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Practical Application of Linear Form of Pythagorus Formula

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Abstract

Pythagorus formula is well known for estimating hypotenuse of a right angle triangle. It has been established earlier that there is some linear relationship, by which hypotenuse can be estimated for all Pythagorean triples of integers. This could not be used for practical purpose, because all three numbers are known. Here it has been shown that under certain conditions, for any given real number, if the difference between hypotenuse and the other side of the triangle is also given, then both other side and the hypotenuse can be estimated by this linear relationship without performing any square root operation. This is an alternative way of looking at Pythagorus formula.

INTRODUCTION

Estimation of the hypotenuse C of a right angle triangle is generally done by Pythagorus formula

$$C^2 = A^2 + \mathrm{B}^2 \qquad \ldots(1)$$

where A and B are the two sides having a right angle between them.

Hypotenuse C can be estimated by taking the square root of the right hand side of equation (1). Recently three papers by Pal (2022) describe how C can be calculated by a linear equation of the form

$$C = Ax + By \qquad \ldots(2)$$

where coefficients x and y depend on the values of A and B. These linear equations were generated for all Pythagorean triples of integers. It was mentioned that this relationship could not be used for any practical purpose, as all three values are known, once we construct these triples.

Here we describe how the linear relationship (2) can be applied in case of any Pythagorean triple of real numbers. Earlier papers by Pal (2022) have mentioned about various methods of generating Pythagorean triples of integers. Based on the fact that the difference between C and A or B will have some particular values, a paper by Roy and Farjana (2012) described a direct method which can generate all primitive and non-primitive Pythagorean triples. In the latest paper, Pal (2022) has generated the coefficients x and y based on the difference between C and A. These coefficients were also valid for earlier categories of triples mentioned in Pal (2022). Actually these coefficients will also be valid for any real number Pythagorean triples. This fact is used here for estimating two other sides of the triangle, when length of one side and the difference between two other sides are given.

PRACTICAL APPLICATION OF LINEAR METHOD

In the paper by Roy et al (2012), authors have mentioned that the difference between C and B or A can have only certain distinct values depending on the given number A or B. They have constructed the triples of integers based on the difference between C and B and they have given several examples of primitive and non-primitive triples. In the latest paper, Pal (2022) has generated the coefficients x and y for equation (2).

The coefficients can be described as follows

$$x = \frac{A+B}{A+B+z} \quad , y = \frac{2z}{B+z} \qquad \ldots(3)$$

where z = C - A

Now we consider the problem that if real positive numbers B and z are given, whether it is possible to construct the real number triple (A,B,C). As we have C = A + z, from equation (1) we get

$$A^2 + B^2 = (A + z)^2$$

$$B^2 = z(2A + z)$$

$$A = (B^2 - z^2)/2z \qquad \ldots(4)$$

Equation (4) implies the condition that $(B^2 – z^2)$ should be greater than zero, and so B should be greater than z. Once A is estimated by equation (4), C can be calculated by equation (2) using the coefficients x and y as described in (3).

Applying the above method, let us look at the following example. If B = 5.7 and z = 2.1 are given, then from (4) we get A = 6.685 and from (2) we get C = 8.785 . It may be mentioned that Pythagorus formula gives the same value of C for these A and B.

DISCUSSION

Though in earlier papers by Pal (2022) , it was mentioned that the linear form (2) could not be used for practical purpose, here we have described a method for practical application of the linear form of Pythagorus formula. Given the value of one side of a right angle triangle and the difference between the hypotenuse and the other side of the triangle, one can estimate both the other side and the hypotenuse without performing any square root operation. This is the major advantage over the quadratic Pythagorus formula.

This method may be verified for any given real positive number B and z with the condition that B > z . It may be noticed that C can also be calculated from the definition of z, which is also a linear method and more simple. The important thing here is that the coefficients x and y in the linear relationship (2) depend on A,B and z . With given B and z, both A and C can be estimated.

CONCLUSION

In earlier papers Pal (2022) have established a linear relationship between the numbers of all Pythagorean triples of integers, but it was mentioned that it could not be used for any practical purpose. The coefficients for the linear relationship have a general pattern and depended on the difference of the hypotenuse and one of the sides of the triangle. These coefficients were also valid for real number triples. Based on these facts, in this paper, it is shown that this linear method can be used for some practical purpose for estimating hypotenuse and one side, when the other side and the difference between hypotenuse and one side are given. This can be done without performing any square root operation. This is just an alternate way of looking at Pythagorus formula.

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