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Some Aspects of Fuzzy Completely γ-Irresolute Functions

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Abstract

In this paper, we introduce a new class of functions called fuzzy completely γ-irresolute functions between fuzzy topological spaces. We obtain several characterizations of this class and study its properties and investigate the relationship with known functions.

Keywords- fuzzy γ- open set, fuzzy completely γ- irresolute, fuzzy completely weakly γ- irresolute, Fuzzy γ- continuous, fuzzy γ-connected.

AMS Subject Classification 2000: 54A40.

1.Introduction

The concept fuzzy has invaded almost all branches of mathematics with the introduction of fuzzy sets by Zadeh [13] of 1965. The theory of fuzzy topological spaces was introduced and developed by Chang [6] and since then various notions in classical topology have been extended to fuzzy topological spaces. In 1989, M.N. Mukherjee and S.P. Sinha[9]introduced and studied the concept of fuzzy irresolute functions. In 2002 T. Noiri and O. R. Sayed[10], introduced and studied the concept of Fuzzy γ -open sets.In this paper we define fuzzy completely γ-irresolute function and give several characterizations and their properties. We also study these functions comparing with other types of already existing functions.

2. Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, γ) (or simply X, Y and Z) represent non-empty fuzzy topological spaces on which no separation axioms are assumed, unless otherwise mentioned.

***Definition*2.1**
Let (X, τ) be fuzzy topological space and λ be any fuzzy set in X.
λis called a fuzzy α -open set [11] if $\lambda \le int\ cl\ int\lambda$
λis called a fuzzy semi-open set [1] if $\lambda \le cl\ int\lambda$
λis called a fuzzy pre-open set [3] if $\lambda \le int\ cl\,\lambda$
λis called a fuzzy β-open set [2] if $\lambda \le cl\ int\ cl\lambda$
λis called a fuzzy γ –open[10] (or) b- open set [4] if $\lambda \le cl\ int\lambda \cup\ int\,cl\lambda$.

The complement of a fuzzy α -open (fuzzy semi-open, fuzzy β-open, fuzzy γ -open set, respectively) set iscalled a fuzzy α -closed (fuzzy semi-closed, fuzzy β -closed, fuzzy γ -open respectively).

Definition: 2.2 A function $f: X \to Y$ is called fuzzy completely continuous [5] if $f^{-1}(\lambda)$ is fuzzy regular open in X for every fuzzy open set V of Y.

Definition: 2.3A function $f: X \to Y$ is called fuzzy γ-irresolute[4] (resp. fuzzy γ -continuous) $f^{-1}(\lambda)$ is fuzzy γ open in X for every fuzzy γ open (resp. fuzzy open) set V of Y.

Definition: 2.4 A space (X,τ) is called fuzzy nearly compact[8] (resp.fuzzy γ -compact) if every fuzzy regular open (respectively fuzzy γ -open) cover of X has a finite subcover.

Definition: 2.5 A space X is called fuzzy almost normal[12] if for each fuzzy closed set A and each fuzzy regular closed set B such that $A \cap B = \phi$, there exists disjoint fuzzy open sets U and V such that $A \le U$ and $B \le V$.

3. Fuzzy Completely γ - Irresolute Function

Definition3.1Let(X,τ) and (Y,σ) be a fuzzy topological spaces. A function $f:(X,\tau)\to(Y,\sigma)$ is said to be a fuzzy completely γ -irresolute function if $\mathrm{f}^{-1}(\lambda)$ is fuzzy regular open in X for every fuzzy γ -open set λ of Y.

Remark 3.1 Every fuzzy strongly continuous function is fuzzy completely γ-irresolute, but the converse is not true.

***Example* 3.1** Let X = {a, b, c}. Define fuzzy sets $\mu_1, \mu_2, \mu_3, \mu_4 : X \to [0,1]$such that

$\mu_1(a) = 0.6$ $\mu_2(a) = 0.3$ $\mu_3(a) = 0.6$ $\mu_4(a) = 0.3$
$\mu_1(b) = 0.5$ $\mu_2(b) = 0.4$ $\mu_3(b) = 0.5$ $\mu_4(b) = 0.4$

$\mu_1(c) = 0.4$ $\mu_2(c) = 0.5$ $\mu_3(c) = 0.5$ $\mu_4(c) = 0.4$

Let $\tau = \{0, \mu_1, \mu_2, \mu_3, \mu_4, 1\}$ and $\sigma = \{0, \mu_2, \mu_3, 1\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then *f* is fuzzy completely γ- irresolute but not fuzzy strongly continuous.

Remark 3.2 Every completely γ- irresolute function is fuzzy γ-irresolute. But the converse is not true.

Example 3.2 Let X = {a, b, c}. Define fuzzy sets $\mu_1, \mu_2, \mu_3, \mu_4 : X \to [0,1]$ as

$\mu_1(a) = 0.6$ $\mu_2(a) = 0.3$ $\mu_3(a) = 0.6$ $\mu_4(a) = 0.3$
$\mu_1(b) = 0.5$ $\mu_2(b) = 0.4$ $\mu_3(b) = 0.5$ $\mu_4(b) = 0.4$
$\mu_1(c) = 0.4$ $\mu_2(c) = 0.5$ $\mu_3(c) = 0.5$ $\mu_4(c) = 0.4.$

Let $\tau = \{0, \mu_1, \mu_2, \mu_3, \mu_4, 1\}$ and $\sigma = \{0, \mu_1, \mu_4, 1\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then *f* is fuzzy γ - irresolute but not fuzzy completely γ irresolute.

Theorem3.1 If $f : (X, T) \to (Y, S)$ is a fuzzycompletely γ - irresolute function A is any fuzzy open subset of X, then the restriction $f|_A : A \to Y$ is fuzzy completely γ-irresolute.

Proof: Let λ be a fuzzy γ -open subset of Y. By hypothesis, $f^{-1}(\lambda)$ t is fuzzy regular open in X hen. Since A is fuzzy open in X, by previous lemma $(f|_A)^{-1}(\lambda) = f^{-1}(\lambda) \cap A$ is fuzzy regular open in A. Therefore, $f|_A$ is fuzzy completely γ - irresolute.

Theorem3.2 The following hold for functions $f : X \to Y$ and $g : Y \to Z$:

a. If $f : X \to Y$ is fuzzy completely γ- irresolute and $g : Y \to Z$ is fuzzy γ - irresolute, then $g \circ f : X \to Z$ is fuzzy completely γ -irresolute.
b. If function $f : X \to Y$ is fuzzy completely continuous and $g : Y \to Z$ is fuzzy completely γ -irresolute, then $g \circ f : X \to Z$ is fuzzy completely γ -irresolute.
c. If $f : X \to Y$ is fuzzy completely γ -irresolute and $g : Y \to Z$ is fuzzy γ - continuous, then $g \circ f : X \to Z$ is fuzzy completely continuous.

Proof: Obvious.

Definition 3.2 A space X is said to be fuzzy γ –connected, if X cannot be expressed as the union of two nonempty fuzzy γ -open sets.

Theorem3.3 If a mapping $f : X \to Y$ is fuzzy completely γ -irresolute surjection and X is fuzzy almost connected then Y is fuzzy γ -connected.

Proof: Assume that X is fuzzy connected and Y is not fuzzy γ - connected. Then Y can be written as $Y = U \bigcup V$ such that U and V are disjoint nonempty fuzzy γ open sets. Since *f* is fuzzy completely γ –irresolute, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint fuzzy regular open sets and $X = f^{-1}(U) \bigcup f^{-1}(V)$. This shows that X is not fuzzy connected. This is a contradiction.

Definition: 3.3 A space X is called fuzzy almost regular [7](resp. fuzzy strongly γ - regular) if for any fuzzy regular closed (resp. fuzzy γ- closed) set $F \le X$ and any point $x \in X - F$, there exists disjoint fuzzy open (resp. fuzzy γ- open) sets U and V such that $x \in U$ and $F \le V$.

Definition: 3.4 A function $f : X \to Y$ is called fuzzy pre-γ-closed if the image of every fuzzy γ-closed subset of X is fuzzy γ-closed set in Y.

Theorem: 3.4 If a mapping $f : X \to Y$ is fuzzy pre-γ-closed, then for each subset B of Y and a fuzzy γ-open set U of X containing $f^{-1}(B)$, there exists a fuzzy γ-open set V in Y containing B such that $f^{-1}(V) \le U$.

Proof: Obvious.

Theorem: 3.5 If f is fuzzy completely γ-irresolute γ-open from an almost regular space X onto a space Y, then Y is fuzzy strongly γ-regular.

Proof: Let *f*be fuzzy γ- closed set in Y with $y \notin F$ such that *y*=*f*(*x*). Since *f* is fuzzy completely γ-irresolute function, $f^{-1}(F)$ is fuzzy regular closed and so fuzzy closed set in X and hence $x \notin f^{-1}(F)$. By almost regularity of X there exists disjoint fuzzy open sets U and V such that $x \in U$ and $f^{-1}(F) \le V$. We obtain that $y = f(x) \in f(U)$ and $F \le f(V)$ such that *f*(U) and *f*(V) are disjoint fuzzy b-open sets. Thus Y is fuzzy strongly γ- regular.

Definition: 3.5 A space X is called fuzzy strongly γ normal if for every pair of disjoint fuzzy γ closed subsets F_1 and F_2 of X there exists disjoint fuzzy γ open sets U and V such that $F_1 \le U$ and $F_2 \le V$.

Theorem: 3.6 If f is fuzzy completely γ-irresolute injective function from an fuzzy almost normal spaces X onto a space Y then Y is fuzzy strongly γ -normal.

Proof: Let F_1 and F_2 be disjoint fuzzy γ-closed sets in Y. Since f is fuzzy completely γ-irresolute function $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are disjoint fuzzy regular closed and so fuzzy closed set in X. By fuzzy almost normality of X, there exists disjoint fuzzy open sets U and V such that $f^{-1}(F_1) \le U$ and $f^{-1}(F_2) \le V$. We obtain that $F_1 \le f(U)$ and

$F_2 \leq f(V)$ such that *f*(U) and *f*(V) are disjoint fuzzy γ-open. Thus Y is fuzzy strongly γ-normal.

Definition: 3.6 A fuzzy topological space (X,τ) is said to be fuzzy γ- T_1 (resp.fuzzy r-T_1) if for each pair of distinct points x and y of X, there exists fuzzy γ- open (resp. fuzzy regular open) sets U_1 and U_2 such that $x \in U_1$ and $y \in U_2$, $x \notin U_2$ and $y \notin U_1$.

Theorem: 3.7 If $f:(X,\tau) \to (Y,\sigma)$ is fuzzy completely γ- irresolute injective function and Y is fuzzy γ- T_1 then X is fuzzy r-T_1.

Proof: Suppose that Y is fuzzy γ- T_1. For any two distinct points x and y of X, there exists fuzzy γ- open sets F_1 and F_2 in Y such that $f(x) \in F_1$, $f(y) \in F_2$, $f(x) \notin F_2$ and $f(y) \notin F_1$. Since *f* injective fuzzy completely γ- irresolute function, we have X is fuzzy r-T_1.

Definition: 3.7 A fuzzy topological space (X,τ) is said to be fuzzy γ- T_2 (resp. fuzzy r-T_2) if for each pair of distinct points x and y of X, there exists disjoint fuzzy γ- open (resp. fuzzy regular open) sets A and B such that $x \in A$ and $y \in B$.

Theorem: 3.8 If $f:(X,\tau) \to (Y,\sigma)$ is fuzzy completely γ- irresolute injective function and Y is fuzzy γ- T_2 then X is fuzzy r-T_2.

Proof: Suppose that Y is fuzzy γ- T_2. For any two distinct points x and y of X, there exists fuzzy γ- open sets F_1 and F_2 in Y such that $f(x) \in F_1$, $f(y) \in F_2$, $f(x) \notin F_2$ and $f(y) \notin F_1$. Since f injective fuzzy completely γ- irresolute function, we have X is fuzzy r-T_1.

Theorem: *3.9* Let Y be fuzzy $\gamma - T_2$ space.

i. If $f, g: X \to Y$ are fuzzy completely γ- irresolute functions, then the set $A = \{x \in X : f(x) = g(x)\}$ is fuzzy δ -closed in X.
ii. If $f, g: X \to Y$ are fuzzy completely γ-irresolute functions, then the subset $E = \{x, y : f(x) = f(y)\}$ is fuzzy δ -closed in $X \times X$.

Proof: We prove (i) only. Let $x \notin A$ then $f(x) \neq g(x)$. Since Y is $\gamma - T_2$ space, there exists fuzzy β-open sets V_1 and V_2 in Y such that $f(x_1) \in \lambda_1$ and $f(x_2) \in \lambda_2$ and $\lambda_1 \cap \lambda_2 = \phi$. Since f and g are fuzzy completely γ- irresolute, $f^{-1}(\lambda_1)$ and $g^{-1}(\lambda_2)$ are fuzzy regular open sets. Put $U = f^{-1}(\lambda_1) \cap g^{-1}(\lambda_2)$. Then U is fuzzy regular open set containing x and $U \cap A = \phi$. Hence we have $x \notin \delta - Cl(A)$.

4.Fuzzy Completely Weakly γ- irresolute Functions

Definition: 4.1 A function $f:(X,\tau)\to(Y,\sigma)$ is said to be fuzzy completely weakly γ- irresolute if for each fuzzy point $x_\alpha \in X$ and for any γ- open set λ containing $f(x)$, there exists an fuzzy open set μ containing *x* such that $f(\mu)\le\lambda$.

Remark: 4.1 It is obvious that every fuzzy completely γ- irresolute function is fuzzy completely weakly γ-irresolute and every fuzzy completely weakly γ-irresolute function is fuzzy γ- irresolute. However, the converses may not true in general as shown in following example.

Example: 4.1 Let X = {a, b, c} and define fuzzy sets $\lambda_1,\lambda_2,\lambda_3 : X\to[0,1]$ as

$\lambda_1(a)=0.2$ $\lambda_2(a)=0.1$ $\lambda_3(a)=0$

$\lambda_1(b)=0.2$ $\lambda_2(b)=0.4$ $\lambda_3(b)=0.6$

$\lambda_1(c)=0$ $\lambda_2(c)=0.6$ $\lambda_3(c)=0.8$

Let $\tau=\{0,\lambda_1,\lambda_2,\lambda_1\vee\lambda_2,\lambda_1\wedge\lambda_2,1\}$ and $\sigma=\{0,\lambda_2,1\}$. Let $f:(X,\tau)\to(X,\sigma)$ be the identity function. Then f is fuzzy completely weakly γ - irresolute but not fuzzy completely γ irresolute.

Let $\tau_1=\{0,\lambda_2,1\}$ and $\tau_2=\{0,\lambda_3,1\}$. If $f:(X,\tau_1)\to(X,\tau_2)$ is the identity function then *f* is fuzzy γ- irresolute but not fuzzy completely weakly γ- irresolute.

Theorem: 4.1 A function $f:X\to Y$ is fuzzy completely weakly γ-irresolute if the graph function $g:X\to X\times Y$, defined by $g(x)=(x,f(x))$ for each $x\in X$ is fuzzy completely weakly γ-irresolute .

Proof: Let V be any fuzzy γ-open set of Y. Then $1\times V$ is a fuzzy γ-open set of $X\times Y$. Since g is fuzzy completely γ-irresolute, $f^{-1}(V)=g^{-1}(1\times V)$ is fuzzy regular open in X. Thus *f* is fuzzy completely weakly γ-irresolute.

Theorem: 4.2The following hold for functions $f:X\to Y$ and $g:Y\to Z$:

a. If $f:X\to Y$ is fuzzy completely weakly γ- irresolute and $g:Y\to Z$ is fuzzy completelyγ-irresolute, then $g\circ f:X\to Z$ is fuzzy completely weakly γ-irresolute.
b. If function $f:X\to Y$ is fuzzy completely continuous and $g:Y\to Z$ is fuzzy completely weakly γ-irresolute, then $g\circ f:X\to Z$ is fuzzy completely γ-irresolute.
c. If $f:X\to Y$ is fuzzy completely γ-irresolute and $g:Y\to Z$ is fuzzy completely weakly γ -irresolute, then $g\circ f:X\to Z$ is fuzzy completely γ-irresolute.
d. If $f:X\to Y$ is fuzzy γ-continuous and $g:Y\to Z$ is fuzzy completely weakly

γ -irresolute, then $g \circ f : X \to Z$ is fuzzy γ- irresolute.

Proof: Obvious.

Theorem: 4.3 If $f:(X,\tau)\to(Y,\sigma)$ is fuzzy completely weakly γ- irresolute injective function and Y is fuzzy γ- T_2 then X is fuzzy Hausdorff.

Proof: Let x, y be any two distinct points of X. Since f is injective, we have $f(x)\neq f(y)$. Since Y is fuzzy $\gamma - T_2$, there exists V and W are γ open sets in Y such that V $\wedge$ W = 0. Since *f* is fuzzy completely weakly γ- irresolute, there exists fuzzy open sets G and H in X such that $f(G)\leq V$ and $f(H)\leq W$. Hence we obtain G $\wedge$ H = 0. This shows that X is fuzzy Hausdorff.

Theorem: 4.4 If a function $f : X \to Y$ is a fuzzy completely weakly γ- irresolute surjection and X is fuzzy connected, then Y is fuzzy γ- connected.

Proof: Suppose that Y is not fuzzy γ-connected. There exists non empty fuzzy γ- open sets V and W of Y such that Y = V $\vee$ W. Since *f* is fuzzy completely weakly γ - irresolute $f^{-1}(V)$ and $f^{-1}(W)$ are fuzzy open sets and $X = f^{-1}(V)\vee f^{-1}(W)$. This shows that X is not fuzzy connected. This is a contradiction.

5. Conclusion

We have defined and proved basic properties of Fuzzy Completely γ-IrresoluteFunctions and Fuzzy CompletelyWeakly γ-Irresolute Function. Many resultshave been established to show how far topological structures are preservedby these γ-Irresolute Functions. We also have provided examples where suchproperties fail to be preserved.

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