Fuzzy $I_g\gamma$–Irresolute Mappings

Anita Singh Banafar and S. S. Thakur

Department of Applied Mathematics, Jabalpur Engineering College
Jabalpur (M. P.) - 482011 INDIA
Email:anita.banafar1@gmail.com

Abstract

In this paper we introduce the concepts of fuzzy $I_g\gamma$-irresolute mappings in fuzzy ideal topological spaces and obtain some of its basic properties and characterizations.

Introduction

In 1945 R. Vaidyanathaswamy [19] introduced the concept of ideal topological spaces. Hayashi [5] defined the local function and studied some topological properties using local function in ideal topological spaces in 1964. Since then many mathematicians studied various topological concepts in ideal topological spaces. After the introduction of fuzzy sets by Zadeh [23] in 1965 and fuzzy topology by Chang [1] in 1968, several researches were conducted on the generalization of the notions of fuzzy sets and fuzzy topology. The hybridization of fuzzy topology and fuzzy ideal theory was initiated by Mahmoud [8] and Sarkar [13] independently in 1997. They [8, 13] introduced the concept of fuzzy ideal topological spaces as an extension of fuzzy topological spaces and ideal topological spaces. Recently the concepts of fuzzy semi-I-open sets [4], fuzzy $\alpha$-I-open sets [21], fuzzy $\gamma$-I-open sets [3], fuzzy pre-I-open sets [10], fuzzy $\delta$-I-open sets [22] and fuzzy $I_g\gamma$-closed sets [14], fuzzy $I_g\gamma$-continuous mappings [15] have been introduced and studied in fuzzy ideal topological spaces. In the present paper we introduce and study the concepts of fuzzy $I_g\gamma$-irresolute mappings in fuzzy ideal topological spaces.

Preliminaries

Let $X$ be a nonempty set. A family $\tau$ of fuzzy sets of $X$ is called a fuzzy topology [1]
on X if the null fuzzy set 0 and the whole fuzzy set 1 belongs to \( \tau \) and \( \tau \) is closed with respect to any union and finite intersection. If \( \tau \) is a fuzzy topology on X, then the pair \((X, \tau)\) is called a fuzzy topological space. The members of \( \tau \) are called fuzzy open sets of X and their complements are called fuzzy closed sets. The closure of a fuzzy set \( A \) of X denoted by \( Cl(A) \), is the intersection of all fuzzy closed sets which contains \( A \). The interior [1] of a fuzzy set \( A \) of X denoted by \( Int(A) \) is the union of all fuzzy open subsets contained in \( A \). A fuzzy set \( V \) in \((X, \tau)\) is called a Q-neighbourhood of a fuzzy point \( x_\beta \) if there exists a fuzzy open set \( U \) of X such that \( x_\beta \in U \leq V \) [3]. A nonempty collection of fuzzy sets \( I \) of a set X satisfying the conditions (i) if \( A \in I \) and \( B \leq A \), then \( B \in I \) (heredity), (ii) if \( A \in I \) and \( B \in I \) then \( A \cup B \in I \) (finite additivity) is called a fuzzy ideal on X. The triplex \((X, \tau, I)\) denotes a fuzzy ideal topological space [8, 13].

**Definition 2.1:** A fuzzy set \( A \) of a fuzzy ideal topological space \((X, \tau)\) is called:
- Fuzzy generalized closed (fuzzy g-closed) set if \( Cl(A) \leq U \) and \( U \) is fuzzy open [17].
- Fuzzy generalized open (fuzzy g-open) set if and only if \( 1-A \) is fuzzy g-closed [17].

**Remark 2.1:** [14] Every fuzzy g-closed (resp. fuzzy \(-\)-closed) set is fuzzy Ig -closed.

**Definition 2.2:** A mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) is said to be:
- Fuzzy g-continuous mapping if the inverse image of every fuzzy closed set of Y is fuzzy g-closed in X [18].
- Fuzzy gc-irresolute if the inverse image of every fuzzy g-closed set of Y is fuzzy g-closed in X [16].

**Definition 2.3:** A fuzzy set \( B \) of a fuzzy topological space \((X, \tau)\) is said to be fuzzy GO-compact relative to X, if for every collection \( \{A_i : i \in \Lambda\} \) of fuzzy g-open subsets of X such that \( B \leq \bigcup \{A_i : i \in \Lambda\} \) there exists a finite subset \( \Lambda_0 \) and \( \Lambda \) such that \( B \leq \bigcup \{A_i : i \in \Lambda_0\} \) [18].

**Definition 2.4:** A fuzzy topological space \((X, \tau)\) is said to be fuzzy GO-connected if
there is no proper fuzzy set of X which is both fuzzy g-open and fuzzy g-closed [18].

**Definition 2.5:** A mapping \( f: (X, \tau, I) \rightarrow (Y, \sigma) \) is said to be:
- Fuzzy Ig-continuous if the inverse image of every fuzzy closed set of Y is fuzzy Ig-closed in X [15].
- Fuzzy IGO-connected if there is no proper fuzzy set of X which is both fuzzy Ig-open and fuzzy Ig-closed [15].

**Remark 2.2:** Every fuzzy continuous mapping is fuzzy Ig-continuous, but the converse may not be true.

**Definition 2.6:** A collection \( \{A_i : i \in \Lambda\} \) of fuzzy Ig-open sets in a fuzzy ideal topological space \( (X, \tau, I) \) is called a fuzzy Ig-open cover of a fuzzy set B of X if \( B \leq \bigcup \{A_i : i \in \Lambda\} \) [15].

**Definition 2.7:** A fuzzy ideal topological space \( (X, \tau, I) \) is said to be fuzzy IGO-compact if every fuzzy Ig-open cover of X has a finite subcover [15].

**Definition 2.8:** A fuzzy set B of a fuzzy ideal space \( (X, \tau, I) \) is said to be fuzzy IGO-compact if for every collection \( \{A_i : i \in \Lambda\} \) of fuzzy Ig-open subsets of X such that \( B \leq \bigcup \{A_i : i \in \Lambda\} \) there exists a finite subset \( \Lambda_0 \) and \( \Lambda \) such that \( B \leq \bigcup \{A_i : i \in \Lambda_0\} \) [15].

**Definition 2.9:** A crisp subset B of a fuzzy ideal topological space \( (X, \tau, I) \) is said to be fuzzy IGO-compact if B is fuzzy IGO-compact as a fuzzy ideal subspace of X [15].

### 3. Fuzzy Igc-Irresolute Mappings

**Definition 3.1:** A mapping \( f \) from a fuzzy ideal topological space \( (X, \tau, I) \) to a fuzzy topological space \( (Y, \sigma) \) is said to be fuzzy Igc-irresolute if the inverse image of every fuzzy g-closed set of Y is fuzzy Ig-closed in X.

**Theorem 3.1:** A mapping \( f: (X, \tau, I) \rightarrow (Y, \sigma) \) is fuzzy Igc-irresolute if and only if the inverse image of every fuzzy g-open set of Y is fuzzy Ig-open in X.

**Proof:** The proof is obvious because \( f^{-1}(1-U) = 1-f^{-1}(U) \) for every fuzzy set U of Y.

**Remark 3.1:** Every fuzzy Igc-irresolute mapping is fuzzy Ig-continuous, but the converse may not be true. For,

**Example 3.1:** Let \( X = \{a, b\} \), \( Y = \{x, y\} \) and the fuzzy sets U and V are defined as follows:
- \( U(a) = 0.5, U(b) = 0.7 \);
- \( V(a) = 0.3, V(b) = 0.2 \);
Let $\tau = \{0, U, 1\}, \sigma = \{0, V, 1\}$ be the fuzzy topologies on $X$ and $Y$ respectively and $I = \{0\}$ be the fuzzy ideal on $X$. Then the mapping $f:(X, \tau, I)\rightarrow(Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is fuzzy continuous and hence fuzzy $I_g$-continuous, but not fuzzy $I_{gc}$-irresolute.

**Example 3.2:** Let $X = \{a, b\}, Y = \{x, y\}$ and let $\tau = \{0, 1\}, \sigma = \{0, U, 1\}$ be the fuzzy topologies on $X$ and $Y$ respectively where,

\[ U(x) = 0.6, U(y) = 0.5 \]

Let $I=\{0\}$ be a fuzzy ideal on $X$. Then the mapping $f:(X, \tau, I)\rightarrow(Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is fuzzy $I_{gc}$-irresolute, but not fuzzy continuous.

**Remark 3.2:** The concepts of fuzzy $I_{gc}$-irresolute mappings and fuzzy continuous mappings are independent.

**Theorem 3.2:** If $f:(X, \tau, I)\rightarrow(Y, \sigma)$ is fuzzy $I_{gc}$-irresolute then for each fuzzy point $x_\beta$ of $X$ and each fuzzy $g$-open set $V$ such that $f(x_\beta) \in V$ there exists a fuzzy $I_g$-open set $U$ such that $x_\beta \in U$ and $f(U) \subseteq V$.

**Proof:** Let $x_\beta$ be a fuzzy point of $X$ and $V$ be a fuzzy $g$-open set such that $f(x_\beta) \in V$. Put $U = f^{-1}(V)$ then by hypothesis $U$ is a fuzzy $I_g$-open set of $X$ such that $x_\beta \in U$ and $f(U) \subseteq V$.

**Theorem 3.3:** If $f:(X, \tau, I)\rightarrow(Y, \sigma)$ is fuzzy $I_{gc}$-irresolute then for each fuzzy point $x_\beta$ of $X$ and each fuzzy $g$-open set $V$ of $Y$ such that $f(x_\beta) q V$, there exists a fuzzy $I_g$-open set $U$ of $X$ such that $x_\beta q U$ and $f(U) = f(f^{-1}(V)) \subseteq V$.

**Proof:** Let $x_\beta$ be a fuzzy point of $X$ and $V$ be a fuzzy $g$-open set such that $f(x_\beta) q V$. Put $U = f^{-1}(V)$. Then by hypothesis $U$ is a fuzzy $I_g$-open set of $X$ such that $x_\beta q U$ and $f(U) = f(f^{-1}(V)) \subseteq V$.

**Theorem 3.4:** If $f : (X, \tau, I)\rightarrow(Y, \sigma)$ is bijective, fuzzy open and fuzzy $I_g$-continuous then $f$ is fuzzy $I_{gc}$-irresolute.

**Proof:** Let $A$ be a fuzzy $g$-closed set in $Y$ and let $f^{-1}(A) \subseteq U$ where $U$ is a fuzzy open set in $X$. Then $A \subseteq f(U)$. Since $f(U)$ is fuzzy open and $A$ is fuzzy $g$-closed set in $Y$, $\overline{Cl}(A) \subseteq f(U)$. Therefore $f^{-1}(\overline{Cl}(A)) \subseteq U$. Since $f$ is fuzzy $I_g$-continuous and $\overline{Cl}(A)$ is fuzzy closed in $Y$, $f^{-1}(\overline{Cl}(A))$ is fuzzy $I_g$-closed set in $X$. Hence, $\overline{Cl}(f^{-1}(\overline{Cl}(A))) \subseteq U$. And so $\overline{Cl}(f^{-1}(A)) \subseteq U$. Which implies that $f^{-1}(A)$ is fuzzy $I_g$-closed set in $X$. Hence $f$ is fuzzy $I_{gc}$-irresolute.

**Theorem 3.5:** If $f : (X, \tau, I)\rightarrow(Y, \sigma)$ is fuzzy $I_{gc}$-irresolute and $g:(Y, \sigma)\rightarrow(Z, \eta)$ is fuzzy $gc$-irresolute. Then $gof : (X, \tau, I)\rightarrow(Z, \eta)$ is fuzzy $I_{gc}$-irresolute.
**Proof:** If A is fuzzy g-closed set in Z, then \( f^{-1}(A) \) is fuzzy g-closed in Y because g is fuzzy g-irresolute. Therefore \((gof)^{-1}(A) = f^{-1}(g^{-1}(A))\) is fuzzy I\(_g\) -closed in X. Hence gof is fuzzy I\(_g\) -continuous.

**Theorem 3.6:** If \( f: (X, \tau, I) \rightarrow (Y, \sigma) \) is fuzzy I\(_g\) -irresolute and \( g: (Y, \sigma) \rightarrow (Z, \eta) \) is fuzzy g-continuous, then \( gof: (X, \tau, I) \rightarrow (Z, \eta) \) is fuzzy I\(_g\) -continuous.

**Proof:** If A is fuzzy closed set in Z, then \( f^{-1}(A) \) is fuzzy g-closed set in Y because g is fuzzy g-continuous. Therefore \((gof)^{-1}(A) = f^{-1}(g^{-1}(A))\) is fuzzy I\(_g\) -closed set in X. Hence gof is fuzzy I\(_g\) -continuous.

**Theorem 3.7:** A fuzzy I\(_g\) -irresolute image of a fuzzy IGO-compact space is fuzzy GO-compact.

**Proof:** Let \( f: (X, \tau, I) \rightarrow (Y, \sigma) \) be a fuzzy I\(_g\) -irresolute mapping from a fuzzy IGO-compact space \( (X, \tau, I) \) onto a fuzzy topological space \( (Y, \sigma) \). Let \( \{A_i : i \in \Lambda\} \) be a fuzzy g-open cover of Y. Then \( \{f^{-1}(A_i) : i \in \Lambda\} \) is a fuzzy I\(_g\) -open cover of X. Since X is fuzzy IGO-compact it has finite subcover say \( \{f^{-1}(A_1), f^{-1}(A_2), f^{-1}(A_3), \ldots, f^{-1}(A_n)\} \) of X. Since \( f(f^{-1}(A_i)) = A_i \) for each i, it follows that \( \{A_1, A_2, A_3, \ldots, A_n\} \) is a finite subcover of \( \{A_i : i \in \Lambda\} \). Hence \( (Y, \sigma) \) is fuzzy GO-compact.

**Theorem 3.8:** If \( f: (X, \tau, I) \rightarrow (Y, \sigma) \) be a fuzzy I\(_g\) -irresolute surjection and X is fuzzy IGO-connected then Y is fuzzy GO-connected.

**Proof:** Suppose Y is not fuzzy GO-connected. Then there exists a proper fuzzy set G of Y which is both fuzzy g-open and fuzzy g-closed. Therefore \( f^{-1}(G) \) is a proper fuzzy set of X, which is both fuzzy I\(_g\) -open and fuzzy I\(_g\) -closed, because f is fuzzy I\(_g\) -irresolute surjection. Hence X is not fuzzy IGO-connected, which is a contradiction.

**References**


