Fuzzy Semi-Pre-Generalized Super Continuous Mapping

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Abstract

In this paper, a new class of sets called fuzzy semi-pre-generalized super continuous mapping is introduced and its properties are studied and we also explore some of the properties.

Key words: Fuzzy topology, fuzzy super closure, fuzzy super interior, fuzzy super closed set, fuzzy super open set, fuzzy super continuity, fuzzy super generalized closed set Fspgs-closed set, fuzzy semi super open set, Fspg super -continuity.

1. Preliminaries

Let X be a non empty set and I = [0, 1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family {Aα: α∈Λ} of fuzzy sets of X is defined by to be the mapping sup Aα (resp. inf Aα). A fuzzy set A of X is contained in a fuzzy set B of X if A(x) ≤ B(x) for each x∈X. A fuzzy point xβ in X is a fuzzy set defined by xβ(y) = β for y = x and x(y) = 0 for y ≠ x, β∈[0, 1] and y ∈ X. A fuzzy point xβ is said to be quasi-coincident with the fuzzy set A denoted by xβqA if and only if β + A(x) > 1. A fuzzy set A is quasi coincident with a fuzzy set B denoted by AqB if and only if there exists a point x∈X such that A(x) + B(x) > 1. A ≤ B if and only if (AqBc).

A family τ of fuzzy sets of X is called a fuzzy topology [2] on X if 0, 1 belongs to τ and τ is super closed with respect to arbitrary union and finite intersection. The members of τ are called fuzzy super open sets and their complement are fuzzy super closed sets. For any fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy super closed super sets of A and the interior of A (denoted by int(A)) is the union of all fuzzy super open subsets of A.
Definition 1.1 [5, 10, 11, 12]: Let \((X, \tau)\) fuzzy topological space and \(A \subseteq X\) then
- Fuzzy Super closure \(scl(A) = \{x \in X: \text{cl}(U) \cap A \neq \emptyset\}\)
- Fuzzy Super interior \(sint(A) = \{x \in X: \text{cl}(U) \subseteq A \neq \emptyset\}\)

Definition 1.2 [5, 10, 11, 12]: A fuzzy set \(A\) of a fuzzy topological space \((X, \tau)\) is called:
- Fuzzy super closed if \(scl(A) \leq A\).
- Fuzzy super open if \(1 - A\) is fuzzy super closed \(sint(A) = A\)

Remark 1.1 [5, 10, 11, 12]: Every fuzzy closed set is fuzzy super closed but the converses may not be true.

Remark 1.2 [5, 10, 11, 12]: Let \(A\) and \(B\) are two fuzzy super closed sets in a fuzzy topological space \((X, \mathcal{I})\), then \(A \cup B\) is fuzzy super closed.

Remark 1.3 [5]: The intersection of two fuzzy super closed sets in a fuzzy topological space \((X, \mathcal{I})\) may not be fuzzy super closed.

Definition 1.3 [1, 5, 6, 7, 10, 11, 12]: A fuzzy set \(A\) of a fuzzy topological space \((X, \tau)\) is called:
- fuzzy semi super open if there exists a super open set \(O\) such that \(O \leq A \leq \text{cl}(O)\).
- fuzzy semi super closed if its complement \(1 - A\) is fuzzy semi super open.

Remark 1.4 [1, 5, 7, 10, 11, 12]: Every fuzzy super open (resp. fuzzy super closed) set is fuzzy semi super open (resp. fuzzy semi super closed) but the converse may not be true.

Definition 1.4 [5, 10, 11, 12]: The intersection of all fuzzy super closed sets which contains \(A\) is called the semi super closure of a fuzzy set \(A\) of a fuzzy topological space \((X, \tau)\). It is denoted by \(scl(A)\).

Definition 1.5 [3, 11, 8, 9, 10, 11, 12]: A fuzzy set \(A\) of a fuzzy topological space \((X, \tau)\) is called:
- fuzzy g- super closed if \(\text{cl}(A) \leq G\) whenever \(A \leq G\) and \(G\) is super open.
- fuzzy g- super open if its complement \(1 - A\) is fuzzy g- super closed.
- fuzzy sg- super closed if \(scl(A) \leq O\) whenever \(A \leq O\) and \(O\) is fuzzy super open.
- fuzzy sg- super open if its complement \(1 - A\) is sg- super closed.
- fuzzy gs- super closed if \(scl(A) \leq O\) whenever \(A \leq O\) and \(O\) is fuzzy super open.
- fuzzy gs- super open if its complement \(1 - A\) is gs- super closed.
Remark 1. 5[10, 11]: Every fuzzy super closed (resp. fuzzy super open) set is fuzzy g- super closed (resp. fuzzy g- super open) and every fuzzy g-super closed (resp. fuzzy g-super open) set is fuzzy gs-super closed (resp. gs –super open) but the converses may not be true.

Remark 1. 6[10, 11]: Every fuzzy semi super closed (resp. fuzzy semi super open) set is fuzzy sg-super closed (resp. fuzzy sg-super open) and every fuzzy sg-super closed (resp. fuzzy sg-super open) set is fuzzy gs-super closed (resp. gs – super open) but the converses may not be true.

Definition 1. 6[10, 11]. : A fuzzy set A of (X, τ) is called:
- Fuzzy semi super open (briefly, Fs- super open) if A ≤ Cl(Int(A)) and a fuzzy semi super closed (briefly, Fs- super closed) if Int(Cl(A)) ≤ A.
- Fuzzy pre super open (briefly, Fp- super open) if A ≤ Int(Cl(A)) and a fuzzy pre super closed (briefly, Fp- super closed) if Cl(Int(A)) ≤ A.
- Fuzzy α super open (briefly, Fα- super open) if A ≤ Cl(Int(A)) and a fuzzy α- super closed (Briefly, Fα- super closed) if Cl Int(Cl(A)) ≤ A.
- Fuzzy semi-pre super open (briefly, Fsp- super open) [16] if A ≤ Cl Int(Cl(A)) and a fuzzy semi-pre super closed (briefly, Fsp- super closed) if IntCl(Int(A)) ≤ A. By FSPO (X, τ), we denote the family of all fuzzy semi-pre super open sets of fts X.

The semi closure [18] (resp α- super closure semi-pre super closure of a fuzzy set A of (X, τ) is the intersection of all Fs- super closed (resp. Fα- super closed, Fsp- super closed) sets that contain A and is denoted by sCl(A) (resp. α Cl(A) and spCl(A)).

Definition 1. 7. [10, 11, 12]: A fuzzy set A of (X, τ) is called:
- Fuzzy generalized super closed (briefly, Fg- super closed) if ClA ≤ H, whenever A ≤ H and H is fuzzy super open set in X;
- Generalized fuzzy semi super closed (briefly, gFs- super closed) [4] if sCl(A) ≤ H, whenever A ≤ H and H is Fs- super open set in X.
- Fuzzy generalized semi super closed (briefly, Fgs- super closed) [13] if sCl(A) ≤ H, whenever A ≤ H and H is fuzzy super open set in X;
- Fuzzy α generalized super closed (briefly, Fαg- super closed) if α Cl(A) ≤ H, whenever A ≤ H and H is fuzzy super open set in X;
- Fuzzy generalized α- super closed (briefly, Fg- super closed) if α Cl(A) ≤ H, whenever A ≤ H and H is Fα- super open set in X;
- Fuzzy generalized semi-pre super closed (briefly, Fgsp- super closed) if spCl(A) ≤ H, whenever A ≤ H and H is fuzzy super open set in X.

Definition 1. 8[10, 11, 12]. : A mapping f: (X, τ) → (Y, σ) is said to be:
- Fs- super continuous if f⁻¹(V) is Fs- super open in X, for each fuzzy super
open set $V$ in $Y$.
- Fuzzy- irresolute if $f^{-1}(V)$ is Fs- super open in $X$, for each Fs- super open set $V$ in $Y$.
- Fp- super continuous if $f^{-1}(V)$ is Fp- super open in $X$, for each fuzzy super open set $V$ in $Y$.
- $\alpha$- super continuous if $f^{-1}(V)$ is $\alpha$- super open in $X$, for each fuzzy super open set $V$ in $Y$.
- gFs- super continuous if $f^{-1}(V)$ is gFs- super closed in $X$, for each fuzzy super closed set $V$ in $Y$.
- Fgs- super continuous if $f^{-1}(V)$ is Fgs- super closed in $X$, for each fuzzy super closed set $V$ in $Y$.
- Fsp- super continuous if $f^{-1}(V)$ is Fsp- super open in $X$, for each fuzzy super open set $V$ in $Y$.
- Fuzzy M-semi pre super continuous if $f^{-1}(V)$ is Fsp- super open in $X$, for each Fsp super open set $V$ in $Y$.
- Fgs-irresolute if $f^{-1}(V)$ is Fgs- super closed in $X$, for every fuzzy super closed set $V$ in $Y$.
- Fsp-irresolute if $f^{-1}(V)$ is Fsp- super closed in $X$, for every Fsp super closed set $V$ in $Y$.
- Fuzzy M-semi-pre super closed if $f(V)$ is Fsp- super closed set in $Y$, for every Fsp super closed set $V$ in $Y$.

**Definition 1.** 9[10, 11, 12]. : A fuzzy point $x_p \in A$ is said to be quasi-coincident with the fuzzy set $A$ denoted by $x_p q A$ iff $p + A(x) > 1$. A fuzzy set $A$ is quasi-coincident with a fuzzy set $B$ denoted by $A q B$ iff there exists $x \in X$ such that $A(x) + B(x) > 1$. If $A$ and $B$ are not quasi-coincident then we write $A q B$. Note that $A \leq B$, $A q (\neg B)$.

**Definition 1.** 10[10, 11, 12]. : A fuzzy topological space $(X, \tau)$ is said to be fuzzy semi connected (briefly, Fs-connected) iff the only fuzzy sets which are both Fs- super open and Fs- super closed sets are 0 and 1.

**Definition 1.** 11. [6, 10, 11, 12]: Let $f$ be a mapping from $X$ into $Y$. If $A$ is a fuzzy set of $X$ and $B$ is a fuzzy set of $Y$ then
- $f(A)$ is a fuzzy set of $Y$, where $f(A) = (\sup x 2 f^{-1}(y) A(x), if f^{-1}(y) \neq 0 0, otherwise for every y \in Y$.
- $f^{-1}(B)$ is fuzzy set of $X$, where $f^{-1}(B)(x) = B(f(x))$ for each $x \in X$.
- $f^{-1}(\neg B) = \neg f^{-1}(B)$.

**2. Fspg- super continuous and Fspg-irresolute mappings**

**Definition 2.** 1. : A mapping $f: (X, \tau) \to (Y, \sigma)$ is called fuzzy semi-pre-generalized super continuous (briefly, Fspg-continuous) if $f^{-1}(V)$ is Fspg- super closed in $(X, \tau)$ for every fuzzy super closed set $V$ of $(Y, \sigma)$. 
Definition 2.2: A mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) is called fuzzy semi-pre-generalized irresolute (briefly, Fspg-irresolute) if \( f^{-1}(V) \) is Fspg-super closed in \( (X, \tau) \) for every Fspg-super closed set \( V \) of \( (Y, \sigma) \).

Theorem 2.1: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be gFs-super continuous. Then \( f \) is Fspg-super continuous.

Proof. - Let \( V \) be a fuzzy super closed set of \( Y \). Since \( f \) is gFs-super continuous, then \( f^{-1}(V) \) is gFs-super closed in \( X \). Since every gFs-super closed set is Fspg-super closed, then \( f^{-1}(V) \) is Fspg-super closed. Thus, \( f \) is Fspg-super continuous.

Lemma 2.1: The converse of the above theorem is not true in general. For,

Example 2.1: Let \( X = \{a, b, c\} \), \( Y = \{x, y, z\} \). Fuzzy sets \( A \) and \( B \) are defined as:
\[
A(a) = 0.1, \quad A(b) = 0.2, \quad A(c) = 0.7; \quad B(x) = 0.1, \quad B(y) = 0.8, \quad B(z) = 0.5.
\]
Let \( \tau = \{0, A, 1\} \) and \( \sigma = \{0, B, 1\} \). Then the mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) defined by \( f(a) = x, \ f(b) = y \) and \( f(c) = z \) is Fspg-super continuous but not gFs-super continuous.

Theorem 2.2: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be Fspg-irresolute, then \( f \) is Fspg-super continuous.

Proof. Proof is immediate as every fuzzy super closed set is Fspg-super closed and \( f \) is Fspg-irresolute map.

Observation 2.2: The converse of the above theorem is not true in general as it can be seen from the following example.

Example 2.2: Let \( X = \{a, b\}, \ Y = \{x, y\} \). The fuzzy set \( A \) is defined as: \( A(a) = 0.3, \ A(b) = 0.7 \). Let \( \tau = \{0, A, 1\} \) and \( \tau = \{0, 1\} \). Then the mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) defined by \( f(a) = x \) and \( f(b) = y \) is Fspg-super continuous but not Fspg-irresolute.

Theorem 2.3: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be Fspg-super continuous. Then \( f \) is Fgsp-super continuous but not conversely.

Proof. Let \( V \) be a fuzzy super closed set of \( Y \). Since \( f \) is Fspg-super continuous, then \( f^{-1}(V) \) is a Fspg-super closed set of \( X \). Since every Fspg-super closed set is Fgsp-super closed, \( f^{-1}(V) \) is also a Fgsp-super closed set of \( X \). Thus, \( f \) is Fgsp-super continuous.

Following example shows that the converse is not true in general:

Example 2.3: Let \( X = \{a, b\}, \ Y = \{x, y\} \). Fuzzy sets \( A \) and \( B \) are defined as:
\[
A(a) = 0.3, \ A(b) = 0.7; \quad B(x) = 0.3, \ B(y) = 0.4.
\]
Let \( \tau = \{0, A, 1\} \) and \( \sigma = \{0, B, 1\} \). Then the mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) defined
by \( f(a) = x \) and \( f(b) = y \) is Fgsp- super continuous but not Fspg- super continuous.

**Theorem 2.4.** Let \( f: (X, \tau) \to (Y, \sigma) \) be Fp- super continuous, then \( f \) is Fspg- super continuous. The following example shows that the converse of the above theorem is not true in general:

**Example 2.4.** Let \( X = \{a, b\} \), \( Y = \{x, y\} \). Fuzzy sets \( A \) and \( B \) are defined as:
\[
A(a) = 0.3, \ A(b) = 0.4; \quad B(x) = 0.6, \ B(y) = 0.5.
\]
Let \( \tau = \{0, A, 1\} \) and \( \sigma = \{0, B, 1\} \). Then the mapping \( f: (X, \tau) \to (Y, \sigma) \) defined by \( f(a) = x \) and \( f(b) = y \) is Fspg- super continuous but not Fp- super continuous. Every Fs- super continuous function is gFs- super continuous but not conversely.

**Theorem 2.5.** Let \( f: (X, \tau) \to (Y, \sigma) \) be Fgs- super continuous. Then \( f \) is Fgsp super continuous but not conversely.

**Proof:** Obvious

**Theorem 2.6.** A mapping \( f: (X, \tau) \to (Y, \sigma) \) is Fspg- super continuous iff inverse image of each fuzzy super open set of \( Y \) is Fspg- super open in \( X \).

**Proof:** It is obvious because \( f^{-1}(1-H) = 1-f^{-1}(H) \) for each fuzzy open set \( H \) of \( Y \).

**Theorem 2.7.** If \( f: (X, \tau) \to (Y, \sigma) \) is Fspg- super continuous then for each fuzzy point \( x_p \) of \( X \) and each \( A \in \sigma \) such that \( f(x_p) \in A \), there exists a Fspg- super open set \( B \) of \( X \) such that \( x_p \in B \) and \( f(B) \leq A \).

**Proof:** Let \( x_p \) be a fuzzy point of \( X \) and \( A \in \sigma \) such that \( f(x_p) \in A \). Put \( B = f^{-1}(A) \). Then by hypothesis \( B \) is a Fspg- super open set of \( X \) such that \( x_p \in B \) and \( f(B) = f(f^{-1}(A)) \leq A \).

**Theorem 2.8.** Let \( f: (X, \tau) \to (Y, \sigma) \) is Fspg- super continuous, then for each fuzzy point \( x_p \) of \( X \) and each \( A \in \sigma \) such that \( f(x_p)qA \), there exists a Fspg- super open set \( B \) of \( X \) such that \( x_pqB \) and \( f(B) \leq A \).

**Proof:** Let \( x_p \in X \) and \( A \in \sigma \) such that \( f(x_p)qA \). Put \( B = f^{-1}(A) \). Then by hypothesis \( B \) is a Fspg- super open set of \( X \) such that \( xpqB \) and \( f(B) = f(f^{-1}(A)) \leq A \). Recall that a fuzzy topological space \((X, \tau)\) is fuzzy T\(_{1/2}\) -space if every Fs- super closed set in \( X \) is fuzzy closed.

**Theorem 2.9.** If \( f: (X, \tau) \to (Y, \sigma) \) is Fspg- super continuous and \( g: (Y, \sigma) \to (Z, \gamma) \) is Fs-continuous and \( Y \) is a fuzzy T\(_{1/2}\) -space. Then \( g \circ f: (X, \tau) \to (Z, \gamma) \) will be Fspg- super continuous.

**Proof:** Let \( A \) is a fuzzy super closed set in \( Z \), then \( g^{-1}(A) \) is Fs- super closed in \( Y \).
Since $Y$ is a fuzzy T1/2-space, $g^{-1}(A)$ is F$_g$- super closed in $Y$ implies $f^{-1}(g^{-1}(A))$ is a F$_{spg}$- super closed set in $X$. Thus, $g \circ f$ is F$_{spg}$- super continuous.

**Definition 2.3.** If every F$_{spg}$- super closed set in $X$ is F$_{sp}$- super closed in $X$, then the space can be denoted as F$_{sp}$ T1/2-space. Next, we prove the following:

**Theorem 2.10.** A fuzzy topological space $(X, \tau)$ is F$_{sp}$ T1/2-space iff $FSPO(X, \tau) = FSPGO(X, \tau)$.

**Proof. (Necessity):** Let $(X, \tau)$ be F$_{sp}$ T1/2-space. Let $A \in FSPGO(X, \tau)$. Then, $1-A$ is a F$_{spg}$- super closed. By hypothesis, $1-A$ is a F$_{sp}$- super closed set and thus $A \in FSPO(X, \tau)$. Hence, $FSPO(X, \tau) = FSPGO(X, \tau)$.

**Sufficiency:** Let $FSPO(X, \tau) = FSPGO(X, \tau)$. Let $A$ is a F$_{spg}$- super closed. Then, $1-A$ is a F$_{spg}$- super open. Hence, $1-A \in FSPO(X, \tau)$. Thus, $A$ is a F$_{sp}$- super closed set. Therefore, $(X, \tau)$ is a F$_{sp}$ T1/2-space.

**Theorem 2.11.** Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \gamma)$ be any two functions. Then;

- $g \circ f: (X, \tau) \rightarrow (Z, \gamma)$ is F$_{spg}$- super continuous, if $g$ is fuzzy super continuous and $f$ is F$_{spg}$- super continuous.
- $g \circ f$ is F$_{spg}$-irresolute, if $f$ and $g$ both are F$_{spg}$-irresolute.
- $g \circ f$ is F$_{spg}$- super continuous, if $g$ is F$_{spg}$- super continuous and $f$ is F$_{spg}$-irresolute.
- Let $Y$ be a F$_{sp}$ T1/2-space. Then, $g \circ f$ is F$_{spg}$- super continuous, if $g$ is F$_{spg}$ super continuous and $f$ is fuzzy M-semi-pre- super continuous.

**Proof.** Obvious.

**Theorem 2.12.** Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be F$_{spg}$- super continuous. Then $f$ is fuzzy semipro super continuous if $(X, \tau)$ is F$_{sp}$ T1/2-space.

**Proof.** Let $V$ be a fuzzy super closed set of $Y$. Since $f$ is F$_{spg}$- super continuous, $f^{-1}(V)$ is F$_{spg}$ super closed set of $X$. Again, $X$ is F$_{sp}$ T1/2-space and hence $f^{-1}(V)$ is F$_{sp}$- super closed set of $X$. This implies that $f$ is fuzzy semi-pre super continuous.

**Theorem 2.13.** Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be fuzzy irresolute and fuzzy M-semi-pre super closed. Then for every F$_{spg}$- super closed set $A$ of $X$, $f(A)$ is a F$_{spg}$- super closed in $Y$.

**Proof.** Let $A$ be a F$_{spg}$- super closed set of $X$. Let $V$ be a fuzzy semi super open set of $Y$ containing $f(A)$. Since $f$ is fuzzy irresolute, $f^{-1}(V)$ is a fuzzy semi super open set of $X$. As $A \leq f^{-1}(V)$ and $A$ is a F$_{spg}$- super closed in $X$, then $spCl(A) \leq f^{-1}(V)$ implies that $f(spCl(A)) \leq V$. Since $f$ is fuzzy M-semi-pre super closed, then $f(spCl(A)) = f(A)$. Since $f(A)$ is a F$_{spg}$- super closed set in $Y$ containing $f(A)$, $f(A)$ is a F$_{spg}$- super closed set in $Y$. Thus, $f$ is fuzzy M-semi-pre super continuous.
spCl(f(spCl(A))). Then, spCl(f(A)) ≤ spCl(f(spCl(A))) = f(spCl(A)) ≤ V. Therefore, f(A) is a Fspg- super closed set in Y.

**Theorem 2.14.:** Let f: (X, τ) → (Y, σ) be onto Fspg- irresolute and fuzzy M-semi pre super closed. If X is Fsp T1/2-space, then (Y, σ) is also Fsp T1/2-space.

**Proof.** Let A be a Fspg- super closed set of Y. Since f is Fspg- irresolute, then f⁻¹(A) is Fspg- super closed set in X. As X is a Fsp T1/2-space and hence f⁻¹(A) is Fsp- super closed in X. Again, f is a fuzzy M-semi-pre super closed map, f(f⁻¹(A)) is a Fsp- super closed set in Y. Since f is onto, f(f⁻¹(A)) = A. Thus, A is a Fsp- super closed set in Y or equivalently, (Y, σ) is Fsp T1/2-space. 

**Theorem 2.15.:** If the bijective mapping f: (X, τ) → (Y, σ) is fuzzy pre-semi- super open and fuzzy M-semi-pre- super continuous, then f is Fspg-irresolute. On Fuzzy Semi-Pre-Generalized super Closed Sets.

**Proof.** Let V be a Fspg- super closed set in Y and let f⁻¹(V) ≤ H where H is a fuzzy semi super open set in X. Clearly, V ≤ f(H). Since f is a fuzzy pre-semi- super open map, f(H) is a fuzzy semi super open set in Y and V is a Fspg- super closed set in Y then spCl(V) ≤ f(H) and thus f⁻¹(spCl(V)) ≤ H. Again, f is a fuzzy M-semi-pre- super continuous, and spCl(V) is Fsp- super closed set, then f⁻¹(spCl(V)) is a Fsp- super closed set in X. Thus, spCl(f⁻¹(V)) ≤ spCl(f⁻¹(spCl(V)) = f⁻¹(spCl(V)) ≤ H. So f⁻¹(V) is a Fspg- super closed set in X. Hence, f is Fspg-irresolute map.

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