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Effect of Electrification of Particles on Boundary Layer Flow and Convective Heat Transfer over a Vertical Permeable Stretching Surface with Thermal Radiation

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Abstract

This paper focuses on electrification of particles, terms related to the heat added to the system to slip-energy flux and heat due to conduction and viscous dissipation in the energy equation of the particle phase in modeling the steady boundary layer free convective flow of a dusty fluid past a vertical permeable stretching surface. The governing partial differential equations of the flow field are reduced into first order ordinary differential equations using similarity transformations. These ordinary differential equations are solved numerically using Runge kutta fourth order method with shooting technique. The effects of physical parameters like fluid-particle interaction parameter, Grashof number, suction parameter, Prandtl number, radiation parameter, Eckert number and diffusion parameter on the flow and heat transfer characteristics are computed and presented graphically and also in tabular form. The rate of heat transfer at the surface and skin friction increase with increasing value of electrification parameter.

Keywords: Electrification of particle, Thermal radiation, Volume fraction, Dusty fluid, Suction parameter, Boundary layer flow.

AMSW classification 76T10,76T15

NOMENCLATURE

Ec Eckert number

q_r Radiation heat flux

q_{r_p} Radiation heat flux of particle phase

Fr Froud number

Gr Grashof number

Pr Prandtl number

T_∞ Temperature at large distance from the wall.

T_p Temperature of particle phase.

T_w Wall temperature

$U_w(x)$ Stretching sheet velocity

c_p Specific heat of fluid

c_s Specific heat of particles

k_s Thermal conductivity of particle

(up,vp) Velocity component of the particle along x-axis and y-axis

A Constant

Ra Thermal radiation

c Stretching rate

f0 Suction parameter

g Acceleration due to gravity

k Thermal conductivity of fluid

l Characterstic length

T Temperature of fluid phase.

u,v Velocity component of fluid along x-axis and y-axis

x,y Cartesian coordinate

K* Mean absorption co-efficient

Greek Symbols :

φ Volume fraction

β Fluid particle interaction parameter

β^* Volumetric coefficient of thermal expansion

σ^* The Stefan Bolzman constant

ρ Density of the fluid

ρ_p Density of the particle phase

ρ_s Material density

η Similarity variable

θ Fluid phase temperature

θp Dust phase temperature

μ Dynamic viscosity of fluid

ν Kkinematic viscosity of fluid

γ Ratio of specific heat

τ Relaxation time of particle phase

τ_T Thermal relaxation time i.e. the time required by the dust particle to adjust its temperature relative to the fluid.

τ_p Velocity relaxation time i.e. the time required by the dust particle to adjust its velocity relative to the fluid.

ε Diffusion parameter

ω Density ratio

INTRODUCTION

The behavior of boundary layer flow over a continuous moving flat surface immersed in an otherwise quiescent fluid was first studied by Sakiadis [50,51],who investigated it theoretically by both exact and approximate methods. Tsou et.al [57] verified the analysis of Sakiadis [50,51] experimentally and extended the same work to heat transfer problem .Crane [13] extended the work of Sakiadis [50,51] to a stretching sheet with linear surface velocity and obtain a similarity solution to the problem .Since then ,research area of stretching sheet has been flooded with many research articles with multiple dimensions enriched by the innovative researchers [8,11,26,32,45,47,48,53,54,60]. In summery the various concepts of the phenomenon are [5, 12, 16, 20, 33, 35, 36, 39, 40, 56], heat and mass transfer on horizontal/vertical plates , on inclined plates ,with or without suction or blowing ,steady flow, unsteady flow due to sudden stretching of sheet or by changing the temperature of the sheet, wall temperature, magnetic field, effect of diffusion –thermo and thermal diffusion of heat, uniform/non-uniform heat source/sink [1,6,8,12,16,20,21,34,40,43] , thermal radiation [1,6,8,12,19,30,35,36,39,40,41,45] , heat transfer over a porous stretching surface[12,17,23,26,29,32,38,42,45], flow through porous media [2,33,41,44,46,55,58], All the above investigations restricted their analysis to the flow induced by a linear/vertical stretching sheet under different physical situations and in the absence of fluid particle suspensions.

It is worth mentioning here that the two phase flows, in which solid spherical particles distributed in a fluid ,are of interest in a wide range of technical problems ,such as flow through packed beds, sedimentation, environmental pollution , nuclear reactor cooling, powder technology, rain erosion ,paint spraying, centrifugal separation, combustion and purification of crude oil , flowing rockets and blood rheology etc. The study of the boundary layer of fluid- particle suspension flow is important in determining the particle accumulation and impingement of particles on the surface [6,14,18,21,49,55].To date an enormous amount of work has been done on the boundary layer flow and heat transfer with consideration of the stretching sheet problem [1,2,3,7,9,10,12,15,24,25,27,28,31,38,42,44].The engineering applications of the stretching sheet problems includes polymer sheet extrusion from a dye, drawing ,tinning and annealing of copper wires ,glass fiber and paper production ,the cooling of metallic plate in a cooling bath and so on. A quick review of the above mentioned literature shows that the investigation is based upon the various physical concepts already told above. B.J. Gireesha et al [4] have studied an MHD boundary layer flow and heat transfer of a dusty fluid over a stretching sheet in presence of viscous dissipation. They have not considered heat energy due to Lorentz force in energy equation. They have observed that the increasing Q (Chandrasekaer number) escalates the magnitude of the Lorentz retarding hydro-magnetic body force which serves to retard the flow considerably but, in non-dimensional particle velocity, it is in contrast. Also their analysis shows that $f'(0)$ negative means the solid surface exerts a drag force on the fluid and it is due to the development of velocity boundary layer on the stretching sheet. They have studied the problem in PST and PHF case. They have observed that the transverse magnetic field produces a body force, Lorentz force which opposes the motion. The resistance offered to the flow is responsible on increasing temperature for both phases..

Even though the study relating to flow and heat transfer in MHD dusty boundary layer flow over stretching sheet [4,5,16,19,29,37,43,59]are available, hardly any study is taken up by considering the base fluid as non –conducting and the particles are electrified. No consulted effort has been made to show the effect of electrification of particles and/or contribution of various physical aspects on two phase flow and heat transfer .Since tribo electrification occurs due to collision of particles with each other or impingement of particles with walls and since the electrification of particles have a pronounced effect on boundary layer characteristics like skin friction, heat transfer etc, it is essential to include this phenomena in the modeling of flow over a stretching sheet The forces and moments acting on a solid particle consist of those due to the net charge in the electric field due to the charged particles. As a general statement, any volume element of charge species, with charge "e" experiences an instantaneous force given by the Lorentz force law given by $\overrightarrow{f} = e\,\vec{E} + \overrightarrow{J} \times \vec{B}$ where $\vec{B}$ is the magnetic flux density. The current densities in corona discharge are so low that the magnetic force term $\overrightarrow{J} \times \vec{B}$ can be omitted, as this term is many orders of magnitude smaller than the Coulomb term$e\vec{E}$.The ion drift motion arises from the interaction of ions, constantly subject to the Lorentz force with the dense neutral fluid medium. This interaction produces an effective drag force on the ions. The drag force is in equilibrium with the Lorentz force so that the ion velocity in a field $\vec{E}$ is limited to $k_m\vec{E}$, where k_m is the mobility of the ion species. The drag force on the ions has an equal and opposite reaction force acting on the neutral fluid molecules via this ion-neutral molecules interaction, the force on the ions is transmitted directly to the fluid medium, so the force on the fluid particles is also given by $\overrightarrow{f} = e\,\vec{E}$, Soo [52].

G. K. Ramesh et al [17] have investigated the convective heat transfer in a dusty fluid over a vertical permeable surface with thermal radiation. From their study, it is observed that the effect of increasing the fluid particle interaction parameter and also suction parameter f0 are to decrease the wall temperature gradient θ(0). It is evident from this study that increasing value of Gr results in thinning of the thermal boundary layer associated with an increase in the wall temperature gradient and hence produces an increase in the heat transfer rate. They have not considered the terms related to the heat added to the system to slip-energy flux and heat due to conduction and viscous dissipation in the energy equation of the particle phase. Also they have not considered the effect of viscous dissipation.

The above analyses motivated to the study of the present paper. Here the particles will be allowed to diffuse through the carrier fluid i.e. the random motion of the particles shall be taken into account because of the small size of the particles. This can be done by applying the kinetic theory of gases and hence the motion of the particles across the streamline due to the concentration and pressure diffusion. The effects of electrification, radiation effect, and volume fraction of particles on skin friction, heat transfer and other boundary layer characteristics also have been studied. Further in the present study, the behavior of incompressible, laminar boundary-layer flow of a dusty fluid over a permeable vertical stretching sheet in presence of electrification of particles is examined. The governing equations are reduced into a system of ODE and are solved using well known Runge Kutta Fourth order method and shooting technique. The effects of volume fraction and electrification of particles on skin friction, heat transfer and other boundary layer characteristics also have been studied.

FLOW ANALYSIS OF THE PROBLEM AND SOLUTION

Consider a steady two dimensional laminar boundary layer of an incompressible viscous dusty fluid over a vertical stretching sheet .The flow is generated by the action of two equal and opposite forces along the x-axis and y-axis being normal to the flow .The sheet being stretched with the velocity $U_w(x)$ along the x-axis, keeping the origin fixed in the fluid of ambient temperatureT_∞. Both the fluid and the dust particle clouds are suppose to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size and number density of the dust particle is taken as a constant throughout the flow.

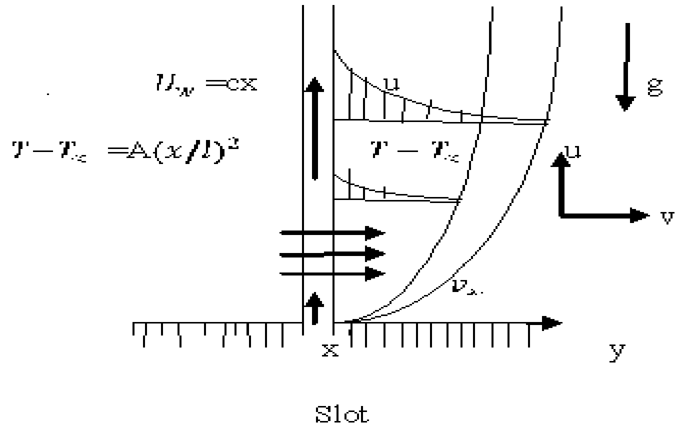


Figure-1 Schematic diagram of the flow

The governing equations of steady two dimensional boundary layer incompressible flows of dusty fluids are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$\frac{\partial}{\partial x}\left(\rho_p up\right)+\frac{\partial}{\partial y}\left(\rho_p vp\right)=0 \quad (2)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\rho}\,\mu\frac{\partial^2 u}{\partial y^2}-\frac{1}{(1-\varphi)\rho}\frac{1}{\tau_p}\varphi\rho_s\,(u-up)+g\beta^*\,(T-T_\infty)+\frac{1}{1-\varphi}\frac{\rho_p}{\rho}\left(\frac{e}{m}\right)E \quad (3)$$

$$\varphi\rho_s\,(up\frac{\partial up}{\partial x}+vp\frac{\partial u_p}{\partial y})=\frac{\partial}{\partial y}\left(\varphi\mu_s\,\frac{\partial up}{\partial y}\right)+\frac{1}{\tau_p}\varphi\rho_s\,(u-up)+\varphi\left(\rho_s-\rho\right)g+\rho_p\left(\frac{e}{m}\right)E$$

(4)

$$\varphi\rho_s\left(up\frac{\partial v_p}{\partial x}+vp\frac{\partial v_p}{\partial y}\right)=\frac{\partial}{\partial y}\left(\varphi\mu_s\,\frac{\partial v_p}{\partial y}\right)+\frac{1}{\tau_p}\varphi\rho_s\,(v-vp) \quad (5)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\varphi\rho_s\,c_s}{(1-\varphi)c_p}\frac{1}{\rho\,\tau_T}\left(T_p-T\right)+\frac{\varphi\rho_s}{(1-\varphi)}\frac{1}{\rho c_p}\frac{1}{\tau_p}(up-u)^2+$$

$$\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{1}{1-\varphi}\frac{1}{\rho c_p}\ \rho_p\left(\frac{e}{m}\right)Eup \tag{6}$$

$$up\frac{\partial T_p}{\partial x}+vp\frac{\partial T_p}{\partial y}\ =-\frac{1}{\tau_p}\left(T_p-T\right)+\frac{1}{\varphi\rho_s\,c_s}\frac{\partial}{\partial y}\left(\varphi k_s\frac{\partial T_p}{\partial y}\right)-\frac{1}{\tau_p}\frac{1}{c_s}(u-up)^2+$$

$$\frac{\mu_s}{\rho_s\,c_s}\left[up\frac{\partial^2 up}{\partial y^2}+\left(\frac{\partial up}{\partial y}\right)^2\right]-\varphi\ \frac{\partial q_{r_p}}{\partial y}+\ \rho_p\left(\frac{e}{m}\right)Eup \tag{7}$$

Where (u,v) and (up,vp) are the velocity components of the fluid and dust particle phases along x and y directions respectfully. μ , ρ and ρ_p, N are the co-efficient of viscosity of the fluid, density of the fluid and particle phase, number density of the particle phase respectfully.

With boundary conditions

$$\left.\begin{aligned}&u=U_\omega(x)=cx\,,v=-v_\omega(x)\,T=T_w=T_\infty+A\left(\frac{x}{l}\right)^2\ at\ y=0\\&\rho_p=\omega\rho,u=0\,,up=0\,,vp\rightarrow v\,,T\rightarrow T_\infty\ ,T_p\rightarrow T_\infty\ as\ y\rightarrow\infty\end{aligned}\right\} \tag{8}$$

Where ω is the density ratio in the main stream and A is a positive constant,

$l=\sqrt{\frac{v}{c}}$ is a characteristic length.

Using the Rosseland approximation for radiation heat flux is simplified as

$$q_r=\frac{4\sigma^*}{3K^*}\frac{\partial T^4}{\partial y} \tag{9}$$

Where σ^* and K^* are the Stefan Bolzman constant and the mean absorption co-efficient respectfully.

Assuming that the temperature differences within the flow such that term T^4 may be expressed as a linear function of the temperature. We expand T^4 in a Taylor series about T_∞ and neglecting the higher order terms beyond the first degree in $(T-T_\infty)$we get

$$T^4\cong 4T_\infty^4\text{T-}3T_\infty^4 \tag{10}$$

For most of the gases $\tau_p \approx \tau_T$, $k_s = k\frac{c_s}{c_p}\frac{\mu_s}{\mu}$ if $\frac{c_s}{c_p} = \frac{2}{3Pr}$, $\varphi\rho_s = \rho_p$ Introducing the following non dimensional variables in equation (1) to (7)

$$u = cxf'(\eta)\,, v = -\sqrt{cv}f(\eta)\,, \eta = \sqrt{\frac{c}{v}}\,y\,, up = cxF'(\eta)\,, vp = \sqrt{cv}\,G(\eta)\,, \varphi\rho_r = H(\eta)$$

$$\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}\,, \theta p(\eta) = \frac{T_p-T_\infty}{T_w-T_\infty}\,, \beta = \frac{1}{c\tau_p}\,, \epsilon = \frac{v_s}{v}\,, Pr = \frac{\mu c_p}{k}\,, Ec = \frac{c^2l^2}{Ac_p}, Ra = \frac{16T_\infty^3\sigma^*}{3K^*k},$$

$$\frac{\partial q_{r_p}}{\partial y} = -\frac{16T_\infty^3\sigma^*}{3K^*}\frac{\partial^2 T_p}{\partial y^2}, \mathrm{M} = \frac{E}{c^2x}\left(\frac{e}{m}\right) \text{Where } T - T_\infty = A\left(\frac{x}{l}\right)^2\theta\,, T_p - T_\infty = A\left(\frac{x}{l}\right)^2 \theta p \quad (11)$$

C is the stretching rate and being a positive constant, c_p is the specific heat of fluid phase.

K is the thermal conductivity, β is the fluid particle interaction parameter.

β^* is the volumetric coefficient of thermal expansion. We get the following non dimensional form.

$$HF + HG' + GH' = 0 \quad (12)$$

$$f'''(\eta) + f(\eta)f''(\eta) - [f'(\eta)]^2 + \frac{1}{(1-\varphi)}\beta\, H(\eta)[F(\eta) - f'(\eta)] + Gr\,\theta + \frac{H(\eta)}{1-\varphi}M = 0 \quad (13)$$

$$G(\eta)F'(\eta) + [F(\eta)]^2 = \epsilon\, F''(\eta) + \beta[f'(\eta) - F(\eta)] + \frac{1}{Fr}\left(1 - \frac{1}{\gamma}\right) + \mathrm{M} \quad (14)$$

$$GG' = \in G'' - \beta[\,f + G\,] \quad (15)$$

$$\theta'' = \left(Pr(2f'\theta - f\theta') - \frac{2}{3}\frac{\beta}{1-\varphi}H[\theta p - \theta] - \frac{1}{1-\varphi}PrEc\beta H[F - f']^2 - PrEcf''^2 - \frac{H(\eta)}{1-\varphi}MPrEc\,F(\eta)\right) / (Ra + 1) \quad (16)$$

$$\theta p''(\eta) = (2F\theta_p + G\theta p' + \beta[\theta p - \theta] + \beta EcPr[f' - F]^2 - \frac{3}{2}\epsilon EcPr[FF'' + (F')^2\,] - \frac{3}{2}MEcPrF(\eta)) / \left(\frac{\epsilon}{Pr} + \frac{3}{2}\frac{Ra}{\gamma}\right) \quad (17)$$

With boundary conditions

$$G'(\eta) = 0, f(\eta) = f_0, f'(\eta) = 1, F'(\eta) = 0, \theta(\eta) = 1\,, \theta p' = 0 \;as\; \eta \to 0$$
$$f'(\eta) = 0, F(\eta) = 0, G(\eta) = -f(\eta)\,, H(\eta) = \omega\,, \theta(\eta) \to 0\,, \theta p(\eta) \to 0 \;as\; \eta \to \infty \quad (18)$$

Where f0 $=\frac{V_w}{(\vartheta C)^{1/2}}$ is the suction parameter and $Gr = g\frac{\beta^*(T_\omega - T_\infty)}{c^2x}$ is the local Grashof number.

SOLUTION OF THE PROBLEM:

Here in this problem the value of $f''(0), F(0), G(0), H(0), \theta'(0), \theta_p(0)$ are not known but $f'(\infty) = 0, F(\infty) = 0, G(\infty) = -f(\infty), H(\infty) = \omega, \theta(\infty) = 0, \theta_p(\infty) = 0$ are given. We use Shooting method to determine the value of $f''(0), F(0), G(0), H(0), \theta'(0), \theta_p(0)$ .We have supplied $f''(0) = \propto_0$ and $f''(0) = \propto_1$. The improved value of $f''(0) = \propto_2$ is determined by utilizing linear interpolation formula. Then the value of $f'(\propto_2, \infty)$ is determined by using Runge-Kutta method. If $f'(\propto_2, \infty)$ is equal to$f'(\infty)$ up to a certain decimal accuracy, then $\propto_2$ i.e $f''(0)$ is determined, otherwise the above procedure is repeated with $\propto_0 = \propto_1$ *and* $\propto_1 = \propto_2$ until a correct $\propto_2$ is obtained. The same procedure described above is adopted to determine the correct values of $F(0), G(0), H(0), \theta'(0), \theta_p(0)$.

The essence of shooting technique to solve a boundary value problem is to convert the boundary value problem into initial value problem. In this problem the missing value of $\theta'(0)$ and $f''(0)$ for different set of values of parameter are chosen on hit and trial basis such that the boundary condition at other end i.e. the boundary condition at infinity (η_∞)are satisfied.A study was conducting to examine the effect of step size as the appropriate values of step size Δη was not known to compare the initial values of $\theta'(0)$ and $f''(0)$ .If they agreed to about 6 significant digits, the last value of η_∞used was considered the appropriate value; otherwise the procedure was repeated until further change in η_∞did not lead to any more change in the value of $\theta'(0)$ and $f''(0)$ .The step size Δη =0.1 has been found to ensure to be the satisfactory convergence criterion of 1×10^{-6}.The solution of the present problem is obtained by numerical computation after finding the infinite value for η. It has been observed from the numerical result that the approximation to $\theta'(0)$ and$f''(0)$ are improved by increasing the infinite value of η which is finally determined as η =5.0 with a step length of 0.1 beginning from η = 0.0 Depending upon the initial guess and number of steps N. the values of $f''(0)$ and $\theta'(0)$ are obtained from numerical computations which are given in table -2 for different parameters.

Table-1; Comparison results for the wall temperature gradient $-\theta'(0)$ in case of Ec=0,β=0,Ra=0 ,Gr=0, f0=0

Pr	Chen	Grubka and Bobba	Able and Mahesha	G.K. Ramesh	Present study $-\theta'(0)$
0.72	1.0885	1.0885	1.0885	1.0886	1.0884
1.0	1.3333	1.3333	1.3333	1.3333	1.3332
10.0	4.7969	4.7969	4.7968	4.7968	4.7969

RESULTS AND DISCUSSION

The equations (12) to (17) with boundary conditions (18) were solved numerically, in double precision, by shooting method using the Runge-Kutta fourth order algorithm. The computations were done by the computer language FORTRAN-77.The results of heat transfer and skin friction coefficient characteristics are shown in Table-2, which shows that it is a close agreement with the existing literature. The effect of various parameters on the velocity profiles and temperature profiles also demonstrated graphically. In order to check the accuracy of our present numerical solution procedure used a comparison of wall temperature gradient -$\theta'(0)$ is made with those reported by with Chen[10], Grubka and Bobba[27], Able and Mahesha[1], G.K. Ramesh[17] for various values of Pradtl number Pr absence of other parameters which are given in table-1.Our present results are in a good agreement with the previous results.

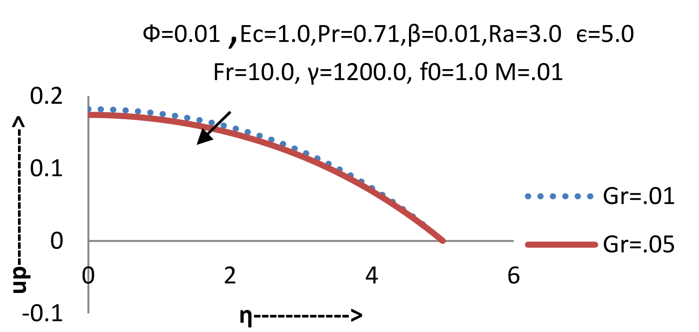


Fig-2: Variation of up w.r.t Gr

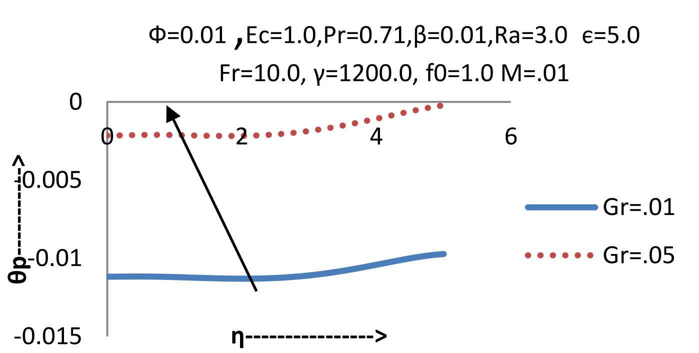


Fig-3: Variation of θp w.r.t Gr

Fig-4: Variation of up w.r.t M

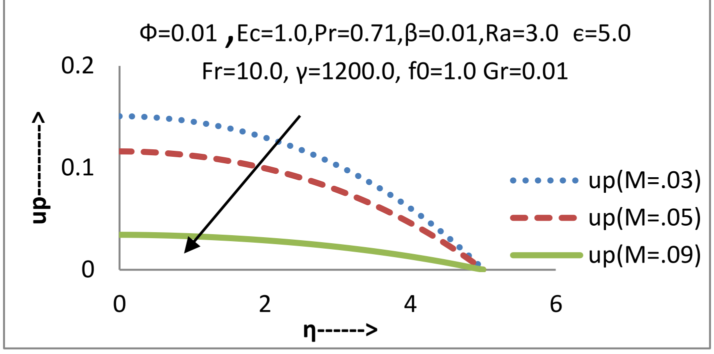


Fig-5: Variation of θp w.r.t M

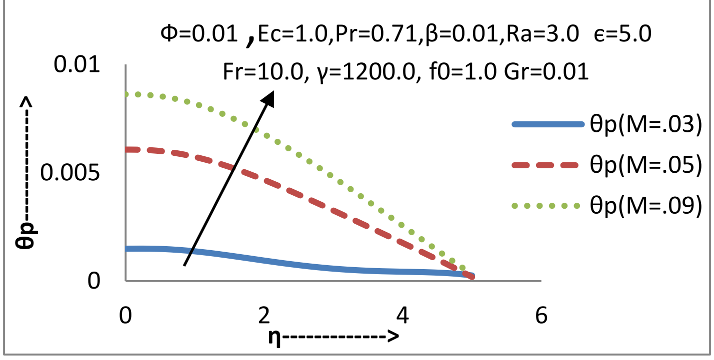


Fig-6: Variation of θ w.r.t Pr

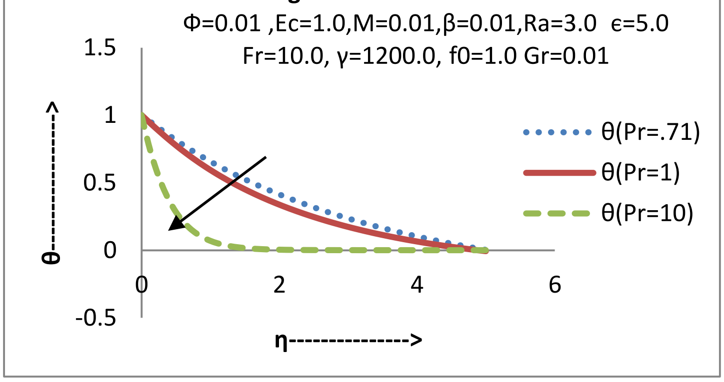
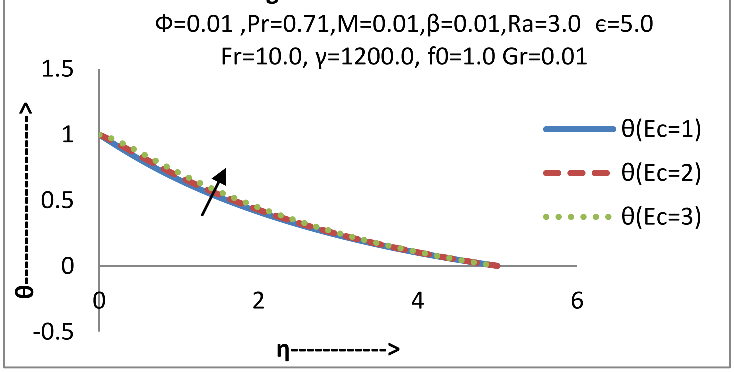


Fig-7: Variation of θ w.r.t Ec



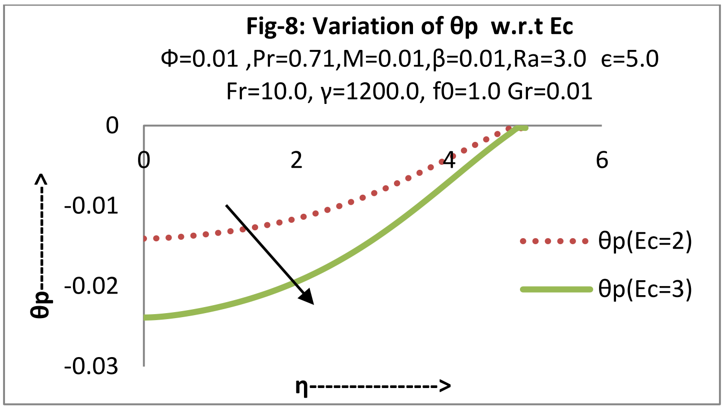


Fig-8: Variation of θp w.r.t Ec
Φ=0.01 ,Pr=0.71,M=0.01,β=0.01,Ra=3.0 ϵ=5.0
Fr=10.0, γ=1200.0, f0=1.0 Gr=0.01

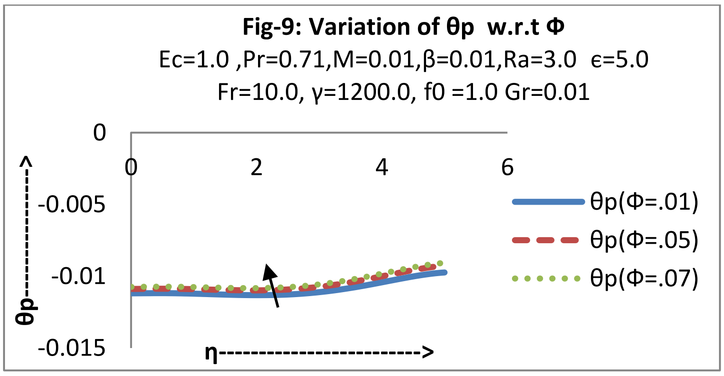


Fig-9: Variation of θp w.r.t Φ
Ec=1.0 ,Pr=0.71,M=0.01,β=0.01,Ra=3.0 ϵ=5.0
Fr=10.0, γ=1200.0, f0 =1.0 Gr=0.01

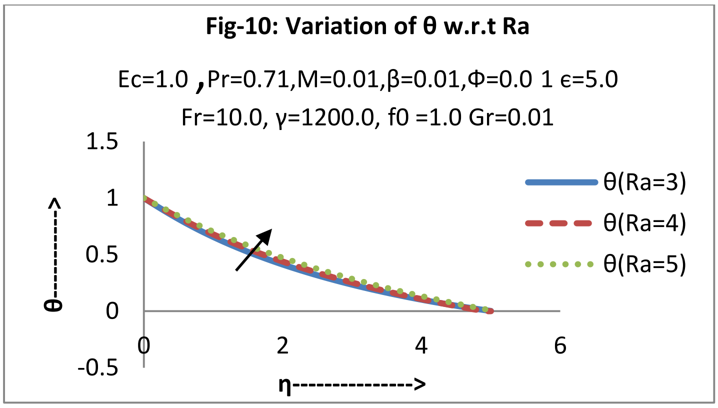


Fig-10: Variation of θ w.r.t Ra
Ec=1.0 ,Pr=0.71,M=0.01,β=0.01,Φ=0.0 1 ϵ=5.0
Fr=10.0, γ=1200.0, f0 =1.0 Gr=0.01

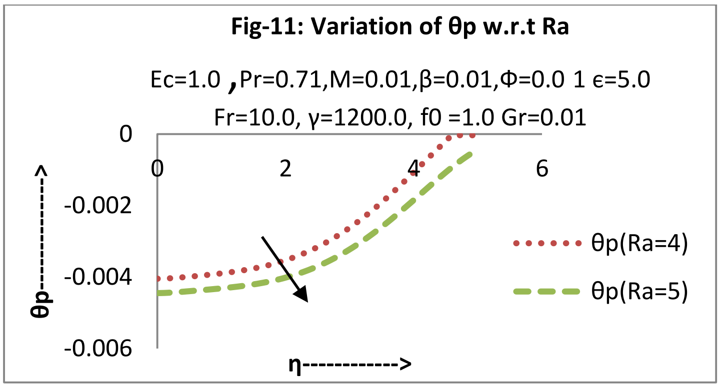


Fig-11: Variation of θp w.r.t Ra
Ec=1.0 ,Pr=0.71,M=0.01,β=0.01,Φ=0.0 1 ϵ=5.0
Fr=10.0, γ=1200.0, f0 =1.0 Gr=0.01

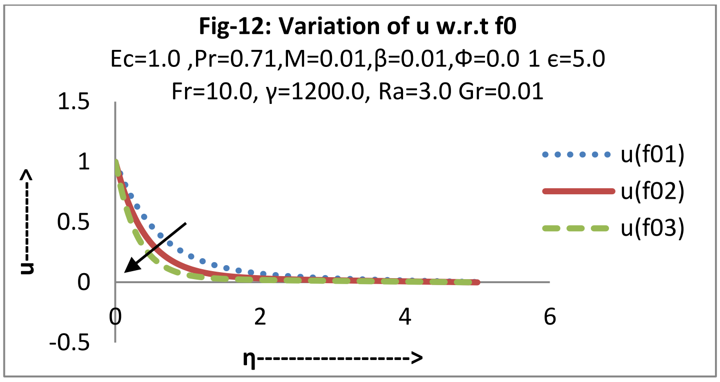


Fig-12: Variation of u w.r.t f0
Ec=1.0 ,Pr=0.71,M=0.01,β=0.01,Φ=0.0 1 ϵ=5.0
Fr=10.0, γ=1200.0, Ra=3.0 Gr=0.01
u---------->
η------------------>
u(f01)
u(f02)
u(f03)

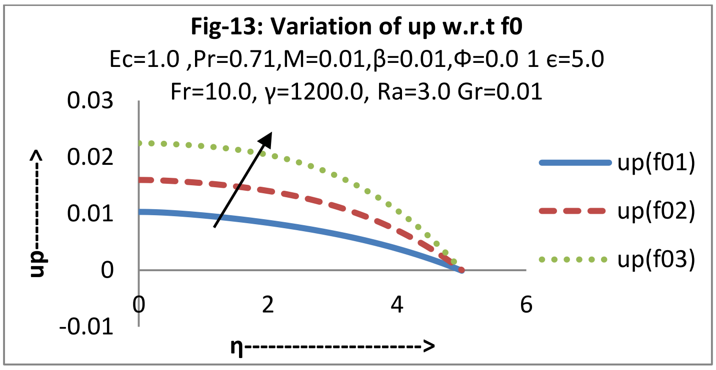


Fig-13: Variation of up w.r.t f0
Ec=1.0 ,Pr=0.71,M=0.01,β=0.01,Φ=0.0 1 ϵ=5.0
Fr=10.0, γ=1200.0, Ra=3.0 Gr=0.01
up---------->
η---------------------->
up(f01)
up(f02)
up(f03)

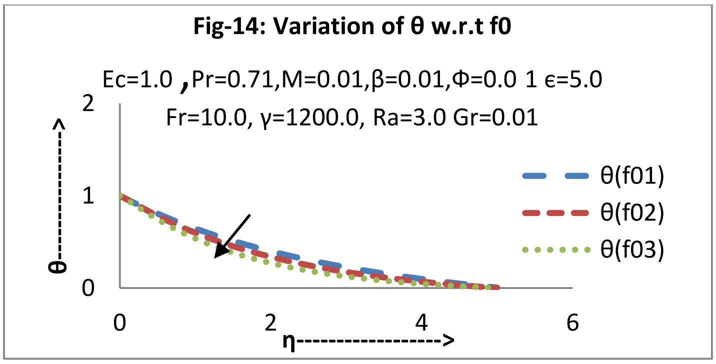


Fig-14: Variation of θ w.r.t f0
Ec=1.0 ,Pr=0.71,M=0.01,β=0.01,Φ=0.0 1 ϵ=5.0
Fr=10.0, γ=1200.0, Ra=3.0 Gr=0.01
θ------------->
η------------------>
θ(f01)
θ(f02)
θ(f03)

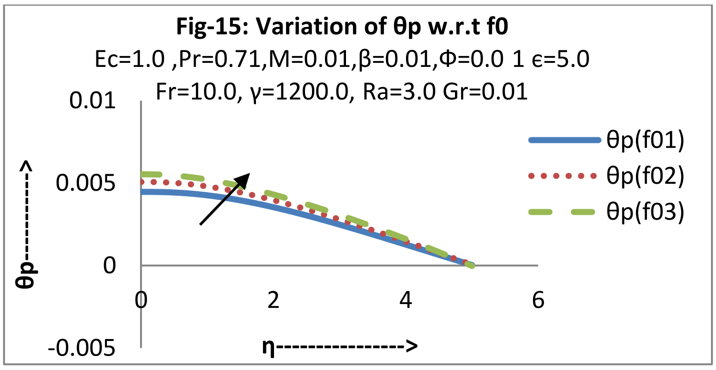
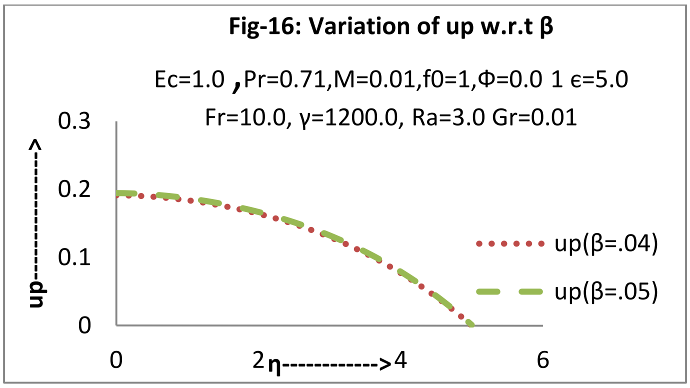


Fig-15: Variation of θp w.r.t f0
Ec=1.0 ,Pr=0.71,M=0.01,β=0.01,Φ=0.0 1 ϵ=5.0
Fr=10.0, γ=1200.0, Ra=3.0 Gr=0.01
θp---------->
η---------------->
θp(f01)
θp(f02)
θp(f03)



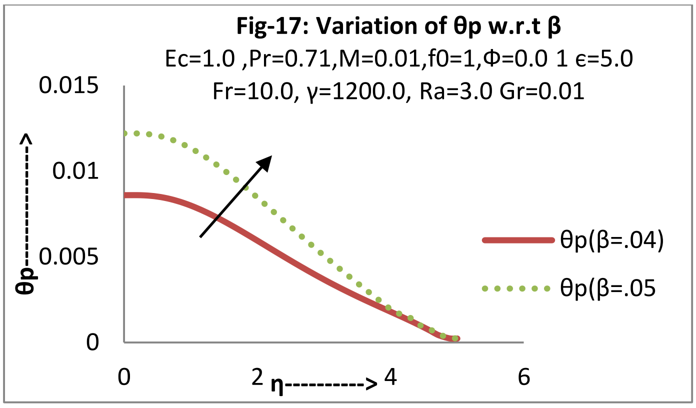


Figure-2 & 3 illustrates the velocity profiles (up) and temperature profiles θp versus η for various values of local Grashof number Gr. It is observed from the figures that the effect of increasing values of local Grashof number Gr is to decrease the velocity distribution (up) and increase the temperature distribution θp Figures-4&5 illustrate velocity distribution (up) and temperature profiles θp with η for various values of electrical parameter M. It shows that velocity of dust phase decreases and temperature of fluid phase increases for increase value of electrical parameter M. Figure -6 depicts the variation in the temperature profiles θ for the selected values of Prandtl number Pr versus η. This figure indicates that the temperature θ decreases for increasing values of Pr.Figures.-7&8 depict the graph of fluid temperature θ and the dust temperature θp w.r.t. η for the selected values of Eckert number Ec. It is observed that the effect of increasing values of Ec, slightly increase in the temperature θ but the temperature θ_p decrease.Figure.-9 depicts the graph of temperature θp w.r.t η for various values of volume fraction φ . It is observed that the temperature θp slightly increases with increase of volume fraction φ.Figures-10&11, illustrate the variation of temperature profiles θ and θp of both phases versus η for the selected values of Ra It is observed that the fluid temperature θ slightly increases but dust temperature θp decreases for the of increasing values of Ra.Fig-12 & 14 depict the velocity profiles u & temperature profiles θ versus η for the effect of suction parameter f0. Fluid phase velocity profiles u and temperature profiles θ decreases asymptotically for increasing value of suction parameter f0 .Fig-13 & 15 depict the velocity profiles (up) and temperature profiles $\theta_p$versus η for the effect of suction parameter f0. Dust phase velocity profiles (up) and the temperature profiles θp increase with increase of suction parameter f0. Fig.-16 & 17 presents the velocity distribution (up) and the temperature distribution θp with η for various values of fluid particle interaction parameter β. It is clearly observed from this figure -16 that no significant change in velocity (up) and from figure-17, the temperature θp increases for increasing value of β.

CONCLUSION

Some of the important findings of the present analysis are as follows.

1. For increasing value of f0 the velocity of fluid phase decreases.
2. Velocity of dust phase (up) increases with increasing value of f0 but decreases with increasing value of Gr and M
3. Temperature of fluid decreases as Pr and f0 increases but increases as Ra increases.
4. The temperature θp decreases for increasing value of Gr and φ but increases for increasing value of Ec, Ra, β, f0 and M.

   The effects of all the physical parameters on the skin friction coefficient$f''(0)$, the wall temperature gradient $\theta'(0)$, are analyzed in Table 2.
5. The effect of increase in electrification parameter M, Fluid particle interaction parameter β and the volume fraction φ are to decrease the skin friction coefficient and to increase in the rate of heat transfer.
6. Both Coefficient of skin friction and rate of heat transfer decrease with the increase of radiation parameter Ra, and Eckert number Ec .
7. The effect of Gr, the Grashof number and ε ,the diffusion parameter are to increase the skin friction and decrease the rate of heat transfer, where as an increase in Prandtl number Pr and suction parameter f0 increase both the skin friction and the rate of heat transfer.
8. Radiation should be at its minimum in order to facilitate the cooling process.

TABLE-2; Values of wall velocity gradient -f^{u}(0), F(0), -G(0), H(0), temperature gradient -$\theta^{\iota}(0)$ and θp (0) for different values of β, Ec , Gr, Pr,M ,Ra , φ and f0 where $\gamma = 1200$

β	Ec	Gr	Pr	Ra	f0	M	φ	ε	$-f^{u}(0)$	F(0)	-G(0)	H(0)	$-\theta^{\iota}(0)$	θp(0)
0.03	1.0	0.01	0.71	3.0	1.0	0.01	.01	5	1.605152	0.188190	1.091958	0.132085	0.385504	0.007824
0.04								5	1.603132	0.191284	1.104388	0.131114	0.388778	0.008586
0.05								5	1.603280	0.194401	1.115960	0.129672	0.387538	0.012206
0.01	1.0	0.01	0.71	3.0	1.0	0.01	.01	5	1.608025	0.181953	1.06918	0.136667	0.383674	-0.011184
	2.0							5	1.602079	0182076	1.070127	0.131885	0.257831	-0.014057
	3.0							5	1.601691	0.182088	1.070294	0.131615	0.128644	-0.023888
0.01	1.0	0.01	0.71	3.0	1.0	0.01	.01	5	1.608025	0.181953	1.06918	0.136667	0.383674	-0.011184
		0.05						5	2.189575	.174077	1.053012	0.133249	0.220501	-0.002155
		0.07						5	1.535403	0.182504	1.071160	0.131646	0.409585	-0.004565
0.01	1.0	0.01	0.71	3.0	1.0	0.01	.01	5	1.608025	0.181953	1.06918	0.136667	0.383674	-0.011184
			1.0					5	1.608691	0.182132	1.069823	1.137367	0.471694	-0.016568
			10.0					5	1.611568	0.182009	1.069669	0.127543	2.310627	-0.280226
0.01	1.0	0.01	0.71	3.0	1.0	0.01	.01	5	1.608025	0.181953	1.06918	0.136667	0.383674	-0.011184
				4.0				5	1.601506	0.181969	1.070321	0.131974	0.352027	-0.004047
				5.0				5	1.600657	0.182011	1.069978	0.130883	0.320693	-0.004449
0.01	1.0	0.01	0.71	3.0	1.0	0.01	.01	5	1.608025	0.181953	1.06918	0.136667	0.383674	-0.011184
					2.0			5	2.394478	0.185626	1.092548	0.131500	0.389793	-0.005415
					3.0			5	3.282218	0.189544	1.117087	0.131791	0.416650	-0.005415
0.01	1.0	0.01	0.71	3.0	1.0	0.03	.01	5	1.594789	0.150576	1.060289	0.141505	0.391182	0.001484
						0.05		5	1.585119	0.115945	1.049421	0.152503	0.393610	0.006072
						0.09		5	1.566027	0.0342118	1.023859	0.184870	0.404825	0.008615
0.01	1.0	0.01	0.71	3.0	1.0	0.1	.01	5	1.608025	0.181953	1.06918	0.136667	0.383674	-0.011184
							.05	5	1.607868	0.181952	1.069617	0.136704	0.383683	-0.010870
							.07	5	1.607770	0.181951	1.069617	0.136673	0.383687	-0.010729
0.05	1.0	0.01	1.0	3.0	1.0	0.01	0.01	1.0	1.61681	0.115511	1.164683	0.182311	0.547737	-0.1118591
0.05	1.0	0.01	1.0	3.0	1.0	0.01	0.01	2.0	1.619947	-0.379955	0.882584	0.433935	0.483187	-0.1179231
0.05	1.0	0.01	1.0	3.0	1.0	0.01	0.01	3.0	1.625529	-0.477509	0.870251	0.561127	0.472316	-0.3365600

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BIOGRAPHICAL NOTES

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