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Detecting Speculative Bubble: Power Comparison of Unit Root Tests

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Abstract

The recent wave of financial globalization causes the dramatic changes within a short period to the economy of a country. The sudden changes in the price of an asset without any intuitive reasons will lead to market imbalance. For different reasons, an investor trusts that the demand for a stock will continue to rise or that the stock will become profitable within a short period. Particularly, a situation in which the price of a stock rises far above from its actual value would motivate the investor to invest on it. This craze continues until he realizes that there is no profit in investment. Consequently the market value of the asset which was hiked to an unjustified level is pull down and which forms a bubble structure in the time series. In the literature this type of bubble is commonly known as financial bubble.

The formation and burst of a speculative bubble creates huge damages in the economy. Hence in order to prevent the market from facing damages, the detection and modeling of financial bubble is very essential. In this direction, a model to generate bubbles proposed by Evans (1991) created a bench mark in the economic literature. Further it is proved that the unit root tests play an important role in detection of such bubbles. Here a simulation study is carried out to observe the effects of variations in the parameters of Evans model on the powers of different unit root tests.

Keywords: Collapsible Bubble, Unit root, ADF test.

1. INTRODUCTION

The recent wave of financial globalization causes the dramatic changes within a short period to the economy of a country. These changes in the price of an asset without any intuitive reasons will lead to market imbalance. For different reasons, an investor trusts that the demand for a stock will continue to rise or that the stock will become profitable within a short period. For example, a situation in which the price of a stock rises far above from its actual value may motivate the investor to invest on it. This craze continues until he realizes that there is no profit in investment. Then onwards the market value of the asset which was hiked to an unjustified level starts to fall which forms a bubble structure in the time series. In the literature this phenomena is commonly known as financial bubble.

Bubbles in the price series of a stock will not only (directly or indirectly) affect the economy of a country but may also create huge damages in the economy of another country. Hence in order to prevent the market from facing damages, the detection and modeling of financial bubble is one of the fundamental problems of economic literature.

One can identify the presence of a bubble in the market through the observations on the features of distribution of asset price. Specifically, the negative skewness and leptokurtosis of return series gives an evidence of bubble. But these variables are based on the assumption of stationarity and linearity of returns. Also, these variables are not unique for bubbles and are often associated with fundamentals [1]. Thus these evidences are usually considered as diagnostic tools for examining the existence of bubble in the market under study.

One of the main characteristic of financial bubble is that if bubbles are present in the market, they should possess explosive behavior in the asset price so that stationarity cannot be attained even after taking multiple differences. As a result, from 1980s onwards bubbles were investigated with the application of econometric analysis. The majority of empirical research examines the existence of stock market bubbles by applying conventional unit root tests on the time series of asset price [2].

However [3] proved that the conventional unit root tests are unable to detect a class of financial bubble, namely periodically collapsing bubble. He noted that the explosive behaviour is only temporary when the bubbles periodically collapse. Moreover, such type of bubbles behave much like an I(1) process or even like a stationary linear AR process provided that the probability of collapse of bubble is not negligible. This criticism leads to look forward improved unit root test procedures to detect financial bubbles. In the next section we summarize the procedures of improved unit root tests which were developed to overcome the Evans critique.

2. IMPROVED UNIT ROOT TESTS

2.1. Max conventional unit root test

Leybourne [4] proposed a test, Max conventional unit root test, by taking the maximum of the ADF statistic (based on the data in forward direction) and ADF statistic (based on reversed direction). He showed that this test performs better as compared to conventional ADF test via Monte Carlo simulation. However, the test failed to capture the collapsible nature of bubbles.

2.2 Supremum ADF Tests (SADF Tests)

Further [5] observed that the periodical collapsing behavior of a speculative bubble can be captured by applying the ADF unit root test on the subsets of the sample. This observation created a mail stone in the field of detecting collapsible bubble. The tests proposed by Phillips are commonly known as Supremum ADF (SADF) unit root tests. Mainly there are two versions of SADF tests namely; Forward recursive SADF test and Forward rolling window SADF test. The testing procedure for both tests is essentially same. But they differ in the subsample selection criteria.

a) Forward recursive SADF Test

In this procedure, first compute a sequence of ADF statistic on the subsamples which expands by one at each pass so that the statistic based on last subsample is nothing but the conventional ADF statistic. Then find the supremum of obtained sequence. Formally one can represent the test statistic as follows.

$$\underset{r_w \in [r_0,1]}{SADF} = \sup_{r_w \in [r_0,1]} \left(ADF_{r_w}^{f} \right) \tag{1}$$

where r_0 is the size of initial subsample. Note that there is no strict rule to select r_0. A part of sample say, $[r_0]$ where [.] denotes the greatest integer part of r_0 is decided as initial subsample size. Usually r_0 lies between 0.1 and 0.3. i.e. selecting initial 10% to 30% of observations from the original sample as the size of first subsample.

b) Forward rolling SADF Test

Here we first determine a series of ADF statistic on the subsamples which rolls ahead with constant sample size so that the starting and ending points of each subsample is incremented by one at each pass. The test statistic for Forward rolling SADF test can be represented as follows.

$$\underset{r_w}{SADF} = \sup_{r_w} \left(ADF_t^{f} \right); t = 1,2,3,... \tag{2}$$

where r_w is the size of rolling window which is fixed at the beginning.

To select r_w, [6] suggested a method based on level of significance (α) and sample size (n). They argued that r_w needs to be chosen according to the total number of sample observations n. If n is small, r_w need to be large enough to ensure that there are enough observations for adequate initial estimation. If n is large, r_w can be set to a smaller number so that the test does not miss any opportunity to detect an early explosive episode. Thus for practical usage they have recommended a formula to determine r_w which is given below.

$$r_w = \alpha + \frac{1.8}{\sqrt{n}} \tag{3}$$

2.3 Max SADF Tests (MSADF Tests)

It is clear that Max conventional unit root test and SADF test are more suitable to detect periodically collapsible bubbles. Thus recently [7] proposed a new set of ADF tests based on SADF tests and Max test. They called the resulting tests as MSADF tests. The rationality behind this procedure is, to take advantage of Max test and SADF test by combining them. The testing procedure of MSADF tests can be explained as follows.

As the name itself suggest, here first we compute the SADF test statistic for the observations in the forward and reverse order separately. Then the maximum among the set of statistic computed is treated as MSADF test statistic. Symbolically, the test statistic for two test procedures can be represented as follows.

$$recursive_{r_w}\ MSADF = \sup_{r_w \in [r_0, 1]} \left(ADF_{r_w}^{f}, ADF_{r_w}^{r} \right) \tag{4}$$

$$rolling_{r_w}\ MSADF = \sup_{r_w} \left(ADF_t^{f}, ADF_t^{r} \right), t = 1,2,3,\ldots \tag{5}$$

where r_0 is the size of initial subsample and r_w is the size of rolling window which is determined as explained earlier. Further they compared power of their tests to detect bubbles with SADF tests proposed by Phillips and found that MSADF tests have better power than the SADF tests.

Here our objective is to observe the effects of variations in the parameters of Evans model on the powers of these unit root tests via Monte Carlo simulation. Thus we carried out a simulation study and results obtained were presented in Section 3

3. DISCUSSION

First we generate the periodically collapsible bubbles by using following Data Generating Process (DGP) as suggested by [3].

$$B_{t+1} = \begin{cases} \rho^{-1} B_t u_{t-1} & \text{if} \quad B_t \le \alpha \\ [\xi + (\lambda\rho)^{-1}\theta_{t+1}(B_t - \rho\xi)]u_{t+1} & \text{if} \quad B_t > \alpha \end{cases} \tag{6}$$

where $\rho = \dfrac{1}{1+g}$, α >0, 0 < ξ < (1+g)α , $u_t \sim \exp(y_t - \tau^2/2)$; $y_t \sim N(0,\tau^2)$ and θ_t is a exogenous independently and identically distributed Bernoulli process which takes value one with probability λ and zero with probability (1-λ). This DGP is sensitive to the choice of the probability of bubble collapse (λ), initial size of bubble (B_0), growth rate of bubble (g) and volatility of bubble (τ) .

To compute the power of different ADF tests, the asset price series P_t is generated by adding fundamental price (which does not contain bubble) series F_t and the bubble price series B_t.The equations used for computing F_t and B_t are as follows.

$$F_t = \frac{1-g}{g} + \frac{1-g}{(1+g)g^2}u + \frac{r_t}{g} \qquad \textit{where} \qquad r_t = c + r_{t-1} + \varepsilon_t, \varepsilon_t \sim N(0,\sigma^2)$$

and $$P_t = F_t + B_t \tag{7}$$

Throughout the analysis, ADF regression model with drift is considered.

$$\Delta y_t = a + \beta y_{t-1} + \sum_{i=1}^{k} \varphi_i \Delta y_{t-i} + \varepsilon_t, \varepsilon_t \sim N(0,\sigma^2)$$

The optimum lag length is found to be zero which is determined using top-down sequential test procedure proposed by [8]. The suitable window length for SADF and MSADF tests were selected as explained above. To compute critical values for the tests, the observations were simulated using simple random walk model as DGP. Table 1 reports the critical values for the different test procedures under study.

Table1: Represents Critical values at 5% significance level for different ADF test

	Conventional ADF		Recursive SADF(w=20)		Rolling SADF(w=35)	
Method	Forward	Max	Forward	Max	Forward	Max
Critical Value	-0.0893	0.2598	1.4404	1.6922	1.7123	1.9263

Note: Observations were simulated using simple random walk model. w represents the window length in respective tests.

One way of quantifying the quality of a statistical test is in terms of its "power". Whenever a new test is proposed or one wish to observe the effects of parameters on the DGP used, a Monte Carlo simulation is carried out. Since our interest is to observe the effects of 2 parameters of Evans bubble generating model namely, growth rate of bubble (g) and volatility of bubble (τ), on the powers of different ADF tests, a simulation of 5000 frequency is carried out. Power of the test is computed by dividing the number of times successful rejection of the null hypothesis from the total number of simulation frequency. Table 2 to Table 7 represents the results of simulation study.

Table2: Represents the power of Forward and Max conventional ADF test at 5% significant level with varying bubble growth rate Vs Collapsible probability.

	Forward Conventional ADF				Max Conventional ADF			
g / ʎ	0.09	0.05	0.02	0.001	0.09	0.05	0.02	0.001
0.90	0.2428	0.4294	0.5028	0.5128	0.2582	0.5746	0.7380	0.7558
0.50	0.4072	0.4832	0.5086	0.5128	0.5034	0.6784	0.7448	0.7558
0.20	0.4402	0.4930	0.5100	0.5128	0.5788	0.7014	0.7478	0.7558
0.10	0.4552	0.4972	0.5082	0.5128	0.6060	0.7072	0.7490	0.7558

Source: Author's calculation

Table 3: Represents the power of Forward and Max recursive SADF test with window length w=20 at 5% significant level with varying bubble growth rate Vs Collapsible probability.

	Forward recursive SADF				Max recursive SADF			
g / ʎ	0.09	0.05	0.02	0.001	0.09	0.05	0.02	0.001
0.90	0.9078	0.6896	0.5466	0.5308	0.8958	0.7136	0.6294	0.6168
0.50	0.6680	05838	0.5380	0.5308	0.6792	0.6410	0.6280	0.6168
0.20	0.5824	0.5466	0.5314	0.5308	0.6068	0.6192	0.6168	0.6168
0.10	0.5554	0.5406	0.5322	0.5308	0.5810	0.6168	0.6168	0.6168

Source: Author's calculation

Table 4: Represents the power of Forward and Max rolling SADF test with window length w=35 at 5% significant level with varying bubble growth rate Vs Collapsible probability.

	Forward rolling SADF				Max rolling SADF			
g / ʎ	0.09	0.05	0.02	0.001	0.09	0.05	0.02	0.001
0.90	0.9746	0.9222	0.8736	0.8686	0.9718	0.9338	0.9026	08986
0.50	0.9038	0.8830	0.8710	0.8686	0.9020	0.9028	0.8998	0.8986
0.20	0.8570	0.8790	0.8680	0.8686	0.8724	0.9012	0.8978	0.8986
0.10	0.8394	0.8676	0.8700	0.8686	0.8520	0.8964	0.8998	0.8986

Source: Author's calculation

Table 5: Represents the power of Forward and Max conventional ADF test at 5% significant level with varying bubble volatility rate Vs Collapsible probability.

	Forward Conventional ADF				Max Conventional ADF			
τ / ʎ	0.05	0.01	0.005	0.0025	0.05	0.01	0.005	0.0025
0.90	0.4330	0.4296	0.4296	0.4294	0.5850	0.5752	0.5746	0.5746
0.50	0.4834	0.4832	0.4832	0.4832	0.6806	0.6784	0.6784	0.6784
0.20	0.4948	0.4930	0.4930	0.4930	0.7018	0.7014	0.7014	0.7014
0.10	0.4986	0.4972	0.4972	0.49972	0.7074	0.7076	0.7072	0.7072

Source: Author's calculation

Table 6: Represents the power of Forward and Max recursive SADF test with window length w=20 at 5% significant level with varying bubble volatility rate Vs Collapsible probability.

	Forward recursive SADF				Max recursive SADF			
τ / ʎ	0.05	0.01	0.005	0.0025	0.05	0.01	0.005	0.0025
0.90	0.6774	0.6860	0.6860	0.6896	0.7068	0.7100	0.7100	0.7136
0.50	0.5710	0.5734	0.5736	0.5838	0.6334	0.6322	0.6324	0.6410
0.20	0.5416	0.5420	0.5420	0.5466	0.6152	0.6132	0.6132	0.6192
0.10	0.5368	0.5362	0.5362	0.5406	0.5150	0.6132	0.6140	0.6168

Source: Author's calculation

Table 7: Represents the power of Forward and Max rolling SADF test with window length w=35 at 5% significant level with varying bubble volatility rate Vs Collapsible probability.

	Forward rolling SADF				Max rolling SADF			
τ / κ	0.05	0.01	0.005	0.0025	0.05	0.01	0.005	0.0025
0.90	0.9728	0.9144	0.9142	0.9222	0.9716	0.9272	0.9270	0.9338
0.50	0.8764	0.8762	0.8764	0.8830	0.9018	0.8982	0.8982	0.9028
0.20	08696	0.8690	0.8688	0.8790	0.8890	0.8898	0.8898	0.9012
0.10	0.8636	0.8634	0.8634	0.8676	0.8892	0.8892	0.8894	0.8964

Source: Author's calculation

<u>*Observations*</u>

i) It is interesting to see that the power of the test procedures (Conventional, SADF and MSADF) remains constant for all values of κ when g=0.001. Phillips et. al (2011) observed that the power of the Conventional ADF test drastically decreases as κ decreases (even we compare κ=0.99 and 0.95) due to the presence of collapsible bubble in the series. But in our simulation study, when the growth rate of bubble is less than or equals to 0.001, the power of the test remains same irrespective of value of κ. Thus we can say that the model proposed by Evans not generating collapsible bubbles if growth rate is very low.

ii) In Forward and Max conventional tests, the power of the test increases as κ or g or τ decreases (Table 2 and 5).

iii) In Forward and Max recursive tests, power of the test decreases as κ or g decreases and in most of the cases the power increases with decrease in τ (Table 3 and 6).

iv) In Forward and Max rolling tests, power of the test decreases as κ or g decreases (Table 4 and 7).

v) In all the cases, the power of the Max SADF test is superior to Forward SADF tests.

vi) Among recursive and rolling ADF procedures, the later performs better than the former.

4. CONCLUSION

In this paper the effects of variations in the parameters of Evans model on the powers of different unit root tests is observed via 5000 Monte Carlo simulation. We found that the power of Forward and Max conventional tests increases as κ or g or τ decreases. Further the power of Forward and Max recursive as well as rolling tests decreases as ʎ or g decreases. It is interesting to see that the power of all the test procedures (Conventional, SADF and MSADF) remains constant for all values of ʎ when g=0.001. It indicates that the model proposed by Evans is not generating collapsible bubbles if growth rate is very low.

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LIST OF ABBREVIATIONS

ADF: Augmented Dickey Fuller

SADF: Supremum Augmented Dickey Fuller

Max SADF: Maximum Supremum Augmented Dickey Fuller

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