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On Information Generating Functions for Single and Double Probability Distributions

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Abstract

In this communication we introduce the various generating functions that produce existing directed divergence and entropy measures.

Keywords: Measure of entropy, Directed divergence, Generating function

1. Introduction

In this paper we have introduced some generating functions that produce existing directed divergence and entropy measures.

In 1948 C.E. Shannon [13] gave the measure

$$S(P) = -\sum_{i=1}^{n} p_i \, ln \, p_i$$

to measure its entropy.

Later in 1972, J.P. Burg [1] gave the measure

$$B(P) = \sum_{i=1}^{n} lnp_i$$

After that Kullback and Liebler[11] evaluate that measure of information associated with the two probability distribution p_i and q_i of discrete random variable, is given as

$$D(p//q_i) = \sum_{i=1}^{n} p_i \, ln \frac{p_i}{q_i}$$

Which is known as the directed divergence.

Renyi [12] also gave the measure

$$S_q\,(P) = \frac{1}{1-q}\ln\sum_{i=1}^{n} p_i^q$$

to measures its directed divergence.
The generating functions are also derived for various measure of information and its corresponding measure of directed-divergence in both classical and in fuzzy cases.
So for entropy

$$f_{\alpha,\beta,\gamma}(t) = \frac{1}{\beta-\alpha}\left[\sum_{i=1}^{n}\left(p_i^{\alpha/\gamma}\right)^t - \beta\right], \alpha \neq \beta,\ \alpha, \beta, \gamma > 0$$

and for directed divergence

$$g_{\alpha,\beta,\gamma}(t) = \frac{1}{\alpha-\beta}\left[\left(\sum_{i=1}^{n} p_i^{\alpha}\, q_i^{2-\overline{\alpha+\beta}}\right)^t - \gamma\right], \alpha \neq \beta, \alpha, \beta, \gamma > 0.$$

All these generating functions are helpful for finding the variance as well as the moments about the mean of the distribution. All these outcomes are may be useful for developing the new ideas and thoughts in the domain of science and the message transmission through the channel in the presence of noise.
In 1997, Kapur[5],[8],[9] defined the generating function

$$f_\alpha\,(t) = \frac{1}{1-\alpha}\left(\sum_{i=1}^{n}(p_i)^t - 1\right), \alpha \neq 1$$

With the property

$$f_\alpha\,(1) = \frac{1}{1-\alpha}\left(\sum_{i=1}^{n} p_i^{\alpha} - 1\right), \alpha \neq 1$$

and

$$f_\alpha(0) = \frac{1}{1-\alpha}\, ln\sum_{i=1}^{n} p_i^{\alpha}\ , \alpha \neq 1$$

Additionally, Kapur[5],[8],[9] defined the generating function for P = $(p_{1,}p_2 \ldots\ldots\ldots\ldots p_n)$ from another probability distribution for relative information, cross-entropy, or directed divergence Q = $(q_{1,}q_2 \ldots\ldots\ldots\ldots\ldots q_n)$.

$$g_\alpha(t) = \left[\left(\sum_{i=1}^{n} p_i^{\alpha}\ q_i^{1-\alpha}\right)^t - 1\right], \alpha \neq 1,$$

With the property that

$$g_\alpha\,(1) = \frac{1}{\alpha-1}\left[\sum_{i=1}^{n} p_i^{\alpha}\, q_i^{1-\alpha} - 1\right], \alpha \neq 1$$

And

$$g_\alpha(1) = \frac{1}{\alpha-1}\ln\sum_{i=1}^{n} p_i^{\alpha}\, q_i^{1-\alpha}, \alpha \neq 1$$

In this paper we produce some generating function for measure of directed divergence and entropy.

2. Main Results

2.1 Generating functions for measures of directed divergence

Let

$$D_{\alpha,\beta}(t) = \frac{1}{(\alpha-1)\beta}\ [\sum_{i=1}^{n}(p_i^{\alpha}\, q_i^{1-\alpha})^{\beta} - t], \alpha > 0, \alpha \neq 1, \beta \neq 0$$

(2.1.1)

Therefore

$$D_{\alpha,1}(1) = \frac{1}{(\alpha-1)}\ [\sum_{i=1}^{n}(p_i^{\alpha}\, q_i^{1-\alpha}) \quad - 1], \alpha > 0, \alpha \neq 1,$$

(2.1.2)

This is Havrda-Charvat [4] measure of directed divergence.

$$D_{\alpha,\beta}(1) = \frac{1}{(\alpha-1)\beta}\ [\sum_{i=1}^{n}(p_i^{\alpha}\, q_i^{1-\alpha})^{\beta} - 1], \alpha > 0, \alpha \neq 1, \beta \neq 0$$

(2.1.3)

This is New directed divergence measure [16].

Let

$$D_{\alpha,\beta}(t) = \frac{1}{\alpha-\beta} ln \sum_{i=1}^{n}(p_i^{\frac{\alpha}{\beta}} q_i^{1-\frac{\alpha}{\beta}})^{t} \ \ \alpha \neq \beta, \alpha > 0, \beta > 0$$

(2.1.4)

Hence

$$D_{\alpha,1}(1) = \frac{1}{\alpha-1} ln \sum_{i=1}^{n} p_i^{\alpha} q_i^{1-\alpha}, \alpha \neq 1$$

(2.1.5)

Which is Renyi's [12] measure of Directed divergence.

Let

$$D_{\alpha,\beta}(t) = \sum_{i=1}^{n}(p_i^{\alpha}\, q_i^{1-\alpha})^{t} \left(lnp_i^{\frac{\alpha}{\beta}} - lnq_i^{\frac{\alpha}{\beta}} \right), \alpha \neq 1, \alpha > 0, \beta > 0$$

(2.1.6)

So

$$D_{1,1}(1) = \sum_{i=1}^{n} p_i\, ln \frac{p_i}{q_i}$$

(2.1.7)

This is Kullback – Leibler's[11] measure of Directed divergence.

Let

$$D_{\alpha,\beta}(t) = \frac{1}{(1-\alpha)\beta} [\sum_{i=1}^{n}(p_i^{\frac{\alpha}{\beta}} - q_i^{1-\frac{\alpha}{\beta}})^{t}, \alpha < 1, \beta > 0, \alpha > 0$$

(2.1.8)

Therefore

$$D_{\frac{1}{2},1}(2) = 2\sum_{i=1}^{n}(\sqrt{p_i} - \sqrt{q_i})^2$$

(2.1.9)

Which is Bhattacharya's[2] measure of Directed Divergence.

2.2 Generating functions for measures of Entropy

Let

$$f_{\alpha,\beta}(t) = \frac{1}{\beta - \alpha}\left[\sum_{i=1}^{n}\left(p_i^{\alpha\beta}\right)^t - \beta\right], \alpha \neq \beta, \alpha > 1, \beta > 0$$

(2.2.1)

So

$$f_{\alpha,1}(1) = \frac{1}{1-\alpha}\;(\sum_{i=1}^{n} p_i^{\alpha} - 1), \alpha \neq 1$$

(2.2.2)

$f_{\alpha,1}(1)$ gives Havrda – Charvat's [4] measure of entropy.

$$f_{\alpha,1}\,(t) = \frac{1}{1-\alpha}[(\sum_{i=1}^{n} p_i^{\alpha}\,)^t\; ln \sum_{i=1}^{n} p_i^{\alpha}\,, \alpha \neq 1$$

(2.2.3)

Then

$$f_{\alpha,1}\,(0) = \frac{1}{1-\alpha}\; ln \sum_{i=1}^{n} p_i^{\alpha}, \alpha \neq 1$$

(2.2.4)

$f_{\alpha,1}\,(0)$ gives Renyi's[12] measures of entropy.

$$f_{1,1}\,(1) = f_{\alpha,1}(1) = \frac{1}{1-\alpha}(\sum_{i=1}^{n} p_i^{\alpha} - 1)$$

(2.2.5)

$$f_{1,1}\,(1) = -\sum_{i=1}^{n} p_i^{\alpha}\; lnp_i$$

$$f_{1,1}\,(1) = -\sum_{i=1}^{n} p_i\; ln\; p_i$$

(2.2.6)

Which is trivial generating function for Shannon's [13] measure of entropy.

$$f_{2,1}\,(t) = 1 - (\sum_{i=1}^{n} p_i^{2}\,)^t$$

(2.2.7)

$$f_{2,1}(1) = 1 - \sum_{i=1}^{n} p_i^2$$

(2.2.8)

Hence $f_{2,1}(1)$ gives Vajda's[14] measure of entropy.

$$f_{\alpha,1}(t) = \frac{1}{1-\alpha}\,[\left(\sum_{i=1}^{n} p_i^{\alpha}\right)^{t}\; ln \sum_{i=1}^{n} p_i^{\alpha}],\alpha \neq 1$$

(2.2.9)

Then

$$f_{\alpha,1}(0) = -\frac{1}{\alpha-1}\; ln \sum_{i=1}^{n} p_i^{\alpha},\alpha \neq 1$$

So

$$f_{\alpha,1}(0) = -\ln(1-\alpha)\frac{1}{1-\alpha}(\sum_{i=1}^{n} p_i^{\alpha})^{\frac{1}{\alpha-1}},\alpha \neq 1$$

(2.2.10)

Which is Behara – Chawla[3] measure of entropy.
Let

$$f_{\alpha,\beta,\gamma}(t) = \frac{1}{\beta-\alpha}[\sum_{i=1}^{n}(p_i^{\frac{\alpha}{\gamma}})^{t} - \beta],\alpha \neq \beta,\alpha,\beta,\gamma > 0$$

(2.2.11)

$$f_{\alpha,1,1}(1) = \frac{1}{1-\alpha}(\sum_{i=1}^{n} p_i^{\alpha} - 1),\alpha \neq 1$$

(2.2.12)

Therefore $f_{\alpha,1,1}(1)$ gives Havrda – Charvat's [4] measure of entropy.

$$f_{\alpha,1,1}(t) = \frac{1}{1-\alpha}[\left(\sum_{i=1}^{n} p_i^{\alpha}\right)^{t} \ln \sum_{i=1}^{n} p_i^{\alpha}],\alpha \neq 1$$

(2.2.13)

Then

$$f_{\alpha,1,1}(0) = \frac{1}{1-\alpha}\ln \sum_{i=1}^{n} p_i^{\alpha}\;\; \alpha \neq 1$$

(2.2.14)

$f_{\alpha,1,1}(0)$ gives Renyi's measure[12] of entropy.

$$f_{1,1,1}(1) = f_{\alpha,1,1}(1) = \frac{1}{1-\alpha}(\sum_{i=1}^{n} p_i^{\alpha} - 1)$$

(2.2.15)

$$f_{\alpha,1,1}(1) = -\sum_{i=1}^{n} p_i^{\alpha}\; ln\, p_i$$

$$f_{1,1,1}(1) = -\sum_{i=1}^{n} p_i \ln p_i \tag{2.2.16}$$

Which is trivial generating function for Shannon's[13] measure of entropy.

$$f_{2,1,1}(t) = 1 - \left(\sum_{i=1}^{n} p_i^2\right)^t \tag{2.2.17}$$

$$f_{2,1,1}(1) = 1 - \sum_{i=1}^{n} p_i^2 \tag{2.2.18}$$

$f_{2,1,1}(1)$ gives Vajda's[14] measure of entropy.

$$f_{\alpha,1,1}(t) = \frac{1}{1-\alpha}\left[\left(\sum_{i=1}^{n} p_i^{\alpha}\right)^t \ln \sum_{i=1}^{n} p_i^{\alpha}\right], \alpha \neq 1 \tag{2.2.19}$$

$$f_{\alpha,1,1}(0) = -\frac{1}{1-\alpha} \ln \sum_{i=1}^{n} p_i^{\alpha}, \alpha \neq 1$$

$$f_{\alpha,1,1}(0) = -\ln(1-\alpha)\frac{1}{1-\alpha}\sum_{i=1}^{n} (p_i^{\alpha})^{\frac{1}{\alpha-1}}, \alpha \neq 1 \tag{2.2.20}$$

Which is Behara – Chawla[3] measure of entropy.

Now let us define

$$f_{\alpha,\beta,\gamma}(t) = \frac{1}{\beta-\alpha}\left[\sum_{i=1}^{n}\left(p_i^{\frac{\alpha}{\gamma}}\right)^t - \beta\right], \alpha \neq \beta, \alpha, \beta, \gamma > 0 \tag{2.2.21}$$

$$f_{1,2,1}(t) = \sum_{i=1}^{n} p_i^t - 2 \tag{2.2.22}$$

$$f_{1,2,1}(t) = \sum_{i=1}^{n} p_i^t \ln p_i \tag{2.2.23}$$

$$f_{1,2,1}(0) = \sum_{i=1}^{n} \ln p_i \tag{2.2.24}$$

Which is Burg's[1] measure of entropy.

$$f_{2,1,2}\,(t) = -\sum_{i=1}^{n} p_i^t\, lnp_i \tag{2.2.25}$$

$$f_{2,1,2}\,(1) = -\sum_{i=1}^{n} p_i\, \ln p_i \tag{2.2.26}$$

Which is Shannon's[13] measure of entropy.

$$f_{2,1,2}\,(t) = 1 - \sum_{i=1}^{n} p_i^t \tag{2.2.27}$$

$$f_{2,1,2}\,(2) = 1 - \sum_{i=1}^{n} p_i^2 \tag{2.2.28}$$

Which is Vajda's[14] measure of entropy.
Now let

$$f_{\alpha,1,1}(t) = \frac{1}{1-\alpha}[\sum_{i=1}^{n} (p_i^{\alpha})^t \; - 1], \alpha \neq 1 \tag{2.2.29}$$

$$f_{\alpha,1,1}\,(1) = \frac{1}{1-\alpha}(\sum_{i=1}^{n} p_i^{\alpha} - 1), \alpha \neq 1 \tag{2.2.30}$$

Which is Havrda – Charvat [4]measure of entropy.

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