The Study of Markets and Prices - The Thermodynamics Approach

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Abstracts

Many researchers have attempted to viaduct their fields with others to gain insight into their own. In the past decade or so, physicists have begun to do academic research in economics. Perhaps people are now actively involved in an emerging field often called Econophysics. The scope of this paper is to present a phenomenological analysis for Markets and prices with Thermodynamics approach. The main ambition of this study is fourfold: 1) First we begin our description of a thermodynamics model of economics with the simplest example. 2) To extend the thermodynamics approach to the study of markets and prices. 3) The problem of the market equilibrium for the two markets with two items of goods. 4) Finally we constructed the economic model with the actual market at constant temperature. And this paper end with conclusion.

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Introduction

Lately, a large number of scientists from the field of physics started to study social systems and mostly economic systems, trying to apply methods and formalisms developed for years in their field. This effort was considered successful and gave birth to the correlation between Physics and Economics. Although not being a novelty, the existence of this relationship has been widely acknowledged [1]. However, only in recent years, the interaction of the above two scientific sectors led to a new interdisciplinary research field: what is now referred to as econophysics. High-frequency finance, financial risk and correlations, and complexity in finance are some fields where physics can be used in Economic theory. The tools-models which are
used by physicists in economics are, among others, statistical physics, thermodynamics, solid state physics etc.

In recent years the point of view in which financial markets are defined has started to change. A new aspect is the consideration of financial markets as complex systems [2]. They are systems consisted of many interacting agents. These interactions are of a highly nonlinear nature. This enormous amount of data allows a detailed statistical description of several aspects of the dynamics of asset price in a financial market. The results of studies based on these data show the existence of several levels of complexity in the price dynamics of a financial asset [3-6]. An alternative way of analyzing markets is by using the concepts of thermodynamics [7]. Most of contemporary models on stock market or asset indexing are based, more or less, on the Black-Scholes formula with the assumption of a stochastic process (geometric Brownian motion) for price changes. Basic parameters of these models are the drift coefficient $\mu$ and the diffusion coefficient $\sigma$. More advanced stochastic models assume that the diffusion coefficient, commonly known as volatility, also behaves in a stochastic manner.

The aim of this paper is to introduce a thermodynamics approach to the study of markets and prices with an alternative formalism.

**Entropy and Temperature in Models of Economics**

First we begin our description of a thermodynamics model of economics with the simplest example [8]. Let an economic system consist of $N$ agents, among whom the income, constant for the system as a whole, is distributed. We assume that there are many ways to distribute income, and we are incapable to foresee all possible alternatives. This assumption fully corresponds to Hayek’s concept of market [9] in which the market is described as an arena of discoveries of new procedures and operations. For ultimate simplicity, we assume that the income is quantum, i.e., presented in integers (which is natural, as the smallest unit of currency is operational in economics).

Hence for each value of the total income $E$ it can correspond a quantity of modes of income distribution between the agents, as a characteristic function $n(E,N)$ of this value. This function is called the statistical weight of the state with income $E$.

At this stage we can introduce the concept of equilibrium. The idea is that two systems (under the above hypothesis) are in equilibrium, if the distribution function of income does not change when they enter in a contact, hence, there is no income flow between the systems. By a “contact” we understand here an “open list” of possible modes of redistribution. It is remarkable that it is possible to calculate the statistical weight regardless of uncountable variety of various institutional limitations imposed on the agents’ incomes, so functions $n(E,N)$ may be different for different systems.

The actual income is irrelevant for the analysis, although it is possible that the restrictions on income are present anyway. Let one system with the total income $E_1$ have $N_1$ number of agents and the other, with $E_2, N_2$ number of agents respectively.
The system which is constituted by subsystems $n_1(E_1, N_1)$ and $n_2(E_2, N_2)$, induces total income $E_1 + E_2$ and a number of agents $N_1 + N_2$. The next goal is to find the right conditions which can describe a state of equilibrium. Under the conceptualization of this paper, equilibrium means no flows between the systems.

In order to build up an appropriate theory, we have to adopt one more, extremely important, hypothesis on the nature of the systems under investigation. Namely, we assume that all elementary states of income distribution have the same probability.

The reason for that is symmetry of states. As it is the fact in probability theory and in statistics, it is assumed equal probability of elementary events because there is no reason to prefer one event over other. It is vital for the theory, to include all probable states of distribution. Changes of function $n(E,N)$ will introduce changes to results obtained by this model.

In order to identify the state of equilibrium, it is required to find conditions by which the income is not redistributed between the interacting systems. Let’s consider a redistribution of income as an interaction takes place. Let $\Delta E$ be a certain part of income which, passes from system 1 to system 2. Hence, the states of the systems change and their statistical weights correspond now to: $n_1(E_1 - \Delta E, N_1)$ and $n_2(E_2 - \Delta E, N_2)$.

According to principle of equal probability, the most probable state of the compounded system is the one with the greatest statistical weight. The aim of this paper is to seek the maximum of the function $n_{tot}(E_1, E_2, N_1, N_2)$, under the restriction of constant total income $E_1 + E_2$. Furthermore, by no transfer of agents from one system to another, the statistical weight of the integrated system is given by:

$$n_{tot}(E_1, E_2, N_1, N_2) = n_1(E_1, N_1)n_2(E_2, N_2)$$  (1)

Since $E_1 + E_2 = Cons \tan 1$, it follows that $\Delta E_1 = -\Delta E_2$.

Instead of seeking the maximum of $n_{tot}$, we can seek the maximum of $\ln n_1 + \ln n_2$ we drive the condition on the maximum. It is very simple:

$$\frac{\partial \ln n_1(E_1, N_1)}{\partial E_1} = -\frac{\partial \ln n_2(E_2, N_2)}{\partial E_2}$$  (2)

So, two systems are in equilibrium, if they are characterized by the same value of parameter $\frac{\partial \ln n(E,N)}{\partial E}$.

In thermodynamics, the logarithm of the statistical weight is called entropy (of the system), and its derivative on energy is the temperature,

$$\frac{\partial \ln n(E,N)}{\partial E} = \frac{1}{T}.$$  (3)

In order to reach the state of equilibrium, the interacting system should be at the same temperature [10].

Now, under the above assumptions, it is needed to examine the accuracy of this approach to the economical systems. An economical system is in state of equilibrium.
if it is almost homogeneous and it does not imply flows from one of its subsystems to
another. However, it is supposed, that the homogeneity still exists only if there is no
separation into such small parts so that no major income flows are noted. A likewise
assumption can be made for an economic systems. The thermodynamic model induces
two important parameters; entropy and temperature. With no information about these
variables it is not possible to find the correct equilibrium conditions for the system.
The system is in the state of equilibrium only when its subsystems have the same
temperature, but it is not possible to calculate the temperature cannot without
knowing the entropy.

In physics, in order to measure temperature, thermometers are used. These are
special devices whose equation of state is known and calibrated. By bringing the
thermometer in contact with a body it is possible to find the temperature of the body
by the thermometer’s change of state. The (stock) markets can be considered as
thermometers in economics, as it will be shown below. Someone could possible raise
objection against the thermodynamic approach to economy, referring to the number of
“particles” involved. The number of the market agents, in the example of this study, is
much smaller in comparison with the amount of particles in physical systems. The
number of particles in physical systems is related and comparable, with the Avogadro
number; though, in the economic systems it is usually \( \approx 10^3 - 10^8 \).

In statistics, the order of dispersion is defined by \( 1/\sqrt{N} \), where \( N \) is the number
of particles in the system. Hence, due to the fact that the number of particles in
physical systems is finite, the statistical errors are rather insignificant. In economic
problems much larger errors, \( \approx 3\% \) are expected. But, taking to account the
roughness of economical models, such errors are not large. Contemporary physics
often applies thermodynamic approaches to systems with rather small number of
particles \( 10^3 - 10^8 \) (nuclear physics, cluster physics, etc.) [11]. However, the results
appear to be valid not only qualitatively, but also quantitatively.

The Thermodynamics Approach - Markets and Prices
Let us try now to extend the thermodynamic approach to the study of markets and
prices. For this purpose, consider a simplified model situation. It is necessary,
nevertheless, to simplify with caution, so as not to lose the most essential situational
characteristics.

In the above sections we showed that if the money flow is constant, it is possible
to introduce the concept of equilibrium in the income distribution. To implement this,
we have to know the entropy of the system, hence its temperature. Using temperature
we are able to find out the conditions under which the system stays in equilibrium
with the environment.

Now, we add to this model a flow of goods. It is still possible to calculate the new
entropy and introduce an additional parameter of equilibrium. In what follows we will
discuss how this additional parameter can be interpreted in terms of the price of the
goods. Moreover, it is also possible to deduce the equation of the market state, i.e., to
find dependence between the flow of goods, the price, the number of buyers and the temperature.

The following discussion will reflect an intuitive concept of the market and the market equilibrium, and then try to formalize it. The market is characterized by the presence of goods which are sold, and money spent at purchases.

Let $V(t)$ be the amount of "units of goods" sold per unit of time during which the buyers spend $E(t)$ of units of money. We say that the market is *stationary* if $V(t)$ and $E(t)$ are constants independent of time and all goods are bought, i.e., there is no accumulation of goods in the hands of the sellers. While the market functions, the deals are made, i.e., agreements on exchange of some of the goods for some money.

Now, consider a model with several co-existing markets able to interact: exchange goods and money resources. The intuitive concept of equilibrium of these markets is that the situation in each of these markets remains, in certain essential aspects, the same even after the interaction. Clearly, this cannot take place for any values of the amounts of goods and money $V_1, E_1$ and $V_2, E_2$ in each system, respectively. Besides, the interaction can vary, i.e., include an exchange only of money resources, only of goods, or both.

Consider the simplest case, when the values of the flows of money and goods are discrete, as is the case in real life:

$$E_n = nE_0, \quad V_m = mV_0$$  \hspace{1cm} (4)

In addition, let the values of $n$ and $m$ be sufficiently large, to make it possible to treat small changes of flows as insignificant and differentiate. Let $N$ be the number of buyers in our model. The market is assumed to be stationary in the above sense.

For this model, introduce the entropy in the same way as earlier, namely, as the logarithm of the number of probable states. In this case, the entropy depends on both the total money expenditure, $E$, and the total amount of purchased goods, $V$. Various states of the market can be regarded as legalized by the rules of distribution of the total supply of money and goods among $N$ buyers. Observe that now we study the “market of the buyers”, so, we do not have to include in the entropy the distribution of goods among sellers.

By analogy with the pressure in statistical thermodynamics, the notion of *marginal price* $P$ is introduced as follows:

$$P = T \frac{\partial S}{\partial V}$$  \hspace{1cm} (5)

First, consider the simplest case, referred to as the “free market”. In this model there are no restrictions on goods and money distribution among the buyers. This means that the buyer can pay any price — either infinitesimally low or infinitely high.

The free market is rather easy to study, because the total number of probable market states is the product of the number of possible distributions of goods between the market agents times the number of possible distributions of money between the same agents. Thus, the statistical weights of the system are obtained by multiplying...
the statistical weights determined by the flows of goods \((V)\) and money \((E)\). Consequently, the entropy of the system (the logarithm of the statistical weight) consists of the two components:

1. One is determined by the flow of goods,
2. The other one, by the flow of money.

\[
S \left( E, V \right) = S \left( E \right) + S \left( V \right) \tag{6}
\]

Let the money and goods flows be quantized, as in (4), we drive the equation of state for such a system as follows.

First, compute the number of probable distributions of goods and money flows among \(N\) agents of the market. The statistical weight \(g \left( E_n, N \right)\) of the flow \(E_n\), distributed among \(N\) agents is equal to the number of non-negative integer solutions of the equation

\[
n = x_1 + \ldots + x_N. \tag{7}
\]

The calculation technique for such equations is well-known. Namely, add 1 to each \(x_i\). Then the quantity to be determined is the number of positive integer solutions of the equation

\[
n + N = y_1 + \ldots + y_N. \tag{8}
\]

Now, let us subdivide the integer segment of length \(n + N\) into \(N\) integer subsegments.

Thus, the statistical weight is equal to

\[
g \left( E_n, N \right) = \binom{N - 1 + n}{N - 1} = \frac{(N - 1 + n)!}{n!(N - 1)!} \tag{9}
\]

For sufficiently large \(n\) and \(N\), the Stirling formula gives

\[
g \left( E_n, N \right) \approx \frac{1}{\sqrt{2\pi}} \frac{(N + n - 1)^{N + n - \frac{1}{2}}}{N - \frac{1}{2} (N - 1)^{n + \frac{1}{2}}} \tag{10}
\]

Hence,

\[
S \left( E_n \right) = \ln g \left( E_n, N \right) \approx \frac{(N + n - 1) \ln \left( N + n - \frac{1}{2} \right) - (N - 1) \ln \left( N - \frac{1}{2} \right)}{n} + \frac{1}{2} \ln 2\pi \tag{11}
\]

The Temperature \(T\) is calculated in terms of \(\frac{\partial S}{\partial E}\)

\[
\frac{1}{T} = \frac{1}{\varepsilon} \frac{\partial S \left( E_n \right)}{\partial n} \approx \frac{1}{\varepsilon} \ln \frac{N + n - \frac{1}{2}}{n + \frac{1}{2}}. \tag{12}
\]

For \(n \gg N\), which is a rather natural condition for the free market, the expression for the inverse temperature is:
This means that the temperature \( T = \frac{E}{N} \) is equal to the mean value of the income per capita. To compute \( \frac{\partial S}{\partial V} \) is a completely analogous, matter (since the entropy \( S \) can be described as the sum of two terms, one of which only depends on \( E \), the other one, on \( V \), see (6)).

\[
\frac{P}{T} = \frac{\partial S \left( V_m \right)}{\partial V_m} = \frac{1}{W} \frac{N}{m} = \frac{N}{V}. \tag{14}
\]

Thus, a relation between the marginal, price, the goods flow and the “temperature” of the free, market is of the form

\[
P = T \frac{N}{V}. \tag{15}
\]

As \( TN = E \), it is easy to show that for the free market the marginal price is equal to a median price. This, actually, allows defining the price as \( T \frac{\partial S}{\partial V} \).

In presence of restrictions on price, the marginal price may deviate from the median one. Manifestly, the equation of the free market (15) is totally analogous to the equation of the ideal gas.

**Market Equilibrium**

Consider now the problem of the market equilibrium for the two markets with two items of goods capable to replace each other. The market with replaceable goods poses one of the most known problems of the mathematical economics. Such markets will prompted the marginal list revolution. In the state of equilibrium, the amount of money spent on the last portion of goods per unit of replacement capacity is the same for all items of goods represented at the market.

In terms of the neo-classical theory, to work out this task, it is necessary to define the utility functions of the goods, the function depending on the volume of consignments. When the derivative of the utility function with respect to the volume of consignment decreases as this volume grows, the techniques of the classical analysis show that the summary utility function reaches its maximum at such a volume of the consignment of goods that the increase of utility by one unit of the expenditure is equal for all nomenclature of goods.

Intuitively, this is a plausible statement. If, at the expenditure of one unit of funds per one item of goods, it were possible to increase the total utility of the purchased goods by replacing one item by another, this replacement have been implemented, until the utility had not dropped due to the increase of the goods’ volume. We see that the dependence of utility on volume is indeed very important in terms of maintenance of sustaining the system’s stability: otherwise all funds could have been invested in the most useful goods only.
Let us now study the problem of replaceable goods in the framework of thermodynamic approach.

Clearly, it makes sense to speak about replacement only if there is certain equivalence between goods. If there is no equivalence, we cannot proceed. So, let such an equivalence exist in the model, i.e., \( n \) units of item 1 are equivalent (never mind, in what sense precisely) to \( m \) units of item 2. The equivalence relation can be used to find the maximum of entropy, in exactly the same manner, as it was done before for only one item of goods. It is important to observe that, in our hypothesis, the equivalence relation does not depend on the flow of goods. If such dependence takes place, the theory of statistical equilibrium is, all the same, possible to deduce, but it is a bit more involved. The most probable or, what is the same here, equilibrium, state of the system is attained as the entropy of the system attains its maximum. To find the maximum of entropy, we use the equivalence relation between two items of goods, in completely the same fashion as earlier on these pages.

Thus, consider a market with \( N \) agents, with a flow of money \( E \) and flows of items of two goods, \( V_A \) and \( V_B \). To find the entropy of system, we have to count the number of probable deals, i.e., the number of ways to attribute to each agent four numbers \( (E_{A_i}, E_{B_i}, V_{A_i}, V_{B_i}) \) that show how much money was spent by an agent per unit time on purchasing the goods \( A \) and \( B \), and how many goods had been bought over the same period, respectively. Adding up over all agents we obtain the values of macroscopic parameters of the system.

Observe that in order to estimate the system’s state, it does not suffice to simply fix the amount of money spent by the \( i \) th agent because some microscopic states may be ruled out, owing to, say, a restriction of the prices from below.

In the general case, the dependence of the entropy on macroscopic parameters cannot be represented as the sum of components each depending on one or two parameters. For the free market, however, this is so.

Consider the system with entropy \( S (E_A, E_B, V_A, V_B) \). Replacing \( E_A \) by \( E_B \) and using the fact that \( E_A + E_B = E \) we see that

\[
\delta S = \frac{\partial S}{\partial E_1} \delta E_1 = \frac{\partial S}{\partial E_2} \delta E_2 = \delta E_1 \left( \frac{\partial S}{\partial E_1} - \frac{\partial S}{\partial E_2} \right).
\]

In the state of equilibrium, the derivatives \( \frac{\partial S}{\partial E_1} \) and \( \frac{\partial S}{\partial E_2} \) are equal and \( V_A \) and \( V_B \) are fixed. If there is an equivalence between the items \( A \) and \( B \), we may “unite” these items and consider the equilibrium problem.

Let \( V_A = V_A^0 n_A, V_B = V_B^0 n_B \) where \( n_A \) and \( n_B \) are the amount of goods, purchased per unit time. Having introducing \( V_A \) and \( V_B \) we imply that the replacement relation is known, i.e., we have a common unit of measurement.

So, again, we have
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\[ V_A + V_B = V, \quad \delta V_A = \delta V_B, \]

\[ \delta S = \frac{\partial S}{\partial A} \delta A + \frac{\partial S}{\partial B} \delta B = \delta A \left( \frac{\partial S}{\partial A} - \frac{\partial S}{\partial B} \right) \]  

(17)

and the condition for the equilibrium state can be expressed as:

\[ \frac{\partial S}{\partial V_A} = \frac{\partial S}{\partial V_B}. \]  

(18)

Observe that we made no assumptions on utility, but only on possibility to replace the goods by other goods.

Now consider the free market where we know how the entropy depends on macro-parameters. As we outlined above, the equilibrium conditions with respect to the money flow are of the form:

\[ \frac{\partial S}{\partial E_A} = \frac{N}{E_A}, \quad \frac{\partial S}{\partial E_B} = \frac{N}{E_B} \]  

(19)

As \( E_A + E_B = E \), the equality of \( \frac{\partial S}{\partial E_B} \) and \( \frac{\partial S}{\partial E_A} \) result in \( E_A = E_B = \frac{E}{2} \). Similar calculational techniques for \( \frac{\partial S}{\partial V_A} \) and \( \frac{\partial S}{\partial V_B} \) produce \( \frac{\partial S}{\partial V_A} = \frac{N}{V_A} \) and \( \frac{\partial S}{\partial V_B} = \frac{N}{V_B} \)

with the constraints that \( V_A + V_B = V \), \( V_A = V_B = \frac{V}{2} \). Returning to the condition of substitution, expressed by \( V_A = V_A^0 n_A \), \( V_B = V_B^0 n_B \), it means

\[ \frac{n_A}{n_B} = \frac{V_B^0}{V_A^0}, \]  

(20)

i.e., the volume of goods, purchased in the equilibrium state of the free market, is inversely proportional to its “replacement capacity”.

Recall our earlier assumption that the equilibrium of the money flow is attained. The condition for the equilibrium of the goods flow takes the form

\[ T \left. \frac{\partial S}{\partial V_A} \right|_{E_A, E_B} = T \left. \frac{\partial S}{\partial V_B} \right|_{E_A, E_B} \]  

(21)

If the money flow is constant, \( TdS \) can be interpreted as the expenditure on the purchase of the last, “marginal” portion of the goods. So, we have obtained the well-known marginalists’ formulation.

**Economical Model at A Constant Temperature**

In the case of actual markets, it is of course very difficult to determine how the actual flow of money looks like. Moreover, the flow of money is not a parameter that affects the equilibrium of the system, i.e., even if we know what the flows of money are in two distinctly organized systems, we cannot say if the systems will come to equilibrium after they come into a contact.
Here we construct the economic model with the actual market at constant temperature since the temperature is a parameter of equilibrium.

To pass to new variables, we can apply a mathematical technique called Legendre transformation [12].

Let us look how to work the Legendre transformation sending independent variables \((E,V)\) into independent variables \((T,V)\). If we start from relation for our theory

\[dE = TdS + \mu dN - PdV\]  \hspace{1cm} (21)

then, assuming that the temperature \(T\) and the number of particles \(N\) are constants, we immediately derive by dividing (21) by \(dV\) that

\[P = -\left. \frac{\partial E}{\partial V} \right|_{F,N} + T \left. \frac{\partial S}{\partial V} \right|_{F,N}\]  \hspace{1cm} (22)

If we introduce the function \(F = E - TS\) - Legendre transform, then

\[P = -\left. \frac{\partial F}{\partial V} \right|_{F,N}\]  \hspace{1cm} (23)

Therefore given \(F\), differentiating yields the value of the price at the constant temperature and constant number of market agents.

This (23) is a very essential relation. Let our system interact with a thermostat at temperature \(T\). Now we are not bounded by a particular value of the flow of money: the system exchanges its money with the thermostat and it suffices to assume that the total system plus thermostat satisfy the law of money conservation.

It is not difficult to see that if the flow of goods changes from \(V_1\) to \(V_2\), then at constant temperature the corresponding variation of the flow of money will be equal to

\[\int_{V_1}^{V_2} PdV = -\int_{V_1}^{V_2} \left. \frac{\partial F}{\partial V} \right|_{F,N} dV = F(V_1) - F(V_2)\]  \hspace{1cm} (24)

Under the increase of the flow of goods the flow of money spent in order to make such an increase possible becomes equal to the increment of the function \(F\).

In statistical thermodynamics, \(F\) is called the free energy. We will call \(F\) the free flow of money. It is easy to see that \(F = -\ln Z\) where \(Z\) is the statistical sum of the system.

\[F = E - TS = \sum_i E_i W_i - T \sum_i W_i \ln W_i,\]  \hspace{1cm} (25)

where \(W_i = \frac{e^{E_i/T}}{Z}\) is the probability of the system to be in the state \(i\). Substituting this \(W_i\) in the formula for \(F\) and computing the difference we get

\[F = -\ln Z\]  \hspace{1cm} (26)

Observe that \(F\) is a negative quantity. The relation (24) shows that the free flow of money is exactly the highest expenditures possible in the system to increase the flow of goods.
Let us show now that a free flow of money is extremal at the most possible configuration and at a constant temperature and constant flow of goods. Indeed, in the system which interacts with a thermostat, the total flow of money is preserved:
\[ dE_c + dE_T = 0, \] (27)
where \( dE_c \) is the infinitesimal increment of the flow of money in the system and \( dE_T \) is the infinitesimal increment in the flow of money of the thermostat.

The total entropy of the system \( S_c + S_T \) is maximal, and therefore \( dS_c + dS_T = 0 \) and the temperature \( T \) of the system is determined by the thermostat and
\[ dE_T = TdS_T \] (28)
Therefore \( dE_T = -dE_c = TdS_T = -TdS_c \) (29)
or \( dE_c - TdS_c = 0, \ dF_c = 0 \) (30)

In other words, \( F \) has an extremum at the point of equilibrium between the system and the thermostat.

This extremum is the minimum which follows from the maximality of the entropy. For any change of the situation which leads the system out of the equilibrium, the total sum of increments of the entropies of the system and the thermostat is
\[ \Delta S_c + \Delta S_T \leq 0. \] (31)
But the energy can only sneak into the system due to the diminishing of the thermostat’s entropy:
\[ \Delta E_c = -T \Delta S_T \] (32)
and since \( \Delta S_T \leq -\Delta S_c \), it follows that
\[ \Delta F_c = \Delta F_c - T \Delta S_c \geq 0. \] (33)
This means that the system is in equilibrium the minimum of \( F \) at constant \( V \) and \( T \) [10].

This is a very important property. It means that we can find the equilibrium point by looking for the minimum of a function which can be computed if we know the statistical sum. To find the statistical sum, it suffices to know the statistical weights of the states with distinct admissible values of the money flow and the temperature. All other parameters of the system (the flow of money and the price) can be found from the simple formulas:
\[ E = -T \frac{\partial}{\partial T} \left. \frac{\partial F}{\partial T} \right|_{V,N}, \] (34)
\[ P = -\frac{\partial F}{\partial V} \bigg|_{V,N}. \] (35)

It is easy to see that the Legendre transformations can be applied differently passing to other independent variables, for example, \( T \) and \( P \). In this case in statistical thermodynamics one uses the thermodynamic potential
\[ \Phi := E - TS + PV. \] (36)
We will retain this name in our case as well. It is not difficult to show that the thermodynamic potential attains a minimum in an equilibrium at constant values of $T$ and $P$. For the thermodynamic potential, we have the following relations:

$$d\Phi = -SdT + VdP, \quad S = -\left.\frac{\partial \Phi}{\partial T}\right|_p, \quad V = \left.\frac{\partial \Phi}{\partial P}\right|_T.$$  

These relations imply (thanks to possibility to interchange the order of differentiation) an important relation:

$$-\left.\frac{\partial S}{\partial P}\right|_T = \left.\frac{\partial V}{\partial T}\right|_P$$  

or

$$-\left.\frac{\partial S}{\partial P}\right|_T = \left.\frac{\partial V}{\partial T}\right|_P.$$  

Now we can construct a thermometer to define the absolute temperature of an arbitrary system.

Let $T = T_{arb}$, where $T_{arb}$ is an arbitrarily graded scale of a thermometer. The equation (39) gives that

$$\left.\frac{\partial E}{\partial P}\right|_T = T \left.\frac{\partial S}{\partial P}\right|_T = -T \left.\frac{\partial V}{\partial T}\right|_P.$$  

We can express $\left.\frac{\partial V}{\partial T}\right|_p$ in the ‘empirical scale’ in terms of $T_{arb}$:

$$\left.\frac{\partial V}{\partial T}\right|_p = \left.\frac{\partial V}{\partial T_{arb}}\right|_p \left.\frac{\partial T_{arb}}{\partial T}\right|_p.$$  

this gives that

$$\left.\frac{\partial E}{\partial P}\right|_{arb} = -T \left.\frac{\partial V}{\partial T}\right|_{arb} \left.\frac{\partial T_{arb}}{\partial T}\right|_{arb}$$  

or

$$\left.\frac{\partial T_{arb}}{\partial T}\right|_{arb} = \left.\frac{\partial E}{\partial P}\right|_{arb} \left.\frac{\partial V}{\partial T}\right|_{arb}.$$  

The right-hand side of this relation only involves the functions that can be measured on the conditional scale [13]. If we can measure them we can, therefore, determine the law of dependence of the conditional scale on the relative temperature.

**Conclusion**

From the above discussion we description of a thermodynamics model of economics with the simplest example and its extend to the thermodynamics approach to the study of markets and prices. Then we considered that the problem of the market equilibrium for the two markets with two items of goods. And finally we constructed the economic model with the actual market at constant temperature.
From the above discussion we concluded that, for real economic systems, this would mean the necessity to have a certain model market which one could append to an arbitrary market and measure the variation of the derivative of the flow of goods with respect to the conditional temperature at the fixed price and the derivative of the flow of money with respect to the price at the fixed conditional changes.

To collect such statistical data for a large number of artificially created conditions is hardly possible in reality. Observe, however, that such a device is possible as a thought experiment. In other words, a recovery of the scale of the absolute temperature from a collection of statistical data is possible, in principle. Indeed, the fact that there is an analogue of the gas thermometer in economics — free market — is a favorable circumstance. As we saw above, we can calculate the absolute temperature of the free market in terms of the mean value of the flow of money per market agent. Therefore we have thermostats with different temperatures in the form of free markets of considerable volume and, at favorable circumstances; we might be able to use them as instruments to study non-free markets.

Creation of such favorable circumstances, however, is rather expensive and requires a great number of large-scale economic experiments whose price is difficult even to imagine. They can, however, be replaced, to an extent, by the study of historical cases where such experiments were performed for some reasons but it would be hardly possible to collect a sufficient number of cases for one particular non-free market. It seems that the only possible way to study the properties of non-free markets is mathematical modeling or computer simulation, the results of which can be compared with the results of analysis of specially selected historical cases.

References


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