

On Continuity Property of Grill Topological Spaces into Topological Spaces via $\mathcal{G}_\omega^\alpha$ –Open Sets

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Abstract

In this paper, we study the continuity property for functions of grill topological spaces into topological spaces by using $\mathcal{G}_\omega^\alpha$ –open sets. We give the concept of $\mathcal{G}_\omega^\alpha$ –continuous functions which is weak form of continuous functions in grill topological spaces into topological spaces. We also study the tow properties contra and almost of the class of functions of grill topological spaces into topological spaces by using $\mathcal{G}_\omega^\alpha$ –open sets.

Keywords: Grill topological space, continuous function.

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1. INTRODUCTION

In topological spaces, some studies have introduced weak form of continuous functions such as in 1984, T. Noiri [9] introduced the class of α –continuous functions in topological spaces. In 1989, Hdeib [5] introduced the class of ω –continuous functions as a weak form of class of continuous functions in topological spaces. In 2009, [7], they introduced the class of $\alpha - \omega$ –continuous functions as class of continuous functions in topological spaces. In 2016, [13], they introduced in topological spaces class of semi– ω –continuous functions. In 2011, [14], they introduced the class of $\mathcal{G} - \alpha$ –continuous functions and class of $\mathcal{G}$ –semi continuous functions in grill topological spaces into topological spaces. In 2021, [1], they introduced the class of $\mathcal{G}^\omega$ –continuous functions in grill topological spaces.

In this work, we investigate and introduce new class of functions in grill topological spaces by using $\mathcal{G}_\omega^\alpha$ –open sets, say class of $\mathcal{G}_\omega^\alpha$ –continuous functions, contra $\mathcal{G}_\omega^\alpha$ –continuous functions, almost $\mathcal{G}_\omega^\alpha$ –continuous functions and almost contra $\mathcal{G}_\omega^\alpha$ –continuous functions.

2. PRELIMINARIES

For a topological space (X, τ) and $A \subseteq X$, throughout this paper, we mean $Cl(A)$ and $Int(A)$ the closure set and the interior set of A , respectively. A subset A of a topological space (X, T) is called: r –open set [16] if $A = Int(Cl(A))$. The complement of r –open set is called r –closed set. A subset A of a space X is called ω –open set [4] if for each $x \in A$, there is an open set U_x containing x such that $U_x - A$ is a countable set. The complement of a ω –open set is called a ω –closed set. The set of all ω –closed sets in X denoted by $\omega C(X, \tau)$ and the set of all ω –open sets in X denoted by $\omega O(X, \tau)$.

Theorem 2.1. [4] For a topological space (X, τ) , the pair $(X, \omega O(X, \tau))$ forms a topological space.

In this work, $Int_\omega(A)$ denotes the ω –interior operator of A defined as the union of all ω –open sets which contained in A and $Cl_\omega(A)$ denotes the ω –closure operator of A defined as the intersection of all ω –closed sets which contain A .

Theorem 2.2. [4] Let A be sub set of X in a topological space (X, τ) . Then

1. $Cl_\omega(A) \subseteq Cl(A)$ for all $A \subseteq X$.
2. $Int(A) \subseteq Int_\omega(A)$ for all $A \subseteq X$.

Definition 2.3. [5] A function $f : (X, \tau) \rightarrow (Y, \sigma)$ of a topological space (X, τ) into a space (Y, σ) is called ω –continuous function if $f^{-1}(A)$ is ω –open set with (X, τ) for every open set A in (Y, σ) .

Definition 2.4. A subset A of a topological space (X, T) is called:

1. α –open set [8] if $A \subseteq Int(Cl(Int(A)))$. The complement of α –open set is called α –closed set.
2. α –continuous function [9] if $f^{-1}(A)$ is α –open set with (X, τ) for every open set A in (Y, σ) .

Definition 2.5. [7] Let (X, T) be a topological space.

1. A subset A of X is called $\alpha - \omega$ -open set if $A \subseteq Int_{\omega}(Cl(Int_{\omega}(A)))$. The complement of $\alpha - \omega$ -open set is called $\alpha - \omega$ -closed set.
2. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ of a topological space (X, τ) into a space (Y, σ) is called $\alpha - \omega$ -continuous if $f^{-1}(A)$ is $\alpha - \omega$ -open set with (X, τ) for every open set A in (Y, σ) .

It is clear that every α -continuous function is $\alpha - \omega$ -continuous function set.

Definition 2.6. Let (X, T) be a topological space.

1. A subset A of X is called semi- ω -open set [12] if $A \subseteq Cl(Int_{\omega}(A))$. The complement of semi- ω -open set is called semi- ω -closed set.
2. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ of a topological space (X, τ) into a space (Y, σ) is called semi- ω -continuous function [13] if $f^{-1}(A)$ is semi- ω -open set with (X, τ) for every open set A in (Y, σ) .

Theorem 2.7. [7] Every ω -open set is $\alpha - \omega$ -open set.

Theorem 2.8. [13] Every $\alpha - \omega$ -continuous function is semi- ω -continuous function.

Definition 2.9. [10] A non-null collection $\mathcal{G}$ of subsets of a topological spaces (X, τ) is said to be a *grill* on X if $\mathcal{G}$ satisfies the following conditions:

- (i) $A \in \mathcal{G}$ and $A \subseteq B$ implies that $B \in \mathcal{G}$
- (ii) $A, B \subseteq X$ and $A \cup B \in \mathcal{G}$ implies that $A \in \mathcal{G}$ or $B \in \mathcal{G}$.

For a grill topological space $(X, \tau, \mathcal{G})$, the operator $\Phi : P(X) \rightarrow P(X)$ from the power set $P(X)$ of X to $P(X)$ was defined in [11] in the following manner : For any $A \in P(X)$,

$$\Phi(A) = \{x \in X : U \cap A \in \mathcal{G}, \text{ for each open neighborhood } U \text{ of } x\}.$$

This operator is called the operator associated with the grill $\mathcal{G}$ and the topology τ .

Then the operator $\Psi : P(X) \rightarrow P(X)$, given by $\Psi(A) = A \cup \Phi(A)$, for $A \in P(X)$, was also shown in [11] to be a Kuratowski closure operator. So for a grill topological space $(X, \tau, \mathcal{G})$ there exists an unique topology $\tau_{\mathcal{G}}$ on X defined by

$$\tau_{\mathcal{G}} = \{U \subseteq X : \Psi(X - U) = X - U\},$$

where $\tau \subseteq \tau_{\mathcal{G}}$ and for any $A \subseteq X$, $\Psi(A) = Cl_{\mathcal{G}}(A)$ such that $Cl_{\mathcal{G}}(A)$ denotes the set of all $\mathcal{G}$ -closure points of A . A point $x \in X$ is called a $\mathcal{G}$ -closure point of A if

for every open set U in $(X, \tau_{\mathcal{G}})$ containing x , $U \cap A \neq \emptyset$. A point $x \in A$ is called a $\mathcal{G}$ -interior point of A if there is open set U in $(X, \tau_{\mathcal{G}})$ such that $x \in U \subseteq A$. The set of all $\mathcal{G}$ -interior points of A denoted by $\text{int}_{\mathcal{G}}(A)$.

Theorem 2.10. [11] Let $(X, \tau, \mathcal{G})$ be a grill topological space. Then for $A, B \subseteq X$, the following properties hold:

1. $A \subseteq B$ implies that $\Phi(A) \subseteq \Phi(B)$;
2. $\Phi(A \cup B) = \Phi(A) \cup \Phi(B)$;
3. $\Phi(\Phi(A)) \subseteq \Phi(A) = Cl(\Phi(A)) \subseteq Cl(A)$;
4. If $U \in \tau$ then $U \cap \Phi(A) \subseteq \Phi(U \cap A)$.

Definition 2.11. [1] A subset A of grill topological space $(X, \tau, \mathcal{G})$ is called a $\mathcal{G}^{\omega}$ -open set if $A \subseteq Cl(\text{Int}_{\omega}(\Psi(A)))$. The complement of $\mathcal{G}^{\omega}$ -open set is called $\mathcal{G}^{\omega}$ -closed set.

Definition 2.12. [2] A function $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ of a grill topological space $(X, \tau, \mathcal{G})$ into a space (Y, σ) is called $\mathcal{G}^{\omega}$ -continuous function if $f^{-1}(A)$ is $\mathcal{G}^{\omega}$ -open set in $(X, \tau, \mathcal{G})$ for every open set A in (Y, σ) .

Theorem 2.13. [2] Every semi- ω -continuous function is $\mathcal{G}^{\omega}$ -continuous function is

Definition 2.14. [14] A subset A of a grill topological space $(X, \tau, \mathcal{G})$ is called a $\mathcal{G}\alpha$ -open set if $A \subseteq \text{Int}(\Psi(\text{Int}(A)))$. The complement of $\mathcal{G} - \alpha$ -open set is called $\mathcal{G} - \alpha$ -closed set.

Definition 2.15. [14] A function $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ of a grill topological space $(X, \tau, \mathcal{G})$ into a space (Y, σ) is called $\mathcal{G} - \alpha$ -continuous function if $f^{-1}(A)$ is $\mathcal{G} - \alpha$ -open set in $(X, \tau, \mathcal{G})$ for every open set A in (Y, σ) .

Definition 2.16. [14] A subset A of a grill topological space $(X, \tau, \mathcal{G})$ is called a $\mathcal{G}$ -semi-open set if $A \subseteq \Psi(\text{Int}(A))$. The complement of $\mathcal{G}$ -semi-open set is called $\mathcal{G}$ -semi-closed set.

Definition 2.17. [14] A function $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ of a grill topological space $(X, \tau, \mathcal{G})$ into a space (Y, σ) is called $\mathcal{G}$ -semi-continuous function if $f^{-1}(A)$ is $\mathcal{G}$ -semi-open set in $(X, \tau, \mathcal{G})$ for every open set A in (Y, σ) .

It is clear that every $\mathcal{G}$ -semi-continuous function is semi- ω -continuous function.

Theorem 2.18. [14] Every $\mathcal{G} - \alpha$ -continuous function is $\mathcal{G}$ -semi-continuous function.

Definition 2.19. [3] A subset A of grill topological space $(X, \tau, \mathcal{G})$ is called a $\mathcal{G}_\omega^\alpha$ –open set if $A \subseteq \text{Int}(\Psi(\text{Int}_\omega(A)))$. The complement of $\mathcal{G}_\omega^\alpha$ –open set is called $\mathcal{G}_\omega^\alpha$ –closed set. The set of all $\mathcal{G}_\omega^\alpha$ –open sets in X is denoted by $\mathcal{G}_\omega^\alpha O(X, \tau)$ and the set of all $\mathcal{G}_\omega^\alpha$ –closed sets in X is denoted by $\mathcal{G}_\omega^\alpha C(X, \tau)$.

For a topological space (X, τ) and $A \subseteq X$, throughout this paper, we mean $_{\mathcal{G}_\omega^\alpha} Cl(A)$ and $_{\mathcal{G}_\omega^\alpha} Int(A)$ the $\mathcal{G}_\omega^\alpha$ –closure set and the $\mathcal{G}_\omega^\alpha$ –interior set of A , respectively.

Theorem 2.20. [3] A subset A of a grill topological space $(X, \tau, \mathcal{G})$ is $\mathcal{G}_\omega^\alpha$ –closed set if and only if $Cl(int_{\mathcal{G}}(Cl_\omega(A))) \subseteq A$.

3. $\mathcal{G}_\omega^\alpha$ –CONTINUOUS FUNCTIONS

Definition 3.1. A function $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ of a grill topological space $(X, \tau, \mathcal{G})$ into a space (Y, σ) is called $\mathcal{G}_\omega^\alpha$ –continuous function if $f^{-1}(A)$ is $\mathcal{G}_\omega^\alpha$ –open set in $(X, \tau, \mathcal{G})$ for every open set A in (Y, σ) .

Example 3.2. Let $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function defined by

$$f(x) = \begin{cases} a, & x \in \{2, 3\} \\ b, & x = 1 \end{cases}$$

where $Y = \{a, b\}$, $X = \{1, 2, 3\}$,

$$\tau = \{\emptyset, X, \{2, 3\}\}, \mathcal{G} = P(X) - \{\emptyset\}, \text{ and } \sigma = \{\emptyset, Y, \{a\}\}.$$

The function f is $\mathcal{G}_\omega^\alpha$ –continuous, since $f^{-1}(\emptyset) = \emptyset$, $f^{-1}(\{a\}) = \{2, 3\}$ and $f^{-1}(Y) = X$ are $\mathcal{G}_\omega^\alpha$ –open sets in $(X, \tau, \mathcal{G})$.

For any grill topological space $(X, \tau, \mathcal{G})$ with a countable set X , it is clear that every $\mathcal{G}_\omega^\alpha$ –continuous function is ω –continuous function but the converse of this fact no need to be true.

Example 3.3. Let $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function defined by

$$f(x) = \begin{cases} a, & x \in \{2, 3\} \\ b, & x = 1 \end{cases}$$

where $Y = \{a, b\}$, $X = \{1, 2, 3\}$,

$$\tau = \{\emptyset, X, \{2, 3\}\}, \mathcal{G} = P(X) - \{\emptyset\}, \text{ and } \sigma = \{\emptyset, Y, \{b\}\}.$$

The function f is ω –continuous, since $f^{-1}(b) = \{1\}$ and $f^{-1}(Y) = X$ are ω –open sets, but not $\mathcal{G}_\omega^\alpha$ –continuous, since there exist an open set $\{b\}$ in (Y, σ) but $f^{-1}(\{b\}) = \{1\}$ is not $\mathcal{G}_\omega^\alpha$ –open set in $(X, \tau, \mathcal{G})$.

The following two examples in a grill topological space $(\mathbb{R}, \tau, \mathcal{G})$, there are not relationship between class of ω -continuous function and class of $\mathcal{G}_\omega^\alpha$ -continuous

Example 3.4. Let $f : (\mathbb{R}, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function defined by

$$f(x) = \begin{cases} a, & x \in \mathbb{R} - \{2\} \\ b, & x = 2 \end{cases}$$

where $Y = \{a, b\}$,

$$\tau = \{\emptyset, \mathbb{R}, \{2\}\}, \mathcal{G} = P(\mathbb{R}) - \{\emptyset\}, \text{ and } \sigma = \{\emptyset, Y, \{a\}\}.$$

The function f is ω -continuous, since $f^{-1}(\{a\}) = \mathbb{R} - \{2\}$ and $f^{-1}(Y) = \mathbb{R}$ are ω -continuous but a set $\mathbb{R} - \{2\}$ is not $\mathcal{G}_\omega^\alpha$ -open set in a grill topological space $(\mathbb{R}, \tau, \mathcal{G})$.

Example 3.5. Let $f : (\mathbb{R}, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function defined by

$$f(x) = \begin{cases} a \text{ if } & -\infty < x \leq 0 \\ b \text{ if } & 0 < x < 4 \\ c \text{ if } & 4 \leq x < \infty \end{cases}$$

where $Y = \{a, b, c\}$,

$$\tau = \{\emptyset, \mathbb{R}, [1, 2]\}, \mathcal{G} = P(\mathbb{R}) - \{\emptyset\}, \text{ and } \sigma = \{\emptyset, Y, \{b\}\}.$$

The function f is $\mathcal{G}_\omega^\alpha$ -continuous, since $f^{-1}(\{b\}) = (0, 4)$ and $f^{-1}(Y) = \mathbb{R}$ are $\mathcal{G}_\omega^\alpha$ -open sets in a grill topological space $(\mathbb{R}, \tau, \mathcal{G})$ but a set $(0, 4)$ is not ω -open set in $\mathbb{R}$ with $(\mathbb{R}, \tau)$.

It is clear that every $\mathcal{G} - \alpha$ -continuous function is $\alpha - \omega$ -continuous function and semi- ω -continuous function, but the converse of this fact no need to be true.

Example 3.6. See above example. The function f is $\alpha - \omega$ -continuous, since $f^{-1}(\{a\}) = \mathbb{R} - \{2\}$ and $f^{-1}(Y) = \mathbb{R}$ are $\mathcal{G}_\omega^\alpha$ -continuous and semi- ω -continuous function, but a set $\mathbb{R} - \{2\}$ is not $\mathcal{G} - \alpha$ -open set in a grill topological space $(X, \tau, \mathcal{G})$.

The functions $\mathcal{G}_\omega^\alpha$ -continuous and $\mathcal{G}$ -same-continuous, are independent.

Example 3.7. Let $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function defined by

$$f(x) = \begin{cases} a, & x \in \{1, 2\} \\ b, & x = 3 \end{cases}$$

where $Y = \{a, b\}$, $X = \{1, 2, 3\}$,

$$\tau = \{\emptyset, X, \{2, 3\}\}, \mathcal{G} = P(X) - \{\emptyset\}, \text{ and } \sigma = \{\emptyset, Y, \{a\}\}.$$

The function f is $\mathcal{G}_\omega^\alpha$ -continuous, since $f^{-1}(\emptyset) = \emptyset$, $f^{-1}(\{a\}) = \{1, 2\}$ and $f^{-1}(Y) = X$ are $\mathcal{G}_\omega^\alpha$ -open sets in $(X, \tau, \mathcal{G})$ but a set $\{1, 2\}$ is not $\mathcal{G}$ -same-open set.

Example 3.8. Let $f : (\mathbb{R}, \tau_u, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function defined by

$$f(x) = \begin{cases} a & \text{if } -\infty < x < 0 \\ b & \text{if } 0 \leq x \leq 4 \\ c & \text{if } 4 < x < \infty \end{cases}$$

where $Y = \{a, b, c\}$,

$$\tau = \tau_u, \mathcal{G} = P(\mathbb{R}) - \{\emptyset\}, \text{ and } \sigma = \{\emptyset, Y, \{b\}\}.$$

The function f is $\mathcal{G}_\omega^\alpha$ -continuous, since $f^{-1}(\{b\}) = [0, 4]$ and $f^{-1}(Y) = \mathbb{R}$ are $\mathcal{G}$ -same-open sets in a grill topological space $(\mathbb{R}, \tau_u, \mathcal{G})$ but a set $f^{-1}(\{b\}) = [0, 4]$ is not $\mathcal{G}_\omega^\alpha$ -open set.

Theorem 3.9. Let $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function of a grill topological space $(X, \tau, \mathcal{G})$ into a space (Y, ρ) , then every $\mathcal{G} - \alpha$ -continuous function is $\mathcal{G}_\omega^\alpha$ -continuous function.

The converse of theorem above no need to be true.

Example 3.10. Let $f : (\mathbb{R}, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function defined by

$$f(x) = \begin{cases} a, & x \in \mathbb{R} - \{2\} \\ b, & x = 2 \end{cases}$$

where $Y = \{a, b\}$,

$$\tau = \{\emptyset, \mathbb{R}, \mathbb{R} - \{1\}\}, \mathcal{G} = P(\mathbb{R}) - \{\emptyset\}, \text{ and } \sigma = \{\emptyset, Y, \{a\}\}.$$

The function f is $\mathcal{G}_\omega^\alpha$ -continuous, since $f^{-1}(\{a\}) = \mathbb{R} - \{2\}$ and $f^{-1}(Y) = \mathbb{R}$ are $\mathcal{G}_\omega^\alpha$ -open but a set $\mathbb{R} - \{2\}$ is not $\mathcal{G} - \alpha$ -open set in a grill topological space $(\mathbb{R}, \tau, \mathcal{G})$.

It is clear that every $\mathcal{G}_\omega^\alpha$ -continuous function is $\mathcal{G}^\omega$ -continuous function but the converse of this fact no need to be true. See Example (2). The function f is $\mathcal{G}^\omega$ -continuous, since $f^{-1}(b) = \{1\}$ and $f^{-1}(Y) = X$ are $\mathcal{G}^\omega$ -open sets, but an open set $\{b\}$ in (Y, σ) and $f^{-1}(\{b\}) = \{1\}$ is not $\mathcal{G}_\omega^\alpha$ -open set in $(X, \tau, \mathcal{G})$.

From all the previous relationships in our work and the preliminaries, we have the following figure.

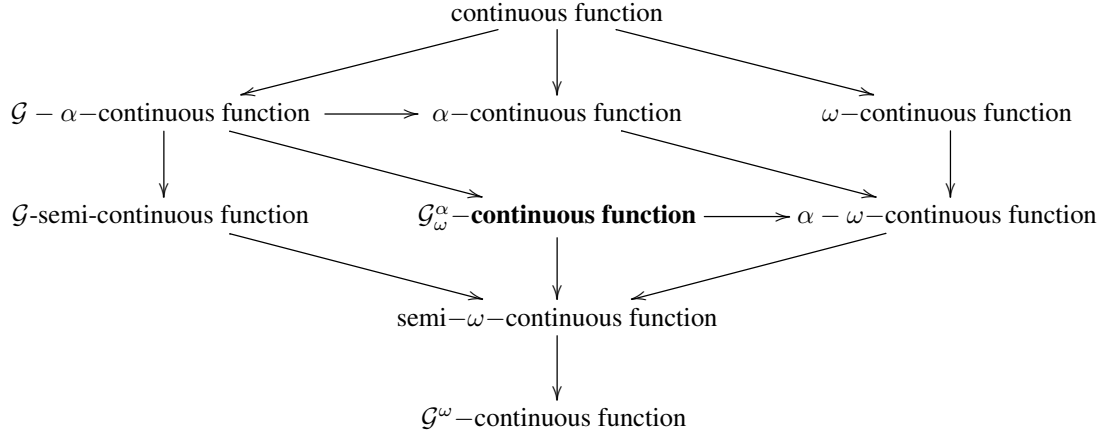


Figure 1: Relation for continuous functions

Theorem 3.11. Let $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function of a grill topological space $(X, \tau, \mathcal{G})$ into a space (Y, σ) . Then f is $\mathcal{G}_\omega^\alpha$ -continuous if and only if $f^{-1}(F)$ is $\mathcal{G}_\omega^\alpha$ -closed set in $(X, \tau, \mathcal{G})$ for every closed set F in (Y, σ) .

Proof. Suppose that f is $\mathcal{G}_\omega^\alpha$ -continuous function. Let F be any closed set in (Y, σ) then $Y - F$ is open set in (Y, σ) . Since f is $\mathcal{G}_\omega^\alpha$ -continuous function. Then $f^{-1}(Y - F) = X - f^{-1}(F)$ is $\mathcal{G}_\omega^\alpha$ -open set in a grill topological space $(X, \tau, \mathcal{G})$. Hence $f^{-1}(F)$ is $\mathcal{G}_\omega^\alpha$ -closed set in $(X, \tau, \mathcal{G})$. Conversely, Let U be any open set in (Y, σ) . Then $Y - U$ is closed set in (Y, σ) . So by the hypothesis $f^{-1}(Y - U) = X - f^{-1}(U)$ is $\mathcal{G}_\omega^\alpha$ -closed set in a grill topological space $(X, \tau, \mathcal{G})$. Therefore $f^{-1}(U)$ is $\mathcal{G}_\omega^\alpha$ -open set in a grill topological space $(X, \tau, \mathcal{G})$ for any open set in space (Y, σ) . Hence f is $\mathcal{G}_\omega^\alpha$ -continuous function. $\square$

Theorem 3.12. A function $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ of a grill topological space $(X, \tau, \mathcal{G})$ into a space (Y, ρ) is $\mathcal{G}_\omega^\alpha$ -continuous if and only if for each $x \in X$ and each open set U in Y with $f(x) \in U$, there exists $\mathcal{G}_\omega^\alpha$ -open set A in $(X, \tau, \mathcal{G})$ such that $x \in A$ and $f(A) \subseteq U$.

Proof. Suppose that f is $\mathcal{G}_\omega^\alpha$ -continuous function. Let $x \in X$ and U be any open set in Y containing $f(x)$. Put $A = f^{-1}(U)$. Since f is a $\mathcal{G}_\omega^\alpha$ -continuous then A is $\mathcal{G}_\omega^\alpha$ -open set in $(X, \tau, \mathcal{G})$ such that $x \in A$ and $f(A) \subseteq U$. Conversely, Let U be any open set in

(Y, σ) . For each $x \in f^{-1}(U)$, $f(x) \in U$. Then by the hypothesis, there exists $\mathcal{G}_\omega^\alpha$ -open set A_x in $(X, \tau, \mathcal{G})$ such that $x \in A_x$ and $f(A_x) \subseteq U$. This mean that, $A_x \subseteq f^{-1}(U)$ and so $f^{-1}(U) = \cup_{x \in f^{-1}(U)} A_x$. Hence,

$$f^{-1}(U) = \cup_{x \in f^{-1}(U)} A_x$$

is $\mathcal{G}_\omega^\alpha$ -open set in $(X, \tau, \mathcal{G})$. That is, f is $\mathcal{G}_\omega^\alpha$ -continuous. $\square$

Theorem 3.13. Let $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function of a grill topological space $(X, \tau, \mathcal{G})$ into a space (Y, σ) , the following properties are equivalent:

1. $\mathcal{G}_\omega^\alpha$ -continuous function is $\mathcal{G} - \alpha$ -continuous function.
2. for each $x \in X$ and for each open set H containing $f(x)$ in space (Y, σ) there exist a set A where A both $\mathcal{G} - \alpha$ -open set and $\mathcal{G}_\omega^\alpha$ -open set such that $x \in f(A) \subseteq H$.

Proof. Suppose that $\mathcal{G}_\omega^\alpha$ -continuous function is $\mathcal{G} - \alpha$ -continuous function. Let x be any point in X and H be any open set containing $f(x)$ in space (Y, σ) then $f^{-1}(H)$ is a both $\mathcal{G}_\omega^\alpha$ -open and $\mathcal{G} - \alpha$ -open set. Put $f^{-1}(H) = B$, $f(B) \subseteq H$ Then by last theorem $x \in f(B) \subseteq f(B) \subseteq H$.

Conversely, Let U be any open set in (Y, σ) . For each $x \in f^{-1}(U)$, $f(x) \in U$. Then by the hypothesis, there exists a set A_x both $\mathcal{G}_\omega^\alpha$ -open and $\mathcal{G} - \alpha$ -open set in $(X, \tau, \mathcal{G})$ such that $x \in A_x$ and $f(A_x) \subseteq U$. This mean, $A_x \subseteq f^{-1}(U)$ and so $f^{-1}(U) = \cup_{x \in f^{-1}(U)} A_x$. Hence by last Theorem and definition(2.15). Hence $\mathcal{G}_\omega^\alpha$ -continuous function is $\mathcal{G} - \alpha$ -continuous function. $\square$

Theorem 3.14. Let $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function of a grill topological space $(X, \tau, \mathcal{G})$ into a space (Y, σ) , if $\mathcal{G}$ is a grill minimal of X , then a $\mathcal{G}_\omega^\alpha$ -continuous function, is $\mathcal{G} - \alpha$ -continuous function.

Theorem 3.15. Let $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a $\mathcal{G}_\omega^\alpha$ -continuous function of a grill topological space $(X, \tau, \mathcal{G})$ into a space (Y, σ) , then every $\mathcal{G}_\omega^\alpha$ -continuous function. is $\mathcal{G}$ -semi-continuous function. for each $x \in X$ and for each open set H containing $f(x)$ in space (Y, σ) there exist a set A where A both $\mathcal{G} - \alpha$ -open set and $\mathcal{G}_\omega^\alpha$ -open set such that $x \in f(A) \subseteq H$.

Theorem 3.16. A function $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ is $\mathcal{G}_\omega^\alpha$ -continuous of grill topological space $(X, \tau, \mathcal{G})$ into a space (Y, σ) if and only if

$$f^{-1}(\sigma \text{Int}(B)) \subseteq \mathcal{G}_\omega^\alpha(\text{Int} f^{-1}(B)) \text{ for all } B \subseteq Y.$$

Proof. Let f be $\mathcal{G}_\omega^\alpha$ -continuous function and B be any subset of Y . Then ${}_\sigma\text{Int}(B)$ is an open set in Y . Since f is $\mathcal{G}_\omega^\alpha$ -continuous then $f^{-1}[{}_\sigma\text{Int}(B)]$ is a $\mathcal{G}_\omega^\alpha$ -open set in $(X, \tau, \mathcal{G})$. That is,

$$\mathcal{G}_\omega^\alpha\text{Int}[f^{-1}[{}_\sigma\text{Int}(B)]] = f^{-1}[{}_\sigma\text{Int}(B)].$$

Since ${}_\sigma\text{Int}(B) \subseteq B$ then $f^{-1}[{}_\sigma\text{Int}(B)] \subseteq f^{-1}(B)$. This implies,

$$f^{-1}[{}_\sigma\text{Int}(B)] = \mathcal{G}_\omega^\alpha\text{Int}[f^{-1}[{}_\sigma\text{Int}(B)]] \subseteq \mathcal{G}_\omega^\alpha\text{Int}(f^{-1}(B)).$$

Hence $f^{-1}({}_\sigma\text{Int}(B)) \subseteq \mathcal{G}_\omega^\alpha\text{Int}[f^{-1}(B)]$.

Conversely, let U be any open set in Y , that is, ${}_\sigma\text{Int}(U) = U$. Since $U \subseteq Y$. Then by the hypothesis,

$$f^{-1}(U) = f^{-1}({}_\sigma\text{Int}(U)) \subseteq \mathcal{G}_\omega^\alpha\text{Int}[f^{-1}(U)].$$

This implies, $f^{-1}(U) \subseteq \mathcal{G}_\omega^\alpha\text{Int}[f^{-1}(U)]$. Hence $f^{-1}(U) = \mathcal{G}_\omega^\alpha\text{Int}[f^{-1}(U)]$, that is, $f^{-1}(U)$ is $\mathcal{G}_\omega^\alpha$ -open set in $(X, \tau, \mathcal{G})$. Hence f is $\mathcal{G}_\omega^\alpha$ -continuous. $\square$

Theorem 3.17. Let $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ and $h : (Y, \sigma) \rightarrow (Z, \rho)$ be two functions. If f is a $\mathcal{G}_\omega^\alpha$ -continuous from a grill topological space $(X, \tau, \mathcal{G})$ into space (Y, σ) and h is continuous from a topological space (Y, σ) into space (Z, ρ) , then $(h \circ f) : (X, \tau, \mathcal{G}) \rightarrow (Z, \rho)$ is a $\mathcal{G}_\omega^\alpha$ -continuous function in a grill topological space $(X, \tau, \mathcal{G})$.

Proof. Let $B \subseteq Z$ and $B \in \rho$. SO $(h \circ f)^{-1}(B) = (f^{-1} \circ h^{-1})(B) = f^{-1}(h^{-1}(B))$. Since h is continuous function, then $h^{-1}(B)$ is open set in space (Y, σ) , also $f^{-1}(h^{-1}(B))$ is a $\mathcal{G}_\omega^\alpha$ -open set in a grill topological space $(X, \tau, \mathcal{G})$. Therefore $(h \circ f)^{-1}(B)$ is a $\mathcal{G}_\omega^\alpha$ -open set in a grill topological space $(X, \tau, \mathcal{G})$. Hence $(h \circ f) : (X, \tau, \mathcal{G}) \rightarrow (Z, \rho)$ is a $\mathcal{G}_\omega^\alpha$ -continuous function $\square$

Theorem 3.18. A function $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ is $\mathcal{G}_\omega^\alpha$ -continuous of grill topological space $(X, \tau, \mathcal{G})$ into a space (Y, σ) if and only if

$$f[\mathcal{G}_\omega^\alpha\text{Cl}(A)] \subseteq {}_\sigma\text{Cl}(f(A)) \text{ for all } A \subseteq X.$$

Proof. Let f be $\mathcal{G}_\omega^\alpha$ -continuous function and A be any subset of X . Then ${}_\sigma\text{Cl}(f(A))$ is a closed set in Y . Since f is $\mathcal{G}_\omega^\alpha$ -continuous then by Theorem (3.11), $f^{-1}[{}_\sigma\text{Cl}(f(A))]$ is $\mathcal{G}_\omega^\alpha$ -closed set in $(X, \tau, \mathcal{G})$. That is,

$$\mathcal{G}_\omega^\alpha\text{Cl}[f^{-1}[{}_\sigma\text{Cl}(f(A))]] = f^{-1}[{}_\sigma\text{Cl}(f(A))].$$

Since $f(A) \subseteq {}_\sigma\text{Cl}(f(A))$ then $A \subseteq f^{-1}[{}_\sigma\text{Cl}(f(A))]$. This implies,

$$\mathcal{G}_\omega^\alpha\text{Cl}(A) \subseteq \mathcal{G}_\omega^\alpha\text{Cl}[f^{-1}[{}_\sigma\text{Cl}(f(A))]] = f^{-1}[{}_\sigma\text{Cl}(f(A))].$$

Hence $f[\mathcal{G}_\omega^\alpha Cl(A)] \subseteq \sigma Cl(f(A))$.

Conversely, let H be any closed set in Y , that is, $\sigma Cl(H) = H$. Since $f^{-1}(H) \subseteq X$. Then by the hypothesis,

$$f[\mathcal{G}_\omega^\alpha Cl[f^{-1}(H)]] \subseteq \sigma Cl[f(f^{-1}(H))] \subseteq \sigma Cl(H) = H.$$

This implies, $\mathcal{G}_\omega^\alpha Cl[f^{-1}(H)] \subseteq f^{-1}(H)$. Hence $\mathcal{G}_\omega^\alpha Cl[f^{-1}(H)] = f^{-1}(H)$, that is, $f^{-1}(H)$ is $\mathcal{G}_\omega^\alpha$ -closed set in $(X, \tau, \mathcal{G})$. Hence by Theorem (3.11), f is $\mathcal{G}_\omega^\alpha$ -continuous. $\square$

Theorem 3.19. A function $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ is $\mathcal{G}_\omega^\alpha$ -continuous of grill topological space $(X, \tau, \mathcal{G})$ into a space (Y, σ) if and only if

$$\mathcal{G}_\omega^\alpha Cl(f^{-1}(B)) \subseteq f^{-1}(\sigma Cl(B)) \text{ for all } B \subseteq Y.$$

Proof. Let f be $\mathcal{G}_\omega^\alpha$ -continuous function and B be any subset of Y . Then $\sigma Cl(B)$ is a closed set in Y . Since f is $\mathcal{G}_\omega^\alpha$ -continuous then by Theorem (3.11), $f^{-1}[\sigma Cl(B)]$ is $\mathcal{G}_\omega^\alpha$ -closed set in $(X, \tau, \mathcal{G})$. That is,

$$\mathcal{G}_\omega^\alpha Cl[f^{-1}[\sigma Cl(B)]] = f^{-1}[\sigma Cl(B)].$$

Since $B \subseteq \sigma Cl(B)$ then $f^{-1}(B) \subseteq f^{-1}[\sigma Cl(B)]$. This implies,

$$\mathcal{G}_\omega^\alpha Cl(f^{-1}(B)) \subseteq \mathcal{G}_\omega^\alpha Cl[f^{-1}[\sigma Cl(B)]] = f^{-1}[\sigma Cl(B)].$$

Hence $\mathcal{G}_\omega^\alpha Cl(f^{-1}(B)) \subseteq f^{-1}[\sigma Cl(B)]$.

Conversely, let F be any closed set in Y , that is, $\sigma Cl(F) = F$. Since $F \subseteq Y$. Then by the hypothesis,

$$\mathcal{G}_\omega^\alpha Cl(f^{-1}(F)) \subseteq f^{-1}(\sigma Cl(F)) = f^{-1}(F).$$

This implies, $\mathcal{G}_\omega^\alpha Cl[f^{-1}(F)] \subseteq f^{-1}(F)$. Hence $\mathcal{G}_\omega^\alpha Cl[f^{-1}(F)] = f^{-1}(F)$, that is, $f^{-1}(F)$ is $\mathcal{G}_\omega^\alpha$ -closed set in $(X, \tau, \mathcal{G})$. Hence by Theorem (3.11), $f^{-1}(F)$ is $\mathcal{G}_\omega^\alpha$ -closed set in $(X, \tau, \mathcal{G})$. That is, f is $\mathcal{G}_\omega^\alpha$ -continuous. $\square$

Definition 3.20. Let $(X, \tau, \mathcal{G})$ be grill topological space and (Y, σ) be topological space. Then the function $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ is called:

- $\mathcal{G}_\omega^\alpha$ -closed function if $f(U)$ is closed set in Y for every $\mathcal{G}_\omega^\alpha$ -closed set U in X .
- $\mathcal{G}_\omega^\alpha$ -open function if $f(U)$ is open set in (Y, σ) for every $\mathcal{G}_\omega^\alpha$ -open set U in $(X, \tau, \mathcal{G})$.

Example 3.21. Let $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma_D)$ be a function defined from a grill topological space $(X, \tau, \mathcal{G})$ into a discrete topological space (Y, σ_D) . Then f be a $\mathcal{G}_\omega^\alpha$ -open and a closed functions. Since for every subset A of Y we get $Int(A) = Cl(A)$, Then f $\mathcal{G}_\omega^\alpha$ -closed functions and $\mathcal{G}_\omega^\alpha$ -open functions.

Theorem 3.22. Let $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function from a grill topological space $(X, \tau, \mathcal{G})$ into a topological space (Y, σ) is $\mathcal{G}_\omega^\alpha$ -open function if and only if $f(\mathcal{G}_\omega^\alpha Int(B)) \subseteq {}_\sigma Int f(B)$ for every $B \in \text{space } (X, \tau, \mathcal{G})$.

Proof. Suppose that f is $\mathcal{G}_\omega^\alpha$ -open function we will prove that $f(\mathcal{G}_\omega^\alpha Int(B)) \subseteq {}_\sigma Int f(B)$ for every $B \in \text{space } (X, \tau, \mathcal{G})$. Let $B \subseteq X$, since $\mathcal{G}_\omega^\alpha Int(B) \subseteq B$. Then $f(\mathcal{G}_\omega^\alpha Int(B)) \subseteq f(B)$. By hypothesis that is implies $f(\mathcal{G}_\omega^\alpha Int(B)) = {}_\sigma Int f(\mathcal{G}_\omega^\alpha Int(B)) \subseteq {}_\sigma Int f(B)$ for any set B in space $(X, \tau, \mathcal{G})$.

Conversely, suppose that $f(\mathcal{G}_\omega^\alpha Int(B)) \subseteq {}_\sigma Int f(B)$ for every $B \in \text{space } (X, \tau, \mathcal{G})$. Let $B \subseteq X$ be any $\mathcal{G}_\omega^\alpha$ -open set in space $(X, \tau, \mathcal{G})$. Then $\mathcal{G}_\omega^\alpha Int(B) = B$. So by hypothesis that is $f(\mathcal{G}_\omega^\alpha Int(B)) = f(B) \subseteq {}_\sigma Int(f(B))$. Then

$$f(B) \subseteq {}_\sigma Int f(B) \quad (1)$$

Since every

$${}_\sigma Int f(B) \subseteq f(B). \quad (2)$$

From (1) and (2). Therefor $f(\mathcal{G}_\omega^\alpha Int(B)) = {}_\sigma Int f(B)$. Hence f is $\mathcal{G}_\omega^\alpha$ -open function. $\square$

Theorem 3.23. Let $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function from a grill topological space $(X, \tau, \mathcal{G})$ into a topological space (Y, σ) is $\mathcal{G}_\omega^\alpha$ -closed function if and only if ${}_\sigma Cl f(B) \subseteq f(\mathcal{G}_\omega^\alpha Cl(B))$ for every $B \in \text{space } (X, \tau, \mathcal{G})$.

Proof. It is similar the proof of above theorem. $\square$

4. CONTRA AND ALMOST $\mathcal{G}_\omega^\alpha$ -CONTINUOUS FUNCTIONS

Definition 4.1. Let $(X, \tau, \mathcal{G})$ be grill topological space and (Y, σ) be topological space. Then the function $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ is called: Contra $\mathcal{G}_\omega^\alpha$ -continuous function if $f^{-1}(V)$ is $\mathcal{G}_\omega^\alpha$ -closed set in $(X, \tau, \mathcal{G})$ for every open set V in (Y, σ) .

Example 4.2. See example(3.3) where $f^{-1}(Y)$, $f^{-1}(\{b\})$ and $f^{-1}(\emptyset)$ are $\mathcal{G}_\omega^\alpha$ -closed sets in $(X, \tau, \mathcal{G})$ for every open sets Y , $\{b\}$, and $\{\emptyset\}$ in (Y, σ) . So f is contra $\mathcal{G}_\omega^\alpha$ -continuous function.

Theorem 4.3. Let $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function from a grill topological space $(X, \tau, \mathcal{G})$ into a topological space (Y, σ) is contra $\mathcal{G}_\omega^\alpha$ -continuous function if and only if $f^{-1}(V)$ is $\mathcal{G}_\omega^\alpha$ -open set in $(X, \tau, \mathcal{G})$ for every closed set V in (Y, σ) .

Proof. Suppose that f is contra $\mathcal{G}_\omega^\alpha$ -continuous function. Let F be any closed sub set of Y then $Y - F$ is open set in space (Y, σ) . Then by the hypothesis $f^{-1}(Y - F) = X - f^{-1}(F)$ is closed set in $(X, \tau, \mathcal{G})$. Hence $f^{-1}(F)$ is open in $(X, \tau, \mathcal{G})$.

Conversely, Let A be any open set in space (Y, σ) so $Y - A$ is closed set in space (Y, σ) . Then by th hypothesis $(f^{-1}(Y - A) = X - f^{-1}(A))$ is $\mathcal{G}_\omega^\alpha$ -open set. Therefor $f^{-1}(A)$ is $\mathcal{G}_\omega^\alpha$ -closed set in $(X, \tau, \mathcal{G})$. Hence f is contra $\mathcal{G}_\omega^\alpha$ -continuous. $\square$

Theorem 4.4. Let $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function from a grill topological space $(X, \tau, \mathcal{G})$ into a topological space (Y, σ) is contra $\mathcal{G}_\omega^\alpha$ -continuous function if and only if for each $x \in X$ and open set A non-containing $f(x)$ in space (Y, σ) . their is $\mathcal{G}_\omega^\alpha$ -closed set B non-containing x such that $f^{-1}(A) \subseteq B$.

Proof. Suppose that f is contra $\mathcal{G}_\omega^\alpha$ -continuous function. Let $x \in X$ be any point and A be any open set in (Y, σ) non-containing $f(x)$. Then $Y - A$ is closed in (Y, σ) . containing $f(x)$. Then by Theorem (4.3) $f^{-1}(Y - A)$ is $\mathcal{G}_\omega^\alpha$ -open set containing x . Put $B \subseteq B = f^{-1}(Y - A)$. So $x \notin X - B = X - f^{-1}(Y - A) \subseteq X - B$. Then exists $\mathcal{G}_\omega^\alpha$ -closed set $X - B$ non-containing x such that $f^{-1}(A) \subseteq X - B$.

Conversely, Let $x \in X$ be any point and B be any closed in (Y, σ) containing $f(x)$. So $Y - B$ is open set in (Y, σ) . non-containing $f(x)$, Then by hypothesis there is $\mathcal{G}_\omega^\alpha$ -closed $A \in (X, \tau, \mathcal{G})$ non-containing x such that $f^{-1}(Y - B) \subseteq A$. Then $(X - A)_x \subseteq X - f^{-1}(Y - B) = f^{-1}(B)$ is $\mathcal{G}_\omega^\alpha$ -open set containing x in $(X, \tau, \mathcal{G})$. Therefor $f^{-1}(B) = \cup\{(X - A)_x : x \in f^{-1}(B)\}$ Hence by last theorem, f is contra $\mathcal{G}_\omega^\alpha$ -continuous function. $\square$

Definition 4.5. Let $(X, \tau, \mathcal{G})$ be grill topological space and (Y, σ) be topological space. Then the function $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ is called:

1. Almost $\mathcal{G}_\omega^\alpha$ -continuous function if $f^{-1}(V)$ is $\mathcal{G}_\omega^\alpha$ -open set in $(X, \tau, \mathcal{G})$ for every r -open set V in (Y, σ) .
2. Almost contra $\mathcal{G}_\omega^\alpha$ -continuous function if $f^{-1}(V)$ is $\mathcal{G}_\omega^\alpha$ -closed set in $(X, \tau, \mathcal{G})$ for every r -open set V in (Y, σ) .

Example 4.6. See, Example (3.2) that is for every r -open sets $Y, \{a\}$ and $\emptyset$ in a space (Y, σ) , $f^{-1}(Y)$, $f^{-1}(\{a\})$ and $f^{-1}(\emptyset)$ are $\mathcal{G}_\omega^\alpha$ -open set in grill topological space $(X, \tau, \mathcal{G})$.

Example 4.7. See, Example (3.3) that is for every r -open sets $Y, \{b\}$ and $\{\emptyset\}$ in space (Y, σ) , $f^{-1}(Y), f^{-1}(\{b\})$ and $f^{-1}(\emptyset)$ are $\mathcal{G}_\omega^\alpha$ -closed set in grill topological space $(X, \tau, \mathcal{G})$.

Theorem 4.8. A function $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ of a grill topological space $(X, \tau, \mathcal{G})$ into a space (Y, σ) , is almost $\mathcal{G}_\omega^\alpha$ -continuous if and only if for each $x \in X$ and each r -open set U of (Y, σ) with $f(x) \in U$, there exists $\mathcal{G}_\omega^\alpha$ -open set A in $(X, \tau, \mathcal{G})$ such that $x \in A$ and $f(A) \subseteq U$.

Proof. Similar the proof of theorem(3.12). □

Theorem 4.9. Let $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function from a grill topological space $(X, \tau, \mathcal{G})$ into a topological space (Y, σ) is almost contra $\mathcal{G}_\omega^\alpha$ -continuous function if and only if $f^{-1}(V)$ is $\mathcal{G}_\omega^\alpha$ -open set in $(X, \tau, \mathcal{G})$ for every r -closed set V in (Y, σ) .

Theorem 4.10. Let $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function from a grill topological space $(X, \tau, \mathcal{G})$ into a topological space (Y, σ) is almost contra $\mathcal{G}_\omega^\alpha$ -continuous function if and only if for each $x \in X$ and each r -closed set A containing $f(x)$ in space (Y, σ) . there is r -open set B containing x such that $f(B) \subseteq A$.

Proof. Suppose f is almost contra almost $\mathcal{G}_\omega^\alpha$ -continuous. Let $x \in X$ be any point and A be any r -closed set with (Y, σ) countable $f(x)$. Then by last Theorem $f^{-1}(A)$ is $\mathcal{G}_\omega^\alpha$ -open set containing x . Put $B = f^{-1}(A) \subseteq A$ that is $f(B) = f(f^{-1}(A)) \subseteq A$. Conversely, Let $x \in X$ be any point and A be any r -closed set with (Y, σ) containing $f(x)$. Then by hypothesis, there is $B = f^{-1}(A)$ containing x . such that $f(B_x) \subseteq A$. Then we get $f^{-1}(A) = \cup\{B_x : x \in f^{-1}(A)\}$. Therefor by last Theorem $f^{-1}(A)$ is $\mathcal{G}_\omega^\alpha$ -open set in $(X, \tau, \mathcal{G})$. Hence $f^{-1}(A)$ is almost contra almost $\mathcal{G}_\omega^\alpha$ -continuous. □

Theorem 4.11. Let $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function from a grill topological space $(X, \tau, \mathcal{G})$ into a topological space (Y, σ) is almost contra $\mathcal{G}_\omega^\alpha$ -continuous function if and only if $Cl(int_{\mathcal{G}}(Cl_\omega(f^{-1}(A)))) \subseteq f^{-1}(A)$ for each r -open set A in space (Y, σ) .

Proof. Suppose that f is almost contra $\mathcal{G}_\omega^\alpha$ -continuous. Let x by any point in X and let A be any r -open containing $f(x)$ in space Y then $f^{-1}(A)$ is $\mathcal{G}_\omega^\alpha$ -closed set in $(X, \tau, \mathcal{G})$. By use theorem (2.20) Hence $Cl(int_{\mathcal{G}}(Cl_\omega(f^{-1}(A)))) \subseteq f^{-1}(A)$. Conversely similar. □

Corollary 4.12. Let $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function from a grill topological space $(X, \tau, \mathcal{G})$ into a topological space (Y, σ) , defined by any definition of it is. If a

grill topological space $(X, \tau, \mathcal{G})$ with countable X , $\tau = \tau_I$ and $\mathcal{G} = P(X) - \emptyset$. Then f $\mathcal{G}_\omega^\alpha$ -continuous, contra $\mathcal{G}_\omega^\alpha$ -continuous, almost $\mathcal{G}_\omega^\alpha$ -continuous and almost contra $\mathcal{G}_\omega^\alpha$ -continuous.

Proof. Let A be any sub set of Y containing $f(x)$. Since $\mathcal{G}_\omega^\alpha O(X, \tau) = P(X)$ then $f^{-1}(A)$ is $\mathcal{G}_\omega^\alpha$ -open and $\mathcal{G}_\omega^\alpha$ -closed set in a grill topological space $(X, \tau, \mathcal{G})$ then above is holed. $\square$

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