Translation of Anti S-fuzzy Subfields of a Field

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of translation of anti S-fuzzy subfield of a field.

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INTRODUCTION

After the introduction of fuzzy sets by L.A.Zadeh[16], several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subgroups, anti-fuzzy subgroups, fuzzy fields and fuzzy linear spaces was introduced by Biswas.R[4, 5]. In this paper, we introduce the some theorems in translation of anti S-fuzzy subfield of a field.

1. PRELIMINARIES:

1.1 Definition: Let X be a non-empty set. A fuzzy subset A of X is a function A: X → [0, 1].

1.2 Definition: A S-norm is a binary operation S: [0, 1]×[0, 1] → [0, 1] satisfying the following requirements;
(i) S(0, x) = x, S(1, x) = 1 (boundary condition)
(ii) S(x, y) = S(y, x) (commutativity)
(iii) S( x, S(y, z) ) = S( S(x, y), z ) (associativity)
(iv) if x ≤ y and w ≤ z, then S(x, w) ≤ S(y, z) (monotonicity).
1.3 Definition: Let \((F, +, \cdot)\) be a field. A fuzzy subset \(A\) of \(F\) is said to be an anti S-fuzzy subfield (anti fuzzy subfield with respect to S-norm) of \(F\) if the following conditions are satisfied:

(i) \(A(x+y) \leq S(A(x), A(y))\), for all \(x\) and \(y\) in \(F\),
(ii) \(A(-x) \leq A(x)\), for all \(x\) in \(F\),
(iii) \(A(xy) \leq S(A(x), A(y))\), for all \(x\) and \(y\) in \(F\),
(iv) \(A(x^{-1}) \leq A(x)\), for all \(x \neq 0\) in \(F\), where 0 is the additive identity of \(F\).

1.4 Definition: Let \((F, +, \cdot)\) and \((F', +, \cdot)\) be any two fields. Let \(f: F \rightarrow F'\) be any function and \(A\) be an anti S-fuzzy subfield in \(F\), \(V\) be an anti S-fuzzy subfield in \(f(F) = F'\), defined by \(V(y) = \inf_{x \in f^{-1}(y)} A(x)\), for all \(x\) in \(F\) and \(y\) in \(F'\). Then \(A\) is called a preimage of \(V\) under \(f\) and is denoted by \(f^{-1}(V)\).

1.5 Definition: Let \(A\) be a fuzzy subset of \(X\). For \(\alpha\) in \([0, 1]\), the lower level subset of \(A\) is the set \(A_\alpha = \{ x \in X: A(x) \leq \alpha \}\).

1.6 Definition: Let \(A\) be an anti S-fuzzy subfield of a field \((F, +, \cdot)\). The lower level subset \(A_\alpha\), for \(\alpha\) in \([0, 1]\) such that \(\alpha \geq A(0), \alpha \geq A(1)\), is called lower level subfield of \(A\).

1.7 Definition: Let \(A\) be a fuzzy subset of \(X\) and \(\alpha \in [0, 1– \sup\{ A(x): x \in X, 0 < A(x) < 1 \}]\). Then \(T = T_\alpha\) is called a translation of \(A\) if \(T(x) = A(x) + \alpha\), for all \(x\) in \(X\).

2–Properties:

2.1 Theorem: Let \(A\) be an anti S-fuzzy subfield of a field \((F, +, \cdot)\). Then for \(\alpha\) in \([0, 1]\) such that \(\alpha \geq A(0), \alpha \geq A(1)\), \(A_\alpha\) is a subfield of \(F\), where 0 and 1 are identity elements of \(F\).

Proof: For all \(x\) and \(y\) in \(A_\alpha\), we have, \(A(x) \leq \alpha\) and \(A(y) \leq \alpha\). Now, \(A(x–y) \leq S(A(x), A(y)) \leq S(\alpha, \alpha) = \alpha\), which implies that, \(A(x–y) \leq \alpha\). And also, \(A(xy^{-1}) \leq S(A(x), A(y)) \leq S(\alpha, \alpha) = \alpha\), which implies that, \(A(xy^{-1}) \leq \alpha\). Therefore, \(A(x–y) \leq \alpha, A(xy^{-1}) \leq \alpha\), we get \(x–y, xy^{-1}\) in \(A_\alpha\). Hence \(A_\alpha\) is a subfield of \(F\).

2.2 Theorem: Let \(A\) be an anti S-fuzzy subfield of a field \((F, +, \cdot)\). Then two lower level subfields \(A_{\alpha_1}\) and \(A_{\alpha_2}\), \(\alpha_1\) and \(\alpha_2\) in \([0, 1]\) and \(\alpha_1 \geq A(0), \alpha_2 \geq A(0), \alpha_1 \geq v_A(1), \alpha_2 \geq v_A(1)\) with \(\alpha_2 > \alpha_1\) of \(A\) are equal if and only if there is no \(x\) in \(F\) such that \(\alpha_1 < A(x) \leq \alpha_2\), where 0 and 1 are identity elements of \(F\).

Proof: Assume that \(A_{\alpha_1} = A_{\alpha_2}\). Suppose there exists \(x \in F\) such that \(\alpha_1 < A(x) < \alpha_2\). Then \(A_{\alpha_1} \subseteq A_{\alpha_2}\), which implies that \(x\) belongs to \(A_{\alpha_2}\), but not in \(A_{\alpha_1}\). This is contradiction to \(A_{\alpha_1} = A_{\alpha_2}\). Therefore there is no \(x \in F\) such that \(\alpha_1 < A(x) < \alpha_2\). Conversely, if there is no \(x \in F\) such that \(\alpha_1 < A(x) < \alpha_2\). Then \(A_{\alpha_1} = A_{\alpha_2}\).
2.3 **Theorem:** Let \((F, +, \cdot)\) be a field and \(A\) be a fuzzy subset of \(F\) such that \(A_0\) be a lower level subfield of \(F\). If \(\alpha\) in \([0, 1]\) satisfying \(\alpha \geq A(0), \alpha \geq A(1)\), then \(A\) is an anti S-fuzzy subfield of \(F\), where 0 and 1 are identity elements of \(F\).

**Proof:** Let \((F, +, \cdot)\) be a field. For \(x\) and \(y\) in \(F\). Let \(A(x) = \alpha_1\) and \(A(y) = \alpha_2\).

**Case (i):** If \(\alpha_1 > \alpha_2\) then \(x\) and \(y\) in \(A_{\alpha_1}\). As \(A_{\alpha_1}\) is a lower level subfield of \(F\), so \(x - y\) and \(xy^{-1}\) in \(A_{\alpha_1}\). Now, \(A(x - y) \leq \alpha_1 = S(\alpha_1, \alpha_2) = S(A(x), A(y))\), which implies that \(A(x - y) \leq S(A(x), A(y))\), for all \(x\) and \(y\) in \(F\). Now, \(A(xy^{-1}) \leq \alpha_1 = S(\alpha_1, \alpha_2) = S(A(x), A(y))\), which implies that \(A(xy^{-1}) \leq S(A(x), A(y))\), for all \(x\) and \(y \neq 0\) in \(F\).

**Case (ii):** If \(\alpha_1 < \alpha_2\) then \(x\) and \(y\) in \(A_{\alpha_2}\). As \(A_{\alpha_2}\) is a lower level subfield of \(F\), so \(x - y\) and \(xy^{-1}\) in \(A_{\alpha_2}\). Now, \(A(x - y) \leq \alpha_2 = S(\alpha_1, \alpha_2) = S(A(x), A(y))\), which implies that \(A(x - y) \leq S(A(x), A(y))\), for all \(x\) and \(y\) in \(F\). Now, \(A(xy^{-1}) \leq \alpha_2 = S(\alpha_1, \alpha_2) = S(A(x), A(y))\), which implies that \(A(xy^{-1}) \leq S(A(x), A(y))\), for all \(x\) and \(y \neq 0\) in \(F\). In all the cases, \(A\) is an anti S-fuzzy subfield of a field \(F\).

2.4 **Theorem:** Let \(A\) be an anti S-fuzzy subfield of a field \((F, +, \cdot)\). If any two lower level subfields of \(A\) belongs to \(F\), then their intersection is also lower level subfield of \(A\) in \(F\).

**Proof:** For \(\alpha_1, \alpha_2\) in \([0, 1]\), \(\alpha_1 \geq A(0)\) and \(\alpha_2 \geq A(0)\), \(\alpha_1 \geq A(1)\) and \(\alpha_2 \geq A(1)\), where 0 and 1 are identity elements of \(F\). Case (i): If \(\alpha_1 > A(x) > \alpha_2\), then \(A_{\alpha_2} \subseteq A_{\alpha_1}\). Therefore, \(A_{\alpha_1} \cap A_{\alpha_2} = A_{\alpha_2}\) but \(A_{\alpha_2}\) is a lower level subfield of \(A\). Case (ii): If \(\alpha_1 < A(x) < \alpha_2\), then \(A_{\alpha_1} \subseteq A_{\alpha_2}\). Therefore, \(A_{\alpha_1} \cap A_{\alpha_2} = A_{\alpha_1}\), but \(A_{\alpha_1}\) is a lower level subfield of \(A\). Case (iii): If \(\alpha_1 = \alpha_2\), then \(A_{\alpha_1} = A_{\alpha_2}\). In all cases, intersection of any two lower level subfields is a lower level subfield of \(A\).

2.5 **Theorem:** Let \(A\) be an anti S-fuzzy subfield of a field \((F, +, \cdot)\). If \(\alpha_i\) in \([0, 1]\), \(\alpha_i \geq A(0)\) and \(\alpha_i \geq A(1)\) and \(A_{\alpha_i}\), \(i\) in \(I\), is a collection of lower level subfields of \(A\), then their intersection is also a lower level subfield of \(A\).

**Proof:** It is trivial.

2.6 **Theorem:** Let \(A\) be an anti S-fuzzy subfield of a field \((F, +, \cdot)\). If any two lower level subfields of \(A\) belongs to \(F\), then their union is also lower level subfield of \(A\) in \(F\).

**Proof:** For \(\alpha_1, \alpha_2\) in \([0, 1]\), \(\alpha_1 \geq A(0)\) and \(\alpha_2 \geq A(0)\), \(\alpha_1 \geq A(1)\) and \(\alpha_2 \geq A(1)\), where 0 and 1 are identity elements of \(F\). Case (i): If \(\alpha_1 > A(x) > \alpha_2\), then \(A_{\alpha_2} \subseteq A_{\alpha_1}\). Therefore, \(A_{\alpha_1} \cup A_{\alpha_2} = A_{\alpha_1}\), but \(A_{\alpha_1}\) is a lower level subfield of \(A\). Case (ii): If \(\alpha_1 < A(x) < \alpha_2\), then \(A_{\alpha_1} \subseteq A_{\alpha_2}\). Therefore, \(A_{\alpha_1} \cup A_{\alpha_2} = A_{\alpha_2}\), but \(A_{\alpha_2}\) is a lower level subfield of \(A\). Case (iii): If \(\alpha_1 = \alpha_2\), then \(A_{\alpha_1} = A_{\alpha_2}\). In all cases, union of any two lower level subfields is a lower level subfield of \(A\).
2.7 Theorem: Let \( A \) be an anti S-fuzzy subfield of a field \( (F, +, \cdot) \). If \( \alpha_i \) in \([0, 1]\), \( \alpha_i \geq A(0), \alpha_i \geq A(1) \) and \( A_{\alpha_i}, i \in I \), is a collection of lower level subfields of \( A \), then their union is also a lower level subfield of \( A \).

Proof: It is trivial.

2.8 Theorem: Any subfield \( H \) of a field \( (F, +, \cdot) \) can be realized as a lower level subfield of some anti S-fuzzy subfield of \( F \).

Proof: It is trivial.

2.9 Theorem: Let \( (F, +, \cdot) \) and \( (F', +, \cdot) \) be any two fields. If \( f: F \rightarrow F' \) is a homomorphism, then the homomorphic image of a lower level subfield of an anti S-fuzzy subfield of \( F \) is a lower level subfield of an anti S-fuzzy subfield of \( F' \).

Proof: Let \( (F, +, \cdot) \) and \( (F', +, \cdot) \) be any two fields and \( f: F \rightarrow F' \) be a homomorphism. That is, \( f(x+y) = f(x)+f(y) \), for all \( x \) and \( y \) in \( F \) and \( f(xy) = f(x)f(y) \), for all \( x \) and \( y \) in \( F \).

Let \( V = f(A) \), where \( A \) is an anti S-fuzzy subfield of \( F \). Clearly \( V \) is an anti S-fuzzy subfield of \( F' \). If \( x \) and \( y \) in \( F \), then \( f(x) \) and \( f(y) \) in \( F' \). Let \( A_{\alpha} \) be a lower level subfield of \( A \). Suppose \( x, y \) and \( x-y, xy^{-1} \) in \( A_{\alpha} \). That is, \( A(x) \leq \alpha \) and \( A(y) \leq \alpha \), \( A(x-y) \leq \alpha \), \( A(xy^{-1}) \leq \alpha \). We have to prove that \( f(A_{\alpha}) \) is a lower level subfield of \( V \).

Therefore, \( f(A_{\alpha}) \leq \alpha \). Hence \( f(A_{\alpha}) \) is a lower level subfield of an anti S-fuzzy subfield \( V \) of a field \( F' \).

2.10 Theorem: Let \( (F, +, \cdot) \) and \( (F', +, \cdot) \) be any two fields. If \( f: F \rightarrow F' \) is a homomorphism, then the homomorphic pre-image of a lower level subfield of an anti S-fuzzy subfield of \( F \) is a lower level subfield of an anti S-fuzzy subfield of \( F' \).

Proof: Let \( (F, +, \cdot) \) and \( (F', +, \cdot) \) be any two fields and \( f: F \rightarrow F' \) be a homomorphism. That is, \( f(x+y) = f(x)+f(y) \), for all \( x \) and \( y \) in \( F \) and \( f(xy) = f(x)f(y) \), for all \( x \) and \( y \) in \( F \).

Let \( V = f(A) \), where \( V \) is an anti S-fuzzy subfield of \( F' \). Clearly \( V \) is an anti S-fuzzy subfield of \( F' \). Let \( x \) and \( y \) in \( F \). Let \( f(A_{\alpha}) \) be a lower level subfield of \( V \). Suppose \( f(x), f(y) \) and \( f(x)-f(y), f(x)(f(y))^{-1} \) in \( f(A_{\alpha}) \). That is, \( V(f(x)) \leq \alpha \) and \( V(f(y)) \leq \alpha \); \( V(f(x)-f(y)) \leq \alpha \), \( V((f(x)(f(y))^{-1}) \leq \alpha \). We have to prove that \( A_{\alpha} \) is a lower level subfield of \( A \). Now, \( A(x) = V(f(x)) \leq \alpha \), \( A(y) = V(f(y)) \leq \alpha \), \( A(x-y) = V(f(x)-f(y)) \leq \alpha \), \( A(xy^{-1}) = V(f(x)(f(y))^{-1}) \leq \alpha \). Hence \( A_{\alpha} \) is a lower level subfield of an anti S-fuzzy subfield \( A \) of \( F \).
2.11 Theorem: Let A be an anti S-fuzzy subfield of a field F, $A^+$ be a fuzzy set in F defined by $A^+(x) = A(x)+1 - A(0)$, for all $x$ in F. Then $A^+$ is an anti S-fuzzy subfield of a field F.

Proof: Let $x$ and $y$ in F. We have, $A^+(x-y) = A(x-y) + 1 - A(0) \leq S(A(x), A(y)) + 1 - A(0) \leq S(A^+(x), A^+(y))$. Therefore, $A^+(x-y) \leq S(A^+(x), A^+(y))$, for all $x, y$ in F. Similarly, $A^+(xy^{-1}) = A(xy^{-1}) + 1 - A(0) \leq S(A(x), A(y)) + 1 - A(0) \leq S(A^+(x), A^+(y))$. Therefore, $A^+(xy^{-1}) \leq S(A^+(x), A^+(y))$, for all $x, y \neq 0$ in F. Hence $A^+$ is an anti S-fuzzy subfield of a field F.

2.12 Theorem: Let A be an anti S-fuzzy subfield of a field F, $A^+$ be a fuzzy set in F defined by $A^+(x) = A(x)+1 - A(0)$, for all $x$ in F. Then there exists 0 in F such that $A(0) = 1$ if and only if $A^+(x) = A(x)$.

Proof: It is trivial.

2.13 Theorem: Let A be an anti S-fuzzy subfield of a field F, $A^+$ be a fuzzy set in F defined by $A^+(x) = A(x)+1 - A(0)$, for all $x$ in F. Then there exists $x$ in F such that $A^+(x) = 1$ if and only if $x = 0$.

Proof: It is trivial.

2.14 Theorem: Let A be an anti S-fuzzy subfield of a field F, $A^+$ be a fuzzy set in F defined by $A^+(x) = A(x)+1 - A(0)$, for all $x$ in F. Then $(A^+)^+ = A^+$.

Proof: Let $x$ and $y$ in F. We have, $(A^+)^+(x) = A^+(x) + 1 - A^+(0) = A(x) + 1 - A(0) + 1 - A(0) = A(x) + 1 - A(0) = A^+(x)$. Hence $(A^+)^+ = A^+$.

2.15 Theorem: Let A be an anti S-fuzzy subfield of a field F. Then $A^0$ is an anti S-fuzzy subfield of the field F.

Proof: For any $x$ in F, we have $A^0(x-y) = A(x-y)A(0) \leq [A(0)] S(A(x), A(y)) \leq S([A(x)]A(0), [A(y)]A(0)) = S(A^0(x), A^0(y))$. that is $A^0(x+y) \leq S(A^0(x), A^0(y))$, for all $x, y$ in F. Similarly, $A^0(xy^{-1}) = A(xy^{-1})A(0) \leq [A(0)] S(A(x), A(y)) \leq S([A(x)]A(0), [A(y)]A(0)) = S(A^0(x), A^0(y))$. That is $A^0(xy^{-1}) \leq S(A^0(x), A^0(y))$, for all $x, y \neq 0$ in F. Hence $A^0$ is an anti S-fuzzy subfield of the field F.

2.16 Theorem: If $M$ and $N$ are two translations of anti S-fuzzy subfield A of a field (F, +, .), then their intersection $M \cap N$ is translation of anti S-fuzzy subfield A.

Proof: Let $x$ and $y$ belong to F. Let $M = T^d_{\alpha} = \{ \langle x, \mu_A(x) + \alpha \rangle / x \text{ in } F \}$ and $N = T^d_{\gamma} = \{ \langle x, \mu_A(x) + \gamma \rangle / x \text{ in } F \}$ be two translations of anti S-fuzzy subfield A of a field (F, +, .). Let $C = M \cap N$ and $C = \{ \langle x, \mu_C(x) \rangle / x \text{ in } F \}$, where $\mu_C(x) = \min \{ \mu_A(x) + \alpha, \mu_A(x) + \gamma \}$,
\[\gamma \}. \text{ Case (i): } \alpha \leq \gamma. \text{ Now, } \mu_c(x-y) = \min \{ \mu_M(x-y), \mu_N(x-y) \} = \min \{ \mu_A(x-y) + \alpha, \mu_A(x-y) + \gamma \} = \mu_A(x-y) + \gamma, \text{ for all } x \text{ and } y \text{ in } F. \text{ And, } \mu_c(xy^{-1}) = \min \{ \mu_M(xy^{-1}), \mu_N(xy^{-1}) \} = \min \{ \mu_A(xy^{-1}) + \alpha, \mu_A(xy^{-1}) + \gamma \} = \mu_A(xy^{-1}) + \gamma = \mu_M(xy^{-1}), \text{ for all } x \text{ and } y \neq 0 \text{ in } F. \text{ Therefore } C = T'_d = \{ (x, \mu_A(x)+\alpha) / x \text{ in } F \} \text{ is a translation of anti S-fuzzy subfield } A \text{ of } F. \text{ Case (ii): } \alpha \geq \gamma. \text{ Now, } \mu_c(x-y) = \min \{ \mu_M(x-y), \mu_N(x-y) \} = \min \{ \mu_A(x-y) + \alpha, \mu_A(x-y) + \gamma \} = \mu_A(x-y) + \gamma = \mu_N(x-y), \text{ for all } x \text{ and } y \text{ in } F. \text{ And, } \mu_c(xy^{-1}) = \min \{ \mu_M(xy^{-1}), \mu_N(xy^{-1}) \} = \min \{ \mu_A(xy^{-1}) + \alpha, \mu_A(xy^{-1}) + \gamma \} = \mu_A(xy^{-1}) + \gamma = \mu_N(xy^{-1}), \text{ for all } x \text{ and } y \neq 0 \text{ in } F. \text{ Therefore } C = T'_d = \{ (x, \mu_A(x)+\gamma) / x \text{ in } F \} \text{ is a translation of anti S-fuzzy subfield } A \text{ of } F. \]

2.17 Theorem: The intersection of a family of translations of anti S-fuzzy subfield A of a field \((F, +, \cdot)\) is also a translation of anti S-fuzzy subfield A.

Proof: Let \(x\) and \(y\) belong to \(F\). Let \(M_i = T'_d = \{ (x, \mu_A(x)+\alpha_i) / x \text{ in } F \} \) be a family of translations of anti S-fuzzy subfield A of the field \((F, +, \cdot)\). Let \(C = \bigcap_{i=1}^{\infty} M_i \) and \(C = \{ (x, \mu_c(x)) / x \text{ in } F \} \), where \(\mu_c(x) = \inf_{i=1} \{ \mu_A(x)+\alpha_i \} = \mu_A(x) + \inf_{i=1} \alpha_i \). Clearly \(C\) is also a translation of anti S-fuzzy subfield A of \(F\).

2.18 Theorem: If \(M\) and \(N\) are two translations of anti S-fuzzy subfield A of a field \((F, +, \cdot)\), then their union \(M \cup N\) is also a translation of anti S-fuzzy subfield A.

Proof: Let \(x\) and \(y\) belong to \(F\). Let \(M = T'_d = \{ (x, \mu_A(x)+\alpha) / x \text{ in } F \} \) and \(N = T'_d = \{ (x, \mu_A(x)+\gamma) / x \text{ in } F \} \) be two translations of anti S-fuzzy subfield A of \(F\). Let \(C = M \cup N \) and \(C = \{ (x, \mu_c(x)) / x \text{ in } F \} \), where \(\mu_c(x) = \max \{ \mu_M(x)+\alpha, \mu_A(x) \} \). Case (i): \(\alpha \leq \gamma. \text{ Now, } \mu_c(x-y) = \max \{ \mu_M(x-y), \mu_N(x-y) \} = \max \{ \mu_A(x-y) + \alpha, \mu_A(x-y) + \gamma \} = \mu_A(x-y) + \gamma = \mu_N(x-y), \text{ for all } x \text{ and } y \text{ in } F. \text{ And, } \mu_c(xy^{-1}) = \max \{ \mu_M(xy^{-1}), \mu_N(xy^{-1}) \} = \max \{ \mu_A(xy^{-1}) + \alpha, \mu_A(xy^{-1}) + \gamma \} = \mu_A(xy^{-1}) + \gamma = \mu_N(xy^{-1}), \text{ for all } x \text{ and } y \neq 0 \text{ in } F. \text{ Therefore } C = T'_d = \{ (x, \mu_A(x)+\gamma) / x \text{ in } F \} \text{ is a translation of anti S-fuzzy subfield A of } F. \text{ Case (ii): } \alpha \geq \gamma. \text{ Now, } \mu_c(x-y) = \max \{ \mu_M(x-y), \mu_N(x-y) \} = \max \{ \mu_A(x-y)+\alpha, \mu_A(x-y)+\gamma \} = \mu_A(x-y)+\gamma = \mu_M(x-y), \text{ for all } x \text{ and } y \text{ in } F. \text{ And, } \mu_c(xy^{-1}) = \max \{ \mu_M(xy^{-1}), \mu_N(xy^{-1}) \} = \max \{ \mu_A(xy^{-1})+\alpha, \mu_A(xy^{-1})+\gamma \} = \mu_A(xy^{-1})+\gamma = \mu_M(xy^{-1}), \text{ for all } x \text{ and } y \neq 0 \text{ in } F. \text{ Therefore } C = T'_d = \{ (x, \mu_A(x)+\alpha) / x \text{ in } F \} \text{ is a translation of anti S-fuzzy subfield A of } F. \text{ Hence all cases, union of any two translations of anti S-fuzzy subfield A of a field } (F, +, \cdot) \text{ is also a translation of anti S-fuzzy subfield A.}

2.19 Theorem: The union of a family of translations of anti S-fuzzy subfield A of a field \((F, +, \cdot)\) is also a translation of anti S-fuzzy subfield A.
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Proof: Let $x$ and $y$ belong to $F$. Let $M_i = T_{\alpha_i} = \{ \langle x, \mu_A(x) + \alpha_i \rangle / x \in F \}$ be a family of translations of anti $S$-fuzzy subfield $A$ of $F$. Let $C = \bigcup_{i \in I} M_i$ and $C = \{ \langle x, \mu_C(x) \rangle / x \in F \}$, where $\mu_C(x) = \sup_{i \in I} \{ \mu_A(x) + \alpha_i \} = \mu_A(x) + \sup_{i \in I} \alpha_i$. Clearly $C$ is also a translation of anti $S$-fuzzy subfield $A$ of $F$.

2.20 Theorem: If $T_{\alpha}^A$ is a translation of anti $S$-fuzzy subfield $A$ of a field $F$, then $T_{\alpha}^A$ is anti $S$-fuzzy subfield of $F$.

Proof: Assume that $T_{\alpha}^A$ is a translation of anti $S$-fuzzy subfield $A$ of a field $F$. Let $x$ and $y$ in $F$. We have, $T_{\alpha}^A(x - y) = A(x - y) + \alpha \leq S(A(x), A(y)) + \alpha \leq S(A(x) + \alpha, A(y) + \alpha) = S(T_{\alpha}^A(x), T_{\alpha}^A(y))$. Therefore, $T_{\alpha}^A(x - y) \leq S(T_{\alpha}^A(x), T_{\alpha}^A(y))$, for all $x$ and $y$ in $F$. And, $T_{\alpha}^A(xy^{-1}) = A(xy^{-1}) + \alpha \leq S(A(x), A(y)) + \alpha \leq S(A(x) + \alpha, A(y) + \alpha) = S(T_{\alpha}^A(x), T_{\alpha}^A(y))$. Therefore, $T_{\alpha}^A(xy^{-1}) \geq S(T_{\alpha}^A(x), T_{\alpha}^A(y))$, for all $x$ and $y \neq 0$ in $F$. Hence $T_{\alpha}^A$ is anti $S$-fuzzy subfield of $F$.

2.21 Theorem: Let $(F, +, \cdot)$ and $(F^l, +, \cdot)$ be any two fields. If $f: F \rightarrow F^l$ is a homomorphism, then the translation of anti $S$-fuzzy subfield $A$ of $F$ under the homomorphic image is anti $S$-fuzzy subfield of $f(F) = F^l$.

Proof: Let $(F, +, \cdot)$ and $(F^l, +, \cdot)$ be any two fields and $f: F \rightarrow F^l$ be a homomorphism. That is $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all $x$ and $y$ in $F$. Let $T_{\alpha}^A$ be a translation of anti $S$-fuzzy subfield $A$ of $F$. Let $V$ be the homomorphic image of $T_{\alpha}^A$ under $f$. We have to prove that $V$ is anti $S$-fuzzy subfield of $f(F) = F^l$.

Now, for $f(x)$ and $f(y)$ in $F^l$, we have $V[f(x) - f(y)] = V[f(x - y)] \leq T_{\alpha}^A(x + y) = A(x - y) + \alpha \leq S(A(x), A(y) + \alpha) = S(T_{\alpha}^A(x), T_{\alpha}^A(y))$, which implies that $V[f(x) - f(y)] \leq S(V(f(x)), V(f(y)))$, for all $f(x)$ and $f(y)$ in $F^l$. And $V[f(x)(f(y)^{-1})] = V[f(xy^{-1})] \leq T_{\alpha}^A(xy^{-1}) = A(xy^{-1}) + \alpha \leq S(A(x), A(y) + \alpha) = S(T_{\alpha}^A(x), T_{\alpha}^A(y))$, which implies that $V[f(x)(f(y)^{-1})] \leq S(V(f(x)), V(f(y)))$, for all $f(x)$ and $f(y) \neq 0$ in $F^l$. Therefore, $V$ is an anti $S$-fuzzy subfield of $F^l$. Hence the homomorphic image of translation of $A$ of $F$ is an anti $S$-fuzzy subfield of $F^l$.

2.22 Theorem: Let $(F, +, \cdot)$ and $(F^l, +, \cdot)$ be any two fields. If $f: F \rightarrow F^l$ is a homomorphism, then the translation of an anti $S$-fuzzy subfield $V$ of $f(F) = F^l$ under the homomorphic pre-image is anti $S$-fuzzy subfield of $F$.

Proof: Let $(F, +, \cdot)$ and $(F^l, +, \cdot)$ be any two fields and $f: F \rightarrow F^l$ be a homomorphism. That is $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all $x$ and $y$ in $F$. Let $T_{\alpha}^V$ be the translation of anti $S$-fuzzy subfield $V$ of $F^l$ and $A$ be the homomorphic pre-image of $T_{\alpha}^V$ under $f$. We have to prove that $A$ is an anti $S$-fuzzy subfield of $F$. Let
x and y be in F. Then, A(x−y) = T^V_α (f(x−y)) = T^V_α (f(x) − f(y)) = V[ f(x) − f(y)] + α ≤ S(V( f(x)), V( f(y))) + α ≤ S((V( f(x))+α, V( f(y))+α) = S(T^V_α ( f(x)), T^V_α ( f(y))) = S(A(x), A(y)), which implies that A(x−y) ≤ S ( A(x), A(y) ), for all x, y in F. And, A(xy−1) = T^V_α ( f(xy−1)) = T^V_α ( f(x)(f(y)−1)) = V[ f(x)(f(y)−1)] + α ≤ S(V(f(x)), V( f(y)))+α ≤ S( (V( f(x))+α, V( f(y))+α) = S (T^V_α ( f(x)), T^V_α ( f(y))) = S (A(x), A(y)), which implies that, A(xy−1) ≤ S(A(x), A(y)), for all x and y ≠ 0 in F. Therefore, A is anti S-fuzzy subfield of F.

REFERENCE