, $\bar{A} \subseteq \bar{U}$. By hypothesis, $\bar{B} \subseteq ivf\text{spcl}(\bar{A})$. Therefore $ivf\text{spcl}(\bar{B}) \subseteq ivf\text{spcl}(ivf\text{spcl}(\bar{A})) = ivf\text{spcl}(\bar{A}) \subseteq \bar{U}$, since \bar{A} is an IVFGSPCS in (X, \mathfrak{I}) . Hence $\bar{B} \in \text{IVFGSPC}(X)$.

2.20 Theorem: An IVFS \bar{A} of an IVFTS (X, \mathfrak{I}) is an IVFGSPCS in (X, \mathfrak{I}) if and only if $\bar{A} \text{ q } \bar{F} \Rightarrow ivf\text{spcl}(\bar{A}) \text{ q } \bar{F}$, for every IVFCS \bar{F} of X .

Proof: Necessity: Let \bar{F} be an IVFCS in (X, \mathfrak{I}) and $\bar{A} \text{ q } \bar{F}$. Then by definition 1.9, $\bar{A} \subseteq \bar{F}^c$, where \bar{F}^c is an IVFOS in (X, \mathfrak{I}) . Then $ivf\text{spcl}(\bar{A}) \subseteq \bar{F}^c$, by hypothesis. Hence again by definition 1.9, $ivf\text{spcl}(\bar{A}) \text{ q } \bar{F}$.

Sufficiency: Let \bar{U} be an IVFOS in (X, \mathfrak{I}) such that $\bar{A} \subseteq \bar{U}$. Then \bar{U}^c is an IVFCS in (X, \mathfrak{I}) and $\bar{A} \subseteq (\bar{U}^c)^c$. By hypothesis, $\bar{A} \text{ q } \bar{U}^c \Rightarrow ivf\text{spcl}(\bar{A}) \text{ q } \bar{U}^c$. Hence by definition 1.9, $ivf\text{spcl}(\bar{A}) \subseteq (\bar{U}^c)^c = \bar{U}$. Therefore $ivf\text{spcl}(\bar{A}) \subseteq \bar{U}$. Hence \bar{A} is an IVFGSPCS in (X, \mathfrak{I}) .

2.21 Theorem: Let (X, \mathfrak{I}) be an IVFTS. Then every IVFS in (X, \mathfrak{I}) is an IVFGSPCS in (X, \mathfrak{I}) if and only if $\text{IVFSPO}(X) = \text{IVFSPC}(X)$.

Proof: Necessity: Suppose that every IVFS in (X, \mathfrak{I}) is an IVFGSPCS in (X, \mathfrak{I}) . Let $\bar{U} \in \text{IVFO}(X)$. Then $\bar{U} \in \text{IVFSPO}(X)$ and by hypothesis, $ivf\text{spcl}(\bar{U}) \subseteq \bar{U} \subseteq ivf\text{spcl}(\bar{U})$. This implies $ivf\text{spcl}(\bar{U}) = \bar{U}$. Therefore $\bar{U} \in \text{IVFSPC}(X)$. Hence $\text{IVFSPO}(X) \subseteq \text{IVFSPC}(X)$. Let $\bar{A} \in \text{IVFSPC}(X)$. Then $\bar{A}^c \in \text{IVFSPO}(X) \subseteq \text{IVFSPC}(X)$. That is $\bar{A}^c \in \text{IVFSPC}(X)$. Therefore $\bar{A} \in \text{IVFSPO}(X)$. Hence $\text{IVFSPC}(X) \subseteq \text{IVFSPO}(X)$. Thus $\text{IVFSPO}(X) = \text{IVFSPC}(X)$.

Sufficiency: Suppose that $\text{IVFSPO}(X) = \text{IVFSPC}(X)$. Let $\bar{A} \subseteq \bar{U}$ and \bar{U} be an IVFOS in (X, \mathfrak{I}) . Then $\bar{U} \in \text{IVFSPO}(X)$ and $ivf\text{spcl}(\bar{A}) \subseteq ivf\text{spcl}(\bar{U}) = \bar{U}$, since $\bar{U} \in \text{IVFSPC}(X)$, by hypothesis. Therefore \bar{A} is an IVFGSPCS in X .

2.22 Theorem: If \bar{A} is an IVFOS and an IVFGSPCS in (X, \mathfrak{S}) , then \bar{A} is an IVFSPCS in (X, \mathfrak{S}) .

Proof: Since $\bar{A} \subseteq \bar{A}$ and \bar{A} is an IVFOS in (X, \mathfrak{S}) , by hypothesis, $ivf_{spcl}(\bar{A}) \subseteq \bar{A}$. But $\bar{A} \subseteq ivf_{spcl}(\bar{A})$. Therefore $ivf_{spcl}(\bar{A}) = \bar{A}$. Hence \bar{A} is an IVFSPCS in (X, \mathfrak{S}) .

2.23 Theorem: Let \bar{A} be an IVFGSPCS in (X, \mathfrak{S}) and \bar{p}_α be an IVFP in X such that $\bar{p}_\alpha q ivf_{spcl}(\bar{A})$. Then $vfcl(\bar{p}_\alpha) q \bar{A}$.

Proof: Let \bar{A} be an IVFGSPCS in (X, \mathfrak{S}) and let $\bar{p}_\alpha q ivf_{spcl}(\bar{A})$. If $ivfcl(\bar{p}_\alpha) q \bar{A}$, then by definition 1.9, $\bar{A} \subseteq (ivfcl(\bar{p}_\alpha))^c$, where $(ivfcl(\bar{p}_\alpha))^c$ is an IVFOS in (X, \mathfrak{S}) . Then by hypothesis, $ivf_{spcl}(\bar{A}) \subseteq (ivfcl(\bar{p}_\alpha))^c \subseteq (\bar{p}_\alpha)^c$. Therefore by definition 1.9, $\bar{p}_\alpha q ivf_{spcl}(\bar{A})$, which is a contradiction to the hypothesis. Hence $ivfcl(\bar{p}_\alpha) q \bar{A}$.

2.24 Theorem: For an IVFS \bar{A} in (X, \mathfrak{S}) , the following conditions are equivalent:
 \bar{A} is an IVFOS and an IVFGSPCS in (X, \mathfrak{S}) ,
 \bar{A} is an IVFROS in (X, \mathfrak{S}) .

Proof: (i) \Rightarrow (ii) Let \bar{A} be an IVFOS and an IVFGSPCS in (X, \mathfrak{S}) . Then $ivf_{spcl}(\bar{A}) \subseteq \bar{A}$. Since $ivf_{spcl}(\bar{A})$ is an IVFSPCS, by definition 1.8, there exists an IVFPCS \bar{B} such that $ivfint(\bar{B}) \subseteq ivf_{spcl}(\bar{A}) \subseteq \bar{B}$ and $ivfcl(ivfint(\bar{B})) \subseteq \bar{B}$. Now $ivfint(ivfcl(ivfint(ivf_{spcl}(\bar{A})))) \subseteq ivfint(ivfcl(ivfint(\bar{B}))) \subseteq ivfint(\bar{B}) \subseteq ivf_{spcl}(\bar{A})$.

Now $ivfint(ivfcl(ivfint(\bar{A}))) \subseteq ivfint(ivfcl(ivfint(ivf_{spcl}(\bar{A})))) \subseteq ivf_{spcl}(\bar{A})$.

Therefore $\bar{A} \cup ivfint(ivfcl(ivfint(\bar{A}))) \subseteq ivf_{spcl}(\bar{A}) \subseteq \bar{A}$. This implies that $ivfint(ivfcl(ivfint(\bar{A}))) \subseteq \bar{A}$. Since \bar{A} is an IVFOS, $ivfint(\bar{A}) = \bar{A}$. Therefore $ivfint(ivfcl(\bar{A})) \subseteq \bar{A}$. Since \bar{A} is an IVFOS, it is an IVFPOS. Hence $\bar{A} \subseteq ivfint(ivfcl(\bar{A}))$. Therefore $\bar{A} = ivfint(ivfcl(\bar{A}))$. Hence \bar{A} is an IVFROS in (X, \mathfrak{S}) .

(ii) \Rightarrow (i) Let \bar{A} be an IVFROS in (X, \mathfrak{S}) . Therefore $\bar{A} = ivfint(ivfcl(\bar{A}))$. Since every IVFROS is an IVFOS, \bar{A} is an IVFOS and $\bar{A} \subseteq \bar{A}$. This implies that $ivfint(ivfcl(\bar{A})) \subseteq \bar{A}$. That is $ivfint(ivfcl(ivfint(\bar{A}))) = ivfint(ivfcl(\bar{A})) \subseteq \bar{A}$. Thus \bar{A} is an IVF β CS. Hence by theorem 2.11, \bar{A} is an IVFGSPCS in (X, \mathfrak{S}) .

2.25 Theorem: For an IVFOS \bar{A} in (X, \mathfrak{S}) , the following conditions are equivalent:
 \bar{A} is an IVFCS in (X, \mathfrak{S}) ,
 \bar{A} is an IVFGSPCS and an IVFQ- set in (X, \mathfrak{S}) .

Proof: (i) \Rightarrow (ii) Since \bar{A} is an IVFCS, it is an IVFGSPCS in (X, \mathfrak{S}) . Now $ivfint(ivfcl(\bar{A})) = ivfint(\bar{A}) = \bar{A} = ivfcl(\bar{A}) = ivfcl(ivfint(\bar{A}))$, by hypothesis. Hence \bar{A} is an IVFQ- set in (X, \mathfrak{S}) .

(ii) \Rightarrow (i) Since \bar{A} is an IVFOS and an IVFGSPCS in (X, \mathfrak{S}) . by theorem 2.24, \bar{A} is an IVFROS in (X, \mathfrak{S}) . Therefore $\bar{A} = ivfint(ivfcl(\bar{A})) = ivfcl(ivfint(\bar{A})) = ivfcl(\bar{A})$, by hypothesis. Hence \bar{A} is an IVFCS in (X, \mathfrak{S}) .

2.26 Theorem: Let (X, \mathfrak{S}) be an IVFTS. Then for every $\bar{A} \in IVFSPC(X)$ and for every IVFS \bar{B} in X , $ivfint(\bar{A}) \subseteq \bar{B} \subseteq \bar{A}$ implies $\bar{B} \in IVFGSPC(X)$.

Proof: Let \bar{A} be an IVFSPCS in X . Then by definition 1.8, there exist an IVFPCS, say \bar{C} such that $ivfint(\bar{C}) \subseteq \bar{A} \subseteq \bar{C}$. By hypothesis, $\bar{B} \subseteq \bar{A}$. Therefore $\bar{B} \subseteq \bar{C}$. Since $ivfint(\bar{C}) \subseteq \bar{A}$, $ivfint(\bar{C}) \subseteq ivfint(\bar{A})$, and $ivfint(\bar{C}) \subseteq \bar{B}$. Thus $ivfint(\bar{C}) \subseteq \bar{B} \subseteq \bar{C}$ and by definition 1.8, $\bar{B} \in IVFSPC(X)$. Hence by Theorem 2.7, $\bar{B} \in IVFGSPC(X)$.

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