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# Generalisation of Linear Equation for Estimating Hypotenuse for all Pythagorean Triples

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**Abstract**

Hypotenuse of a right angle triangle is generally estimated by using the Pythagorus Theorem with a quadratic equation. Here an alternative method of its estimation by a linear equation is described. Earlier it was established for some particular categories of Pythagorean triples. Here it has been established for all such triples. There is some pattern in the coefficients of the linear equation.

## INTRODUCTION

Generally the hypotenuse C of a right angle triangle is calculated by Diophantine equation

$$C^2 = A^2 + \mathrm{B}^2 \qquad \ldots(1)$$

where A and B are the two sides having a right angle between them.

Here to find the value of C, one has to take the square root of the right hand side. Recently two papers by Pal (2022) describe how C can be calculated by a linear equation of the form

$$C = Ax + By \qquad \ldots(2)$$

where coefficients x and y depend on the values of A and B. These linear equations were generated for particular categories of Pythagorean triples, but not for all triples.

Here we give the general form of these coefficients x and y which can be applied for all Pythagorean triples.

Since long Pythagorean triples generation methods are well known. In general, these triples are generated by Euclid's formula, which can be stated as follows. For positive integers, m > n,

$A = m^2 - n^2$, $B = 2mn$, and $C = m^2 + n^2$ will form one Pythagorean triple. The same method was applied by Pal (2022) in his earlier papers to generate different categories of triples. This classical formula can generate primitive triples, but it does not generate all triples, particularly not all non-primitive triples. By generating triples by the above formula, the difference between C and A will be $2n^2$ . Based on the fact that the difference between C and A or B will have some particular values, recently a paper by Roy and Farjana ( 2012) described a direct method which can generate all Pythagorean triples. Here we are generating the coefficients x and y based on the difference between C and A. These coefficients will also be valid for earlier categories of triples mentioned in Pal (2022).

**General coefficients for Linear Method**

Roy et al (2012) mentioned that the difference between C and B or A can have only certain distinct values depending on the given number A or B. They have constructed the triples based on the difference between C and B and they have given several examples of primitive and non-primitive triples. Here we are considering those triples with interchange of A and B, so that the coefficients x and y in equation (2) will depend on the difference between C and A. Here it may be noticed that by interchanging A and B in equation (1), there will be no change in the value of C.

Let us denote $z = C - A$ . It may be noted here that in category I of triples mentioned by Pal (2022), the value of z is 2, whereas in category II of triples the value of z is 1. In another paper of Pal (2022), the coefficients have been constructed for more general categories of triples where z has different values. For details of the method for generation of all triples for a particular value of A and different possible values of z , one can refer to the paper by Roy and Farjana (2012). Here we shall only look at a few examples mentioned by Roy and Farjana (2012). A few examples of primitive triples are (1855, 792, 2017), (1175, 792, 1417) and (9785, 792, 9817) and examples of non-primitive triples are (448, 60, 452), (297, 60, 303) and (175, 60, 185). In all these examples z has different values.

For generating coefficients x and y for equation (2), we follow the similar method as in the earlier papers by Pal (2022). Looking at the patterns of x and y for earlier categories of triples, we arrive at the following general coefficients x and y which depend only on A, B and z. The coefficients can be described as follows

$$x = \frac{A+B}{A+B+z} \quad , y_G = \frac{2z}{B+z} \qquad \ldots(3)$$

where $z = C - A$

Applying the above coefficients in equation (2), it may be verified for all the examples of triples mentioned above that the values of C obtained by equations (1) and (2) are same. It is valid for all Pythagorean triples which may be constructed by the direct method mentioned by Roy and Farjana (2012).

## DISCUSSION

Following the procedure of generating coefficients of linear equation for earlier categories of triples, here it is extended for all Pythagorean triples, constructed based on the difference of C and A or B. These coefficients can be verified for other primitive and non-primitive triples. These are also valid for earlier categories mentioned in papers by Pal (2022). Interesting to note that there is some general pattern in the coefficients of the linear equation (2) which can be used instead of finding the square root from equation (1). As it was mentioned earlier, coefficients remain same also for all real number triples obtained dividing integer triples by integer n.

## CONCLUSION

As the categories of triples in earlier papers by Pal (2022) did not cover all possible Pythagorean triples, here coefficients for linear equation have been generated for all triples. The triples were generated by an alternate method proposed by Roy et al (2012) and coefficients have been verified for a few examples generated by them. It can be verified for any Pythagorean triple, constructed by this method.

## REFERENCES

[1] Pradip Kumar Pal, (2022), Beauty of integers created by nature., *International Journal of Mathematical Education*, V. 12, No.1, p. 1-5.

[2] Pradip Kumar Pal, (2022), Linearisation of Diophantine equation for Pythagorean triples, *International Journal of Mathematical Education*, V. 12, No.1, p. 7-9.

[3] Tanay Roy, Farjana Jaishmin Sonia, (2012), A direct method to generate Pythagorean triples and its generalization to Pythagorean Quadrupoles and n-tuples. *https://www.researchgate.net/publication/51988608*