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Study of precontinuous Mappings in Bitopological space

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Abstract

In this paper, the concept of bitopological precontinuous mapping, algebra of pair wise continuous mapping, pair wise are investigated and discussed several characterization and properties of pair wise precontinuous mapping in bitoplogical space.

Keywords: Bitopological spaces, mappings.

1. INTRODUCTION

Levine (1960) introduced the concept of strongly continuous mappings in topological spaces. These mappings were also considred by Cullen (1961). J. C. Kelly (1963) introduced the study of bitopological space. A non-empty set X equipped with two topologies τ_1 and τ_2 called a bitopological spaces. The concept of continuity in topological spaces was extended to bitopological spaces by Pervin (1967). A function f: $(X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is said to be pairwise continuous (resp. pairwise open, pairwise closed, a pairwise (homeomorphism) if the induced function f: $(X, \tau_1) \rightarrow (Y, \mu_1)$ and f: $(X, \tau_2) \rightarrow (Y, \mu_2)$ are continuous (resp. open, closed, homeomorphism). It has been found true by various authors that several properties which are preserved by continuous mappings remain inverant under much less restrictive types of mappings in topological spaces. A detailed study of these mappings was done by Arya and Gupta (1974). K. C. Rao and S, M. Felix (1992) introduced the concept of pairwise strongly continuous mappings and studied their properties. They extended some results pertaining to topological spaces (with single topology) to bitopological spaces. After the introduction of the definition of a bitopological space a large number of topologists have turned their attention to the generalization of different concepts of a single topological space in this space. A. S. Mashhour et al introduced preopen sets, precontinuous functions and preopen mappings in a single topological space and obtained a number of their properties. A. Kar and P. Bhattacharyya (1992) generalize these notions of mashhour et al in bitoopological spaces. These notions have also been studied by M. Jelic (1991)

2. PRELIMINARY

Definition-2. 1. A function f: $(X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is called pair wise semi continuous (resp. pair wise α - continuous, pairwise continuous) if for each μ_i - open set $V \subset Y$ $f^{-1}(V)$ is an (i, j) – semi open set (resp. (i, j)- α - set, τ_j -open set) in X, for $i \neq j$; i, j = 1, 2.

Definition-2. 2. A mapping f: $(X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is called a pairwise weakly continuous mapping if for each point p in X and each μ_i-open set H containing f (p), there exists a τ_ι- open set G containing p such that $f(G) \subset \mu_i$ - cl (H) for $i \neq j$, i = 1, 2, and j = 1, 2.

Definition-2. 3. A subset A is said to be (i, j)-semiopen if there exists an open set U of X such that $U \subset A \subset cl(U)$.

Definition-2. 4. A mapping f: $(X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ termed (i, j)-semi-continuous iff for $O \in \mu_i$, $f^{-1}[O]$ is (i, j)-semi open in X for $i \neq j$ and i, j = 1, 2. f is bitopologically semicontinuous if f is (i, j)-semi- continuous for $i \neq j$ and i, j = 1, 2.

Definition-2. 5. In bitopological space (X, τ_1, τ_2), $A \subset X$ is said to be (i, j)- α -open iff $A \subset \tau_\iota$ -int (τ_j– cl(τ_ι–int A))for $i \neq j$ and i, j = 1, 2.

Definition-2. 6. In a bitopological space (X, τ_1, τ_2), a net $\{x_\alpha, \alpha \in D, \geq\}$ is said to be converge to a point $x \in X$, denoted by $\{x_\alpha, \alpha \in D, \geq\} \rightarrow x$, if the net is eventually in every τ_ι-neighbourhood of x, i= 1, 2.

3. BITOPOLOGICAL PRECONTINUITY:

In this section the concept of bitopological precontinuity and Characterizations of (i, j) - precontinuous mapping have been studied.

Definition-3. 1. A mapping f: $(X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is termed (i, j) - precontinuous iff for $O \in \mu_i$, $f^{-1}[O]$ is (i, j) -preopen in X, for $i \neq j$ and i, j = 1, 2.

Definition-3. 2. A mapping f: $(X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is Called bitopologically precontinuous iff f is (i, j) – precontinuous for $i \neq j$ and i, j = 1, 2.

Remark 3. 3. If f: $(x, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ ispairwise continuous in the sence of Pervin (1967)then f is obviously bitopologically precontinuous. But the converse is not always true as shown by example 3. 4.

Example 3. 4. Let X=Y={a, b, c, d}, τ_1={ϕ, X, {a}, {b, c}, {a, b, c}} and τ_2={ϕ, X, {a, d}} μ_1={ϕ, X, {a}, {b}, {a, b}} and μ_2={ϕ, X, {a}, {a, c}, {a, d}, {a, c, d}} and f: (X, τ_1, τ_2)→(Y, μ_1, μ_2) be the identity mapping. Then f is bitopologically precontinuous. On the other hand, f^{-1} [{b}]={b} $\notin \tau_1$ for {b}$\in \mu_1$, and so f is not pairwise continuous in the sense Pervin (1967).

Remark 3. 5. The notion of bitopologically precontinuous is not equivalent to pre continuous in individual topological spaces as shown by examples 3. 6. and 3. 7. below.

Example 3. 6. Let (X, τ_1, τ_2) be the bitopological space X= {a, b, c, d} τ_1={ϕ, X, {b}, {a, b}, {b, c}, {a, b, c}} and τ_2={ϕ, X, {d}, {a, d}, {c, d}, {a, c, d}}. Let μ_1={ϕ, Y, {c}} and μ_2={ϕ, Y, {a}} where X=Y if f: (X, τ_1, τ_2)→(Y, μ_1, μ_2) be the identity mapping, then f is bitopologically precontinuous. On the other hand, f^{-1} [{c}] is not precontinuous in (X, τ_1) for {c}$\in \mu_1$. So f is not precontinuous on (X, τ_1). Similar remark applies to{a}$\in \mu_2$ and hence f is not pre continuous on (X, τ_2).

Example 3. 7:. Let X=Y={a, b, c, d} τ_1= {ϕ, X, {a, d}, {b, c}} τ_2={ϕ, X, {b, d}, {a, c}} and μ_1={ϕ, X, {b}, {a, b}, {b, c}, {a, b, c}} μ_2 = {ϕ, X, {b}} If f: (X, τ_1, τ_2)→(Y, μ_1, μ_2) is the identity mapping. Then f is precontinuous on both (X, τ_1) and (X, τ_2). But since f^{-1}[{c}] is not (1, 2)-preopen. For {c}$\in \mu_1$, f is not (1, 2)-precontinuous. Similar remark applies to {b}$\in \mu_2$ and hence f is not (2, l)-precontinuous. So f is not bitopologically precotinuous.

Remark 3. 8. The notion of bitopologically precontinuous is independent of that of bitopologically semicontinuous. This can be seen from examples 3. 9 and 3. 10 below.

Example 3. 9. In example 3. 6. f is bitopologically precontinuous but it is neither (1, 2)-semicontinuous nor (2, l)-semicontinuous and hence not bitopological semicontinuous.

Example 3. 10. Let X=Y={a, b, c, d} τ_1={ϕ, X, {a}, {b}, {a, b}} τ_2={ϕ, X, {a}, {b}, {a, b}, {a, c}, {a, b, c}} μ_1={ϕ, Y, {a}, {a, c}} and μ_2={ϕ, Y, {b}, {b, d}} if f: (X, τ_1, τ_2)→(Y, μ_1, μ_2)is the identity mapping then f is bitopologically semi continuous shown by **Bose.** On the other hand f is not bitopologically precontinuous.

Characterizations for (i, j)- precontinuous are given in the following theorem.

Theorem 3. 11. Let f: (X, τ_1, τ_2)→(Y, μ_1, μ_2) then the following statements are equivalent:

(a) f is (i, j)- precontinuous.

(b) For each x$\in$X and each net {x_α, $\alpha \in$ D, ≥} converging to x, the image net {f (x_α), $\alpha\in$D, ≥} is eventually in every μ_i - neighbourhood of f (x), whose inverse is τ_j-closed in X.

(c) For each $x \in X$ and each μ_i -open set U^* containing f (x), there exists an (i, j) - preopen set $U \subset X$ such that $x \in U$ and $f [U] \subset U^*$.

(d) The inverse image of each μ_i - closed set in Y is (i, j) - preclosed in X.

(e) For each $A \subset X$, $f [\tau_i\text{-cl} (\tau_j\text{-int } A)] \subset \mu_i\text{-cl} (f [A])$

(f) For each $A^* \subset Y$, $\tau_i\text{-cl} (\tau_j\text{-int} (f^{-1}[A^*])) \subset f^{-1}[\mu_i\text{-cl } A^*]$

Proof:

(a) implies (b): Let $x \in X$ and let M be any μ_i-neighbourhood of f (x) such that f^{-1} [M] is τ_i -closed. Then there exists a μ_i-open set U^* such that $f (x) \in U^* \subset M$. So $x \in f^{-1} [U^*] \subset f^{-1} [M]$. By (a) $f^{-1} [U^*]$ is (i, j) - preopen. Hence $x \in f^{-1} [U^*] \subset \tau_i\text{-int} (\tau_j\text{-cl} (f^{-1} [U^*]))$. Since closure and interior respect inclusion it follows from above that $x \in f^{-1} [U^*] \subset \tau_i \text{-int} (\tau_i\text{-cl} (f^{-1} [U^*]) \subset \tau_i \text{-int} (\tau_j\text{-cl} (f^{-1} [M]))$. If $\{\mathbf{x}_\alpha, \alpha \in D, \geq\} \rightarrow x$ then $\{\mathbf{x}_\alpha, \alpha \in D, \geq\}$ is eventually in τ_i -int $(\tau_j\text{-cl} (f^{-1} [M]))$. This implies that there exists $\alpha_o \in D$ such that for $\alpha \geq \alpha_o$, $f (x_\alpha) \in f [\tau_i \text{-int} (\tau_j\text{-cl} (f^{-1} [M]))] \subset f [\tau_j\text{-cl}[f^{-1} (M)]] \subset f f^{-1} [M]$ (Since f^{-1} [M] is τ_j – closed) $\subset$ M. Hence $f (x_\alpha)$ is eventually in M.

(b) implies (c): Let $x \in X$ and U^* be any μ_i-open set containing f (x). If possible, suppose (c) is not true. Then $f [U] \subset U^*$ for any (i, j)- preopen set U containing x. Then $f [U] \cap (y - U^*) \neq \phi$ so that $U \cap f^{-1} (Y - U^*) \neq \phi$ ….. (1). Now, let N (x) be the family of all τ_i - neighbourhoods of x. Then each $N \in N (x)$ is τ_i-open and so (i, j) - preopen therefore by (1) $N \cap f^{-1} (Y - U^*) \neq \phi$ for all $N \in N (x)$ let $x_N \in N \cap f^{-1} (Y - U^*)$. Now $\{x_N, N \in N (x), \leq\}$ is a net in X, converging to x, and $x_N \in f^{-1} (Y - U^*)$ for all $N \in N (x)$. So $f (x_N) \in (Y - U^*)$, i. e., $f (x_N) \notin U^*$ for all $N \in N (x)$. Therefore the image net $\{f (x_N), N \in N (x), \leq\}$ is not eventually in U^*, which contradicts (b). So (c) must be true,

(c) implies (d): Let $U^* \subset Y$ be μ_i-closed. Let $x \in X - f^{-1} [U^*]$ Then$f (x) \in f [X- f^{-1} [U^*]] \subset Y - U^*$ where $Y-U^*$ is μ_i-open. So by (c), there exists an (i, j) - preopen set $U_x \subset X$ such that $x \in U_x$ and $f [U_x] \subset Y - U^*$. Hence $x \in U_x \subset f^{-1} f [U_x] \subset f^{-1} [Y - U^*] = X - f^{-1} [U^*]$ so that $X- f^{-1} [U^*] = U \{U_x: x \in X- f^{-1} [U^*]\}$ whence by theorem 2. 3. 2. $X- f^{-1} [U^*]$ is (i, j) - preopen i. e., $f^{-1} [U^*]$ is (i, j) - preclosed in X.

(d). implies (e): For any $A \subset X$, $A \subset f^{-1} [f [A]] \subset f^{-1} [\mu_i\text{-cl} (f [A])]$ where $f^{-1} [\mu_i \text{- cl} (f [A])]$ is (i, j) - preclosed by (d). Since closure and interior respect inclusion it is clear that $\tau_i \text{- cl} (\tau_j \text{- int } A) \subset \tau_i \text{-cl} (\tau_j\text{-int} (f^{-1}[\mu_i \text{-cl} (f [A])]) \subset f^{-1}[\mu_i\text{-cl} (f [A])]$ (by (3) of theorem 2. 3. 7.). This implies that $f [\tau_i\text{-cl} (\tau_j\text{-int } A) \subset f[f^{-1} [\mu_i\text{-cl} (f [A])]] \subset \mu_i\text{-cl} (f [A])$.

(e) implies (f): For any $A^* \subset Y$, let $A = f^{-1} [A^*]$. Then $A \subset X$ and so by (e) $f[\tau_i\text{-cl} (\tau_j – \text{int } A)] \subset \mu_i \text{- cl} (f [A]$. Hence $\tau_i\text{-cl} (\tau_j – \text{int } A) \subset f^{-1} f [\tau_i\text{-cl} (\tau_j\text{-int } A)] \subset f^{-1} [\mu_i\text{- cl} (f [A])]$, i. e., $\tau_i \text{-cl} (\tau_j – \text{int} (f^{-1} [A^*])) \subset f^{-1} [\mu_i –\text{cl} (f f^{-1} [A^*])] \subset f^{-1} [\mu_i\text{-cl } A]$.

(f) implies (a): $B^* \subset Y$, be μ_i-open, then $A^* = Y - B^*$ is τ_i-closed and $f^{-1}[A^*] = X - f^{-1}[B^*]$ whence $X - f^{-1}[A^*] = f^{-1}[B^*]$. Now by (f) τ_i-cl (τ_j-int ($f^{-1}[A^*]$))$\subset f^{-1}[\mu_i -$ cl $A^*] = f^{-1}[A^*]$. i. e., X- (τ_i-cl (X- $f^{-1}[A^*]$)) (by relations connecting complimation, closure and interior operator) $\supset f^{-1}[A^*]$, i. e.τ_i int (τ_j – cl ($f^{-1}[B^*]$))$\supset f^{-1}[B^*]$ that $f^{-1}[B^*]$ is (i, j) - preopen. Hence f is (i, j) precontinuous.

Definition 3. 12: In (X, τ_1, τ_2), a net $\{x_\alpha, \alpha \in D, \geq\}$ is said to be bitopologically preconverges to a point $x \in X$, denoted by $\{\mathbf{x}_\alpha, \alpha \in D, \geq\} \xrightarrow{P} x$ if the net is eventually in every (i, j) – preopen set containing x for $i \neq j$ and i, j =1, 2. (i, j) - pre continuous establishes an interesting relation between precon-vergence of a net and convergence of its image net. In fact we have

Theorem 3. 13: If a mapping f: (X, τ_1, τ_2) $\rightarrow$(Y, μ_1, μ_2) is precontinuous. Then for each $x \in X$ and each net $\{\mathbf{x}_\alpha, \alpha \in D, \geq\}$ in X preconverging to x, the image net $\{f(x_\alpha), \alpha \in D, \geq\}$ converges to f (x).

Proof: Let f be (i, j) - precontinuous. Let $x \in X$ and $\{x_\alpha, \alpha \in D, \geq\}$ be A net in X such that $\{\mathbf{x}_\alpha, \alpha \in D, \geq\} \rightarrow x$. Let U* be any, μ_i–open neighbourhood of f (x), i = 1, 2. Then $f^{-1}[U^*]$ is an (i, j) - preopen set containing x. Since $\{\mathbf{x}_\alpha, \alpha \in D, \geq\} \rightarrow x$. there exists $\alpha_0 \in D$ such that $x_\alpha \in f^{-1}[U^*]$ for all $\alpha \geq \alpha_0$. Therefore $f(x_\alpha) \in f\, f^{-1}[U^*] \subset U^*$ for all $\alpha \geq \alpha_0$. Hence the image net $\{f(x_\alpha), \alpha \in D, \geq\} \rightarrow f(x)$.

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