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A New Class of Cosmological Models in Lyra Geometry in the Presence of Particle Creation

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Abstract

Cosmological models with constant deceleration parameter in the cosmological theory based on Lyra's geometry in the presence of creation of matter have been discussed. We have presented the modified field equations in the presence of particle creation. Exact solutions have been obtained for a spatially flat FRW model for a particular choice of the particle creation function. We have also discussed the conditions for achieving an accelerated power law expansion of the universe in the present matter dominated era.

Keywords: Cosmology, Lyra Geometry, Open Thermodynamics, Particle creation.

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1. INTRODUCTION

Einstein introduced the cosmological constant into his field equations to obtain a static model of the universe based on the cosmological principle. In the absence of the cosmological term Einstein's field equations allow only non-static cosmological models for nonzero energy density. In Einstein's general theory of relativity gravitation is described in terms of geometry. In 1918 Weyl [1] proposed a more general theory in which electromagnetism is also described geometrically. However, this theory which is based on non-integrability of length transfer was not accepted in general as it had some unsatisfactory features.

Later in 1951 Lyra [2] came up with a modification of Riemannian geometry, which may also be considered as a modification of Weyl's geometry. He introduced a gauge function into the structureless manifold. This removed the non-integrability condition of the length of a vector under parallel transport and also led to the incorporation of the cosmological constant, in a more natural way, from the geometry. Subsequently, Sen [3] and Sen and Dunn [4] proposed a new scalar tensor theory of gravitation, an analog of the Einstein field equations based on Lyra's geometry, which in normal gauge can be written as

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2} \phi_i \phi_j - \frac{3}{4} \phi_k \phi^k = -8\pi G T_{ij} \tag{1}$$

where ϕ_i is the displacement vector and the other symbols have their usual meaning as in Riemannian geometry.

Halford [5-6] pointed out that the constant displacement vector field ϕ_i in Lyra's geometry played the role of a cosmological constant in the normal general relativistic treatment. Many researchers have studied cosmological models based on Lyra's geometry with a constant displacement field vector. However, there is no priori reason for considering the displacement field vector to be a constant.

Singh and Singh [7-10] have studied Bianchi Type I, III, Kantowski- Sachs models based on Lyra's geometry with a time dependent displacement field. (Singh and Desikan, 1997) [11] have studied a class of models in Lyra's geometry based on Einstein's theory with a time dependent displacement field by considering the deceleration parameter to be a constant.

The evolution equations in General Relativity and scalar tensor theories like the Brans-Dicke theory are purely adiabatic and reversible. Hence, they cannot provide an explanation for the origin of cosmological entropy. Irreversible processes during the cosmic expansion might have generated the cosmological entropy. Prigogine et al. [12, 13] investigated the role of irreversible processes in creation of matter out of gravitational energy in the context of General Relativity. Prigogine and Geheniau [14] and Prigogine and Glansdorff [15] showed when thermodynamics of open systems is applied to cosmology it leads to a reinterpretation of the matter-energy stress tensor in the Einstein's equations [12, 13]. They showed that the effect of matter creation is equivalent to adding a supplementary negative pressure term to the thermodynamic pressure and this negative pressure drives the cosmic expansion. Johri and Desikan [16-17] analysed the role of irreversible processes, corresponding to creation of matter out of gravitational energy in the context of both Einstein's theory of gravity and Brans-Dicke theory.

In this paper we have discussed FRW cosmological models with constant deceleration parameter in Lyra geometry with time varying displacement field vector in the presence of creation of matter. The modified field equations in the presence of particle creation are given in section 2. In order to make the system of equations well-defined, an additional assumption in the form of the particle creation function $N(t)$ is made and unique solutions are obtained. Section 3 deals with the solutions of the field equations and their discussion.

2. FIELD EQUATIONS

In equation (1), the time-like displacement vector is given by

$$\phi_i = (\beta(t),0,0,0) \tag{2}$$

In the presence of creation of matter the energy momentum tensor is given by

$$T_{ij} = (\rho + p + p_c)u_i u_j - (p + p_c)g_{ij} \tag{3}$$

where ρ and p are the energy density and pressure respectively, p_c is the creation pressure, u_i the fluid-four velocity, and g_{ij} is the metric tensor.

For the FRW metric

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \tag{4}$$

where $k = 1, 0, -1$, the field equations (1) with equations (2) and (3) become

$$3H^2 + \frac{3k}{R^2} - \frac{3\beta^2}{4} = \chi\rho \tag{5}$$

$$2\dot{H} + 3H^2 + \frac{k}{R^2} + \frac{3\beta^2}{4} = -\chi(p + p_c) \tag{6}$$

where $\chi = 8\pi G$ and $H = \frac{\dot{R}}{R}$ is the Hubble's function. Equations (5) and (6) lead to the continuity equation

$$\chi\dot{\rho} + \frac{3}{2}\beta\dot{\beta} + 3\left[\chi(\rho + p + p_c) + \frac{3\beta^2}{2}\right]H = 0 \tag{7}$$

where the creation pressure is given by $p_c = -(1+\gamma)\frac{\rho}{N}\frac{dN}{dt}\frac{1}{3H}$.

Considering a barotropic equation of state

$$p = \gamma\rho, -1 \le \gamma \le 1 \tag{8}$$

and eliminating $\rho(t)$ from (5) and (6) we obtain

$$2\dot{H} + 3(1+\gamma)H^2 + (1+3\gamma)\frac{k}{R^2} + (1-\gamma)\frac{3\beta^2}{4} = (1+\gamma)\frac{\dot{N}}{N} \tag{9}$$

We note that there are two independent equations in four unknowns viz., $R(t)$, $\rho(t)$, $\beta(t)$ and $N(t)$. In order to obtain a unique solution we require two more relations among the variables. Hence, we have taken the deceleration parameter to be constant and assumed a particular form for the particle source function $\frac{\dot{N}}{N} = \psi(t)$.

3. EXACT SOLUTIONS OF THE FIELD EQUATIONS

We now consider a model with constant deceleration parameter, that is,

$$q = \frac{-R\ddot{R}}{(\dot{R})^2} = b \tag{10}$$

where b is a constant. Equation (10) can be rewritten as

$$\frac{\ddot{R}}{R} + b\left(\frac{\dot{R}}{R}\right)^2 = 0 \tag{11}$$

On integrating the above equation we get the exact solution

$$R(t) = \begin{cases} (D+Ct)^{1/(1+b)} & b \ne -1 \\ R_0 e^{H_0 t} & b = -1 \end{cases} \tag{12}$$

where C, D, R_0 and H_0 are constants of integration.

We shall now discuss the behavior of the model by considering the following form for the rate of creation

$$\frac{1}{N}\frac{dN}{dt} = aH \tag{13}$$

where $a \ge 0$ is a parameter. When $a = 0$, this reduces to the models discussed in (Singh and Desikan, 1997).

Using (10) and (13) in (9) for a flat universe leads to

$$\beta^2 = \frac{4[a(1+\gamma)-(1+3\gamma)+2b]}{[3(1-\gamma)+a(1+\gamma)]} H^2 \tag{14}$$

Now using (14) in (5) for a flat universe yields

$$\chi\rho = \frac{6(2-b)}{[3(1-\gamma)+a(1+\gamma)]} H^2 \tag{15}$$

From (15) we see that $\rho \geq 0$ if $2-b \geq 0$ i.e. $b \leq 2$ as the denominator is always positive.

Similarly, from (14), since the denominator is always positive, we observe that

$$\beta^2 > 0 \text{ if } b > \frac{(1+3\gamma)-a(1+\gamma)}{2} \tag{16}$$

and

$$\beta^2 < 0 \text{ if } b < \frac{(1+3\gamma)-a(1+\gamma)}{2} \tag{17}$$

From equation (14) we notice that when $b = \frac{(1+3\gamma)-a(1+\gamma)}{2}$, we get $\beta^2 = 0$, and the equations reduce to those of the standard FRW flat universe. We can clearly see that the behaviour of the displacement vector β is determined by the rate of creation of particles.

From (17) we note that the universe would undergo accelerated expansion, i.e. $b < 0$ in the present matter dominated era ($\gamma = 0$) if $(1-a) < 0$, that is, $a > 1$. Also, a cannot exceed 3 as we must have $b > -1$ for an expanding universe. So for values of a in the range $1 < a < 3$, the universe will undergo an accelerated power-law expansion and for these values of a we have $\beta^2 < 0$. For values of a in the range $0 < a < 1$, the universe will undergo a power-law expansion but would be decelerating and for these values of a we have $\beta^2 > 0$.

Case (i): $b \neq -1$

For singular models the expression (12a) may be written as

$$R(t) = R_0 t^{1/(1+b)} \tag{18}$$

Using (18) in (14) and (15) gives

$$\beta^2 = \frac{4[a(1+\gamma)-(1+3\gamma)+2b]}{[3(1-\gamma)+a(1+\gamma)]} \frac{1}{(1+b)^2} \frac{1}{t^2} \tag{19}$$

and

$$\chi\rho = \frac{6(2-b)}{[3(1-\gamma)+a(1+\gamma)]}\frac{1}{(1+b)^2}\frac{1}{t^2} \tag{20}$$

From equations (19) and (20) we observe that β^2 and ρ vary as $1/t^2$. When $a=0$, equations (19) and (20) reduce to equations (19) and (20), respectively, of [11].

Case (ii): $b=-1$

In this case equation (10) gives

$$\dot{H} = 0 \text{ i.e. } H = H_0 = \text{constant} \tag{21}$$

Using (21) in (14) and (15) leads to

$$\beta^2 = \frac{4[(a-3)(1+\gamma)]}{[3(1-\gamma)+a(1+\gamma)]}H_0{}^2 = \text{constant} \tag{22}$$

and

$$\chi\rho = \frac{18}{[3(1-\gamma)+a(1+\gamma)]}H_0{}^2 = \text{ constant} \tag{23}$$

From (22) we see that since the denominator is always positive and $(1+\gamma)\geq 0$, $\beta^2 \geq 0$ when $a \geq 3$. And $\beta^2 < 0$ for $0 < a < 3$. From (23), we have $\rho > 0$ for all times.

When $a=0$, equations (22) and (23) reduce to equations (31) and (32), respectively, of [11].

4. CONCLUSION

In this paper we have discussed cosmological models based on Sen's equation in Lyra geometry for constant deceleration parameter in the presence of creation of matter. The behaviour of the displacement field β and the energy density ρ have been examined. We have discussed both power-law expansion and exponential expansion of a flat universe for a particular choice of the creation function. We have analysed the conditions for accelerated power law expansion in the present matter dominated era.

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