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Solution of Reynolds Equation for the Short Journal Bearings Rotation System

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Abstract

The initial and second order rotatory theory of fluid mechanics lubrication was based on the expressions obtained by holding the terms up to first and second powers of rotation number M within the extended generalized Reynolds equation of the classical Reynolds theory. Within this analysis, there are the derivations of the new equations for pressure under the consequences of second order rotation and their reductions into first order rotation of hydrodynamic lubrication. The expression for the exponential and logarithmic variation of the pressure with respect M is obtained. The comparative studies give some new excellent fundamental solutions with the help of geometrical figures, expressions, calculated table and graph for the short journal bearings for second order rotation. The analysis of equations for pressure, table, and graph analyzes that pressure increases with increasing values of the rotation number. The pressure is not independent of viscosity and varies with the viscosity of the fluid.

Keywords: Pressure, Reynolds equation, Rotation, Viscosity, Vorticity.

1. INTRODUCTION

1.1 Short Journal Bearing

In general the bearings [1], [2] is divided in to four categories:

(1) Dry bearings for example; plastic bushings, coated metal bushings etc.

(2) Fluid film bearings for example; shaft bearings etc.

(3) Semi-lubricated bearings for example; oil-impregnated bronze bushings etc.

(4) Rolling component bearings for example; ball, cylindrical, spherical or tapered roller and needle etc.

Except from some radial-configuration craft engines, the majority piston engines use fluid film bearings [3]. This can be true for the shaft and typically within the rotating shaft, though usually the later runs directly within the engine structure. Here we've got to debate the operating of the fluid film operating and to demonstrate however engine designers are reducing friction losses through bearing technology [4]. The fluid film bearings operate by generating, as a by-product of the relative motion between the shaft and also the bearing, a very thin film of lubricant at a sufficiently large pressure to match the applied load, as long as that load is among the bearing capability [5]. Fluid film bearings represent a type of scientific method, by virtue of providing terribly massive load carrying capabilities during a compact, light-weight implementation, and in contrast to the opposite categories, in most cases is designed for infinite life. The fluid film bearings operate in any of the three modes:

(a) Boundary

(b) Fully-hydrodynamic

(c) Mixed.

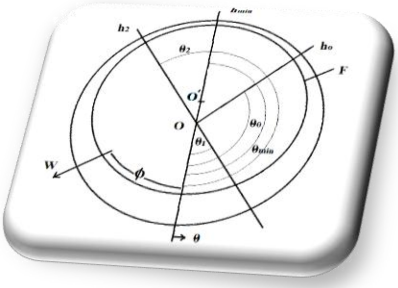

Figure-1 (Geometry of Journal Bearing)

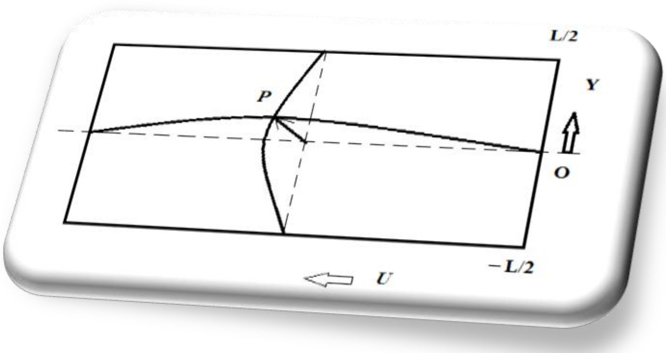

Figure-2 (Geometry of Short Journal Bearing)

In totally hydrodynamic or "full-film" lubrication, the moving surface of the journal is totally separated from the bearing surface by a really thin film of lubricants. The applied load causes the center line of the journal to be displaced from the center line of the bearing. This eccentricity creates a circular "wedge" within the clearance house. The stuff, by virtue of its body, clings to the surface of the rotating journal, and is drawn into the wedge, making a really pressure, that acts to separate the journal from the bearing to support the applied load. The bearing eccentricity is expressed because the center line displacement divided by the radial clearance. The bearing eccentricity will increase with applied load and reduces with larger journal speed and body. The hydrodynamic pressure has no relationship in the least to the engine pressure, except that if there's short engine pressure to deliver the specified copious volume of oil into the bearing, the hydraulics pressure mechanism can fail and therefore the bearing and journal are destroyed. The pressure distribution within the hydraulics region of a fluid film bearing will increase from quite low within the massive clearance zone to its most at the purpose of minimum film thickness for the incompressible fluid like oil is force into the convergence "wedge" zone of the bearing. However, this radial profile doesn't exist homogeneously across the axial length of the bearing. If the bearing has spare breadth, the profile can have a virtually flat from across the hard-hitting region. The second mode of bearing operation is boundary lubrication. In boundary lubrication, the "peaks" of the slippery surfaces i.e., journal and bearing, are touching one another, however there's conjointly an especially thin film of the lubricants solely some molecules thick that is found within the surface " wedge ". That thin film tends to cut back the friction from what it'd be if the surfaces were fully dry. The mixed mode could be a region of transition between boundary and full-film lubrication. The surface peaks on the journal and bearing surfaces part penetrate the fluid film and a few surface contact happens, however the hydraulics pressure is getting down to increase [4], [5].

1.2 Differential Equation for Hydrodynamic Lubrication Theory

The two dimensional classical theory [6] of fluid mechanics lubrication was 1st given by O. Reynolds. In 1886, within the wake of a classical experiment by Beauchamp Tower [7], he developed an equation celebrated as: Reynolds Equation. The formation and basic mechanism of fluid film was analyzed by that experiment by taking some assumptions that the film thickness is extremely tiny as compared to the axial and longitudinal dimensions of fluid film and if the lubricator layer is to transmit pressure between the shaft and therefore the bearing, the layer should have variable thickness.

Osborne Reynolds himself derived “Generalized Reynolds Equation” [6], which depends on density, film thickness, surface and transverse velocities. The equation originally derived by Reynolds was restricted to incompressible fluids, thus it had been developed generally enough to incorporate the results of compressibility and dynamic loading and was aforementioned to be Generalized Reynolds Equation. So the ultimate form of Generalized Reynolds Equation [5], [6] was as given:

$$\left(\frac{\rho h^3}{\mu}\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{\rho h^3}{\mu}\frac{\partial P}{\partial z}\right)=6(U_1-U_2)\frac{\partial(\rho h)}{\partial x}+12\rho V \quad (1)$$

Where ρ is that the density, μ is that the viciousness, h is that the film thickness of fluid film, U_1 and U_2 are the surface velocities and V is that the general velocity. Within the equation (1), the term was because of the bearing velocities on the lubricator film and depends on whether or not the bearing surfaces have angular or translational velocities, whereas the term was because of relative speed of bearing surfaces within the direction traditional to the fluid film. In most cases, the bearing is stationary and solely the runner in thrust bearings and also the shaft within the journal bearings are moving, therefore U_1=U and U_2=0. Currently the ultimate equation for incompressible lubricants was based by Reynolds is as given:

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{\mu}\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{\rho h^3}{\mu}\frac{\partial P}{\partial z}\right)=6U\frac{\partial(\rho h)}{\partial x}+12\rho V_0 \quad (2)$$

Where *U* is that the sliding velocity, V_0 is the motion of journal center.

The rotation [8] of fluid film regarding an axis that lies across the film provides some new ends up in lubrication issues. The origin of rotation is copied by bound general theorems associated with vorticity within the rotating fluid dynamics. The rotation induces a part of vorticity within the direction of rotation and also the effects arising from it area unit predominant, for giant Taylor's no., it ends up in the streamlines changing into confined to plane transversal to the direction of rotation. The extended version of "Generalized Reynolds Equation" is claimed to be "Extended Generalized Reynolds Equation" given by Banerjee et. al., [9] that takes under consideration of the consequences of the uniform rotation regarding an axis that lies across the fluid film and depends on the rotation no., M i.e., the root of the standard Taylor's no. This generalization of the classical theory is understood because the "Rotatory Theory of hydrodynamic Lubrication".

The "First order rotatory theory" and "Second order rotatory theory" of hydrodynamic Lubrication was given by Banerjee et.al. [10] on retentive the terms containing up to 1st and second powers of M severally, and neglecting higher powers of M. This paper analyzes regarding the pressure within the short journal bearings [11] with respect to the impact of second order rotation and comparative analysis with regard to rotation no. and viscosity for second order rotatory theory with classical Reynolds theory and 1st order rotatory theory of hydrodynamic lubrication. The geometries of journal bearings and infinitely short journal bearings are given by the figures-1 and figure-2 severally.

2. GOVERNING EQUATIONS WITH BOUNDARY CONDITIONS

The Extended Generalized Reynolds Equation derived by Banerjee et al., [11], [12] in ascending powers of rotation no. *M* and by retentive the terms containing up to second powers of M and neglecting higher powers of *M*, is written as equation (3). For the case of pure $W^*=0$, and if the bearing is infinitely short then the pressure gradient in x-direction is far smaller than the pressure gradient in y-direction. In y-direction the gradient $\partial_y P$ is of the order of (P/L) and within the *x*-direction, and is of order of (P/B). If $L << B$ then $P/L >> P/B$, so $\partial_x << \partial_y$. Then the terms containing ∂_x can be neglected as compared to the terms ∂_y containing in the expanded form of Generalized Reynolds Equation. Thus we've the equation as given:

$$\partial_y\left[F(h)\rho\partial_y\right] + \partial_x\left[G(h)\partial_y\right] = -\partial_x\left[\frac{\rho U}{2}\{h - M\,G(h)\}\right] - \partial_y\left[\frac{M\rho^2 U}{2}F(h)\right] \quad (3)$$

Where, $$F(h) = \left[-\frac{h^3}{12\mu}\left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right], G(h) = -\frac{M\rho^2h^5}{120\mu^2}\left(1 - \frac{31M^2\rho^2h^4}{3024\mu^2}\right) \quad (4)$$

Where *x, y* and *z* area unit coordinates, *U* is that the sliding velocity, *P* is that the pressure, ρ is that the fluid density, μ is that the body.

Taking $h=h(x)$, $U=-U$, $P=P(y)$, $h=C\,(1+e\cos\theta)$, $x=R\,\theta$, where θ being measured from x-direction. For the determination of pressure the boundary conditions are as follows:

$$P=0,\ y = \pm\frac{L}{2} \quad (5)$$

The solution of the differential equation (1) under the boundary condition (2) gives the pressure for infinite short journal bearing under the effects of second order rotatory theory of hydrodynamic lubrication as follows:

$$P = \alpha + \beta M + \gamma\, M^2 \quad (6)$$

Where, $$\alpha = \frac{(3\mu U e\sin\theta)(L^2-4y^2)}{4C^2(1+e\cos\theta)^3R}, \beta = \frac{\rho U e\sin\theta(L^2y-4y^3)}{8CR(1+e\cos\theta)^2},$$

$$\gamma = \frac{Ue\sin\theta\rho^2C^2}{16\mu R(1+e\cos\theta)}\frac{e^2\sin^2\theta}{R^2}\left[\left(y^4 - \frac{L^2y^2}{2} + \frac{L^2}{16}\right) - \frac{53}{35}(1+e\cos\theta)^2\left(y^2 - \frac{L^2}{4}\right)\right]$$

On neglecting the term containing M^2 in equation (3), we get the pressure equation for first order rotatory theory of hydrodynamic lubrication i.e., as given

$$P = \frac{(3\mu U e\sin\theta)(L^2-4y^2)}{4C^2(1+e\cos\theta)^3R} + \frac{\rho U e\sin\theta(L^2y-4y^3)}{8CR(1+e\cos\theta)^2}M \quad (7)$$

On neglecting the term containing M^2 and M in equation (3), we get the pressure equation for classical Reynolds theory of hydrodynamic lubrication i.e., as given

$$P = \frac{(3\mu Uesin\,\theta)}{C^2(1+ecos\theta)^3 R}\left(\frac{L^2}{4} - y^2\right) \tag{8}$$

3. NUMERICAL SIMULATIONS

By taking the values of different mathematical terms in *C.G.S.* system as follows:

e=0.2, C/R=0.002, θ=30°, μ=0.0002, C=0.0067, ρ=0.9, U=10^2, h=0.02, y=50, L=200; the calculated values of pressure with respect M are given by table-1.

Table-1

S.NO.	M	P(First Order Rotation)	P(Second Order Rotation)
1.	0.1	5647618.375	5647618.061
2.	0.2	11109927.84	11109926.59
3.	0.3	16572237.31	16572234.49
4.	0.4	22034546.78	22034541.77
5.	0.5	27496856.24	27496848.42
6.	0.6	32959165.71	32959154.44
7.	0.7	38421475.18	38421459.84
8.	0.8	43883784.64	43883764.61
9.	0.9	49346094.11	49346068.76

4. CONCLUSIONS

The variation of pressure with respect to rotation number M by taking viscosity as constant; are shown by equations, tables and graphs. These show that in the first and second order rotatory theory of hydrodynamic lubrication, the pressure increases with increasing values of M, when viscosity is taken as arbitrary constant. The equations, tables and graphs for first and second order rotatory theory of hydrodynamic lubrication show that the pressure is not independent of viscosity. The comparative exponential, logarithmic and parabolic variations of pressure with respect to first and second order rotation have very small variations and have the same expressions.

$$P=7E+06\, e^{2.479M},\ P=2E+07\, \log_e M+4E+07,\ P=-31.30\, M^2+5E+07M+18530$$

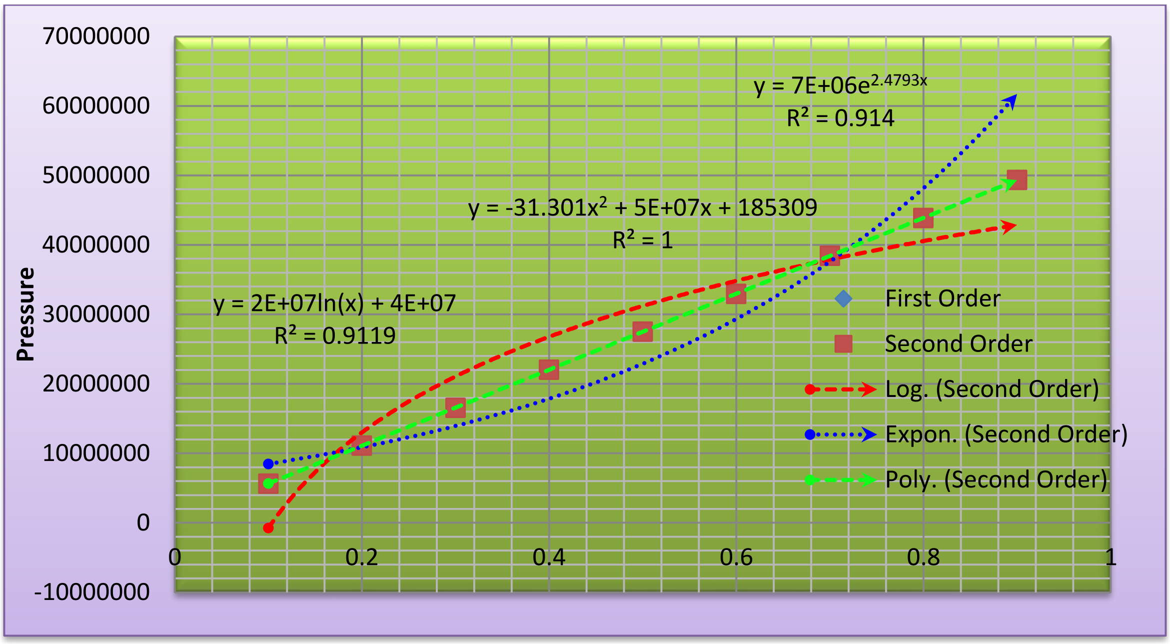

Figure-3. Variation of pressure with respect to *M*

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