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A Family of Unbiased Estimators of Population Mean Using an Auxiliary Variable

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Abstract

In the present study, we have proposed a family of estimators given by Khoshnevisan et al (2007) for population mean of the study variable using information on an auxiliary variable under the jack – knife technique. Its unbiasedness and mean squared error (MSE) are derived. The comparison of the proposed unbiased sampling strategy with the usual unbiased estimator and other estimators available in the literature are carried out. The results obtained are illustrated numerically using an empirical sample considered in the literature.

Keywords: Family of estimators, auxiliary information, bias, MSE.

1. INTRODUCTION

The use of auxiliary information to increase the precision of estimators has been discussed extensively by various authors. The ratio estimator among the most commonly used estimator of the population mean or total of some variable of interest of a finite population with the help of an auxiliary variable when the correlation coefficient between the two variables is positive. In case of negative correlation, product estimator is used. These estimators are more efficient i.e. has smaller variance than the usual estimator of the population mean based on the sample mean of a simple random sample.

In this paper, a class of estimators given by Khoshnevisan et al (2007) is reconsidered by using the generalized jack – knife statistic of Gray and Schucany in section 2. In a finite population, let y denote the variable whose population mean $\bar{Y}$ is to be estimated by using information of the auxiliary variable x. Assuming that the population mean $\bar{X}$ and of the auxiliary variable are known the following class of estimators of the population mean were given by Khoshnevisan et al (2007)

$$\bar{y}_\alpha = \bar{y}\left[\frac{a\bar{X}+b}{\alpha(a\bar{x}+b)+(1-\alpha)(a\bar{X}+b)}\right]^g \quad (1)$$

where $\bar{y}$ denotes the sample mean of the variable of interest y; $\bar{x}, \bar{X}$ respectively denote the sample and the population means of the variables x; a and b are real numbers or the functions of the known parameters of the auxiliary variable.

1.1 Definitions and results

Assuming that the population mean $\bar{X}$ of the auxiliary variables is known. Let $\bar{y}$, $\bar{x}$ respectively denote the sample means of the variables y, x based on a simple random sample without replacement (SRSWOR) of size *n* drawn from the population.

Define:

$e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}, e_1 = \frac{\bar{x}-\bar{X}}{\bar{X}}$, Then $E(e_0) = E(e_1) = 0$, $E(e_0^2) = \frac{f}{n}C_y^2, E(e_1^2) = \frac{f}{n}C_x^2$, $E(e_0 e_1) = \frac{f}{n}\rho_{yx}C_y C_x$

where

$$f = \frac{N-n}{N}, C_y^2 = \frac{S_y^2}{\bar{Y}^2}, C_x^2 = \frac{S_x^2}{\bar{X}^2}, S_y^2 = (N-1)^{-1}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^2,$$

$$\rho_{yx} = \frac{S_{yx}}{\sqrt{S_y^2}\sqrt{S_x^2}} S_{yx} = (N-1)^{-1}\sum_{i=1}^{N}\left(Y_i-\bar{Y}\right)\left(X_i-\bar{X}\right)$$

Also, ρ_{yx} denote the correlation coefficient between Y and X ,respectively and C_y^2, C_x^2 and denote the coefficient of variation of Y , X and respectively.

1.2 Properties of the estimator $\bar{y}_\alpha$

Putting these values, we have

$$\bar{y}_\alpha = \bar{Y}\left\{1 + e_0 - g\alpha\lambda e_1 + \frac{g(g+1)}{2}\alpha^2\lambda^2 e_1^2 - g\alpha\lambda e_1 e_0 + \ldots\right\} \quad \text{where } \lambda = \frac{a\bar{X}}{a\bar{X}+b} \quad (2)$$

Taking expectation of (2), we have

$$\text{E}(\bar{y}_\alpha) = \bar{Y}E\left\{1 + e_0 - g\alpha\lambda e_1 + \frac{g(g+1)}{2}\alpha^2\lambda^2 e_1^2 - g\alpha\lambda e_1 e_0 + \ldots\right\} \quad (3)$$

And the bias of the estimator is given by

$$B(\bar{y}_\alpha) = \frac{f}{n}\bar{Y}\left[\frac{g(g+1)}{2}\alpha^2\lambda^2 C_x^2 - 2\alpha\lambda g\rho C_x C_y\right] \quad (4)$$

And its mean square error is given by

$$MSE(\bar{y}_\alpha) = E(\bar{y}_\alpha - \bar{Y})^2 = \bar{Y}^2\frac{f}{n}\left[C_y^2 + \alpha\lambda g C_x^2 - 2\alpha\lambda\rho g C_x C_y\right] \quad (5)$$

The minimum MSE of the class of estimators is obtained by minimizing with respect to α we get the optimum value $\alpha = \frac{K}{\lambda g} = \alpha_{opt}$ where $K = \rho \frac{C_y}{C_x}$

$$MSE_{\min}(\bar{y}_{\alpha^*}) = \bar{Y}^2 \frac{f}{n}\left(1-\rho^2\right)C_y^2 \tag{6}$$

The $MSE_{\min}(\bar{y}_{\alpha^*})$ is same as that of the approximate variance of the usual linear regression estimator.

2. THE PROPOSED JACK-KNIFE ESTIMATOR $\bar{y}_{\alpha j}$

Let a simple random sample of size n=2m is drawn without replacement from the population of size N. This sample of size n=2m is then split up at random in to two sub samples each of size m

$$\bar{y}_{\alpha} = \bar{y}\left[\frac{a\bar{X}+b}{\alpha(a\bar{x}+b)+(1-\alpha)(a\bar{X}+b)}\right]^g \tag{7}$$

$$\bar{y}_{\alpha_i} = \bar{y}\left[\frac{a\bar{X}+b}{\alpha(a\bar{x}_i+b)+(1-\alpha)(a\bar{X}+b)}\right]^g, \; i=1,\,2 \tag{8}$$

Let us define

$$\bar{y}_{\alpha j} = \frac{\bar{y}_{\alpha} - R\bar{y}'_{\alpha}}{1-R} \text{ where } \bar{y}'_{\alpha} = \frac{\bar{y}_{\alpha_1} + \bar{y}_{\alpha_2}}{2} \text{ and } R = \frac{B(\bar{y}_{\alpha})}{B(\bar{y}'_{\alpha})}$$

By analogy to (4), we have

$$B(\bar{y}_{\alpha}) = \frac{f}{n}\bar{Y}\left[\frac{g(g+1)}{2}\alpha^2\lambda^2C_x^2 - 2\alpha\lambda g\rho C_xC_y\right] = B_1 \text{(say)} \tag{9}$$

Similarly, it can be shown that

$$B(\bar{y}_{\alpha_i}) = \frac{f}{m}\bar{Y}\left[\frac{g(g+1)}{2}\alpha^2\lambda^2C_x^2 - 2\alpha\lambda g\rho C_xC_y\right], \; i=1,\,2$$

Thus, $$B(\bar{y}'_{\alpha}) = \frac{f}{m}\bar{Y}\left[\frac{g(g+1)}{2}\alpha^2\lambda^2C_x^2 - 2\alpha\lambda g\rho C_xC_y\right] \tag{10}$$

Now, taking expectation of $\bar{y}_{\alpha j}$, we have $E(\bar{y}_{\alpha j}) = \frac{E(\bar{y}_{\alpha}) - RE(\bar{y}'_{\alpha})}{1-R} = \bar{Y}$ thereby showing that $\bar{y}_{\alpha j}$ is an unbiased estimator of population mean $\bar{Y}$.

2.1 Mean square error of $\bar{y}_{\alpha j}$

$$MSE(\bar{y}_{\alpha j}) = E(\bar{y}_{\alpha j} - \bar{Y}) = E\left\{\frac{\bar{y}_{\alpha} - R\bar{y}'_{\alpha}}{1-R} - \bar{Y}\right\}^2 = \frac{E(\bar{y}_{\alpha} - \bar{Y})^2 + R^2E(\bar{y}'_{\alpha} - \bar{Y}) - 2R(\bar{y}_{\alpha} - \bar{Y})(\bar{y}'_{\alpha} - \bar{Y})}{(1-R)^2} \tag{11}$$

$$MSE(\bar{y}_\alpha) = E(\bar{y}_\alpha - \bar{Y})^2 = \bar{Y}^2 \frac{f}{n}\left[C_y^2 + \alpha\lambda g C_x^2 - 2\alpha\lambda\rho g C_x C_y\right] \tag{12}$$

$$E(\bar{y}'_\alpha - \bar{Y})^2 = E\left[\frac{\bar{y}_{\alpha_1} + \bar{y}_{\alpha_2}}{2} - \bar{Y}\right]^2 = E\left[\frac{(\bar{y}_{\alpha_1} - \bar{Y}) + (\bar{y}_{\alpha_2} - \bar{Y})}{2}\right]^2$$

$$= \frac{1}{4}\left\{E(\bar{y}_{\alpha_1} - \bar{Y})^2 + E(\bar{y}_{\alpha_2} - \bar{Y})^2 + 2E(\bar{y}_{\alpha_1} - \bar{Y})(\bar{y}_{\alpha_2} - \bar{Y})\right\}$$

$$E(\bar{y}_{\alpha_i} - \bar{Y})^2 = \bar{Y}^2 \frac{f_m}{m}\left[C_y^2 + \alpha\lambda g C_x^2 - 2\alpha\lambda\rho g C_x C_y\right] \quad ; i = 1, 2 \tag{13}$$

$$E(\bar{y}_{\alpha_1} - \bar{Y})(\bar{y}_{\alpha_2} - \bar{Y}) = \bar{Y}^2 (e_0^{(1)} - \alpha\lambda g e_1^{(1)})(e_0^{(2)} - \alpha\lambda g e_1^{(2)})$$

$$= \bar{Y}^2 E(e_0^{(1)} e_0^{(2)} - \alpha\lambda g(e_0^{(1)} e_1^{(2)} + e_1^{(1)} e_0^{(2)}) + \alpha^2\lambda^2 g^2 e_1^{(1)} e_1^{(2)})$$

$$= -\frac{\bar{Y}^2}{N}\left[C_y^2 + \alpha\lambda g C_x^2 - 2\alpha\lambda\rho g C_x C_y\right] \tag{14}$$

Putting these values, we have

$$E(\bar{y}'_\alpha - \bar{Y})^2 = \frac{1}{4}\left\{2\left(\frac{N-m}{mN}\right) - \frac{2}{N}\right\}\bar{Y}^2\left[C_y^2 + \alpha\lambda g C_x^2 - 2\alpha\lambda\rho g C_x C_y\right]$$

$$= \bar{Y}^2 \frac{f_n}{n}\left[C_y^2 + \alpha\lambda g C_x^2 - 2\alpha\lambda\rho g C_x C_y\right] \tag{15}$$

$$E(\bar{y}'_\alpha - \bar{Y})(\bar{y}_\alpha - \bar{Y}) = \left(\frac{\bar{y}_{\alpha_1} + \bar{y}_{\alpha_2}}{2} - \bar{Y}\right)(\bar{y}_\alpha - \bar{Y})$$

$$= \frac{1}{2}\left\{E(\bar{y}_{\alpha_1} - \bar{Y})(\bar{y}_\alpha - \bar{Y}) + E(\bar{y}_{\alpha_2} - \bar{Y})(\bar{y}_\alpha - \bar{Y})\right\}$$

$$E(\bar{y}_{\alpha_i} - \bar{Y})(\bar{y}_\alpha - \bar{Y}) = \bar{Y}^2 (e_0^{(i)} - \alpha\lambda g e_1^{(i)})(e_0 - \alpha\lambda g e_1)$$

$$= \bar{Y}^2 E\left\{e_0 e_0^{(i)} - \alpha\lambda g(e_0 e_1^{(i)} + e_1 e_0^{(i)}) + \alpha^2\lambda^2 g^2 e_1 e_1^{(i)}\right\}$$

$$E(\bar{y}_{\alpha_i} - \bar{Y})(\bar{y}_\alpha - \bar{Y}) = \bar{Y}^2 \frac{f_n}{n}\left[C_y^2 + \alpha\lambda g C_x^2 - 2\alpha\lambda\rho g C_x C_y\right] \quad ; i = 1, 2 \tag{16}$$

Putting all the results in (11), we get

$MSE(\bar{y}_{\alpha j}) = \bar{Y}^2 \frac{f_n}{n}\left[C_y^2 + \alpha\lambda g C_x^2 - 2\alpha\lambda\rho g C_x C_y\right]$ which is same as the MSE of $\bar{y}_\alpha$.

3. COMPARISON OF THE ESTIMATORS

We consider the following known estimators belonging to the considered family of estimators

1. For $\alpha = 0, a = 0, b = 0$ and $g = 0$ in the proposed class of estimators SRS mean $\bar{y}$.
2. The ratio estimator for $\alpha = 1, a = 1, b = 0$ and $g = 1$

$$\bar{Y}_R = \bar{y}\frac{\bar{X}}{\bar{x}}$$

3. Product estimator for $\alpha = 1, a = 1, b = 0$ and $g = -1$

$$\bar{Y}_P = \bar{y}\frac{\bar{x}}{\bar{X}}$$

4. Sisodia and dwivedi (1981) estimator for $\alpha = 1, a = 1, b = C_x$ and $g = 1$,

$$y_{sd} = \bar{y}\left(\frac{\bar{X} + C_x}{\bar{x} + C_x}\right)$$

5. Pandey and Dubey (1988) estimator for $\alpha = 1, a = 1, b = C_x$ and $g = -1$

$$y_{pd} = \bar{y}\left(\frac{\bar{x} + C_x}{\bar{X} + C_x}\right)$$

6. Upadhyaya and Singh (1999) estimator for $\alpha = 1, a = \beta_2(x), b = C_x$ and $g = -1$

$$y_{us1} = \bar{y}\left(\frac{\beta_2(x)\bar{x} + C_x}{\beta_2(x)\bar{X} + C_x}\right)$$

7. Upadhyaya and Singh (1999) estimator for $\alpha = 1, a = C_x, b = \beta_2(x)$ and $g = -1$

$$y_{us2} = \bar{y}\left(\frac{C_x\bar{x} + \beta_2(x)}{C_x\bar{X} + \beta_2(x)}\right)$$

8. G.N.singh(2003) estimator for $\alpha = 1, a = 1, b = \sigma_x$ and $g = -1$

$$y_{GN1} = \bar{y}\left(\frac{\bar{x} + \sigma_x}{\bar{X} + \sigma_x}\right)$$

9. G.N.singh (2003) estimator for $\alpha = 1, a = \beta_1(x), b = \sigma_x$ and $g = -1$

$$y_{GN2} = \bar{y}\left(\frac{\beta_1(x)\bar{x} + \sigma_x}{\beta_1(x)\bar{X} + \sigma_x}\right)$$

10. G.N.singh (2003) estimator for $\alpha = 1, a = \beta_2(x), b = \sigma_x$ and $g = -1$

$$y_{GN3} = \bar{y}\left(\frac{\beta_2(x)\bar{x} + \sigma_x}{\beta_2(x)\bar{X} + \sigma_x}\right)$$

11. Singh,Tailor (2003) estimator for $\alpha = 1, a = 1, b = \rho$ and $g = 1$

$$y_{TL1} = \bar{y}\left(\frac{\bar{X} + \rho}{\bar{x} + \rho}\right)$$

12. Singh,Tailor (2003) estimator for $\alpha = 1, a = 1, b = \rho$ and $g = -1$

$$y_{TL2} = \bar{y}\left(\frac{\bar{x} + \rho}{\bar{X} + \rho}\right)$$

13. Singh,et.al.(2004) estimator for $\alpha = 1, a = 1, b = \beta_2(x)$ and $g = 1$

$$y_{\sin gh1} = \bar{y}\left(\frac{\bar{X} + \beta_2(x)}{\bar{x} + \beta_2(x)}\right)$$

14. Singh, et.al. (2004) estimator for $\alpha = 1, a = 1, b = \beta_2(x)$ and $g = -1$

$$y_{\sin gh2} = \bar{y}\left(\frac{\bar{x} + \beta_2(x)}{\bar{X} + \beta_2(x)}\right)$$

The MSE of the above estimators up to the first order of approximation is given by

$$\text{Var}(\bar{y}) = \frac{f}{n} C_y^2, \ MSE(\bar{y}_R) = \frac{f}{n}\bar{Y}^2(C_y^2 + C_x^2 - 2\rho C_x C_y)$$

$$MSE(\bar{y}_P)=\frac{f}{n}\bar{Y}^2(C_y^2+C_x^2+2\rho C_x C_y)\,,\qquad MSE(\bar{y}_{sd})=\frac{f}{n}\bar{Y}^2(C_y^2+\theta_1^2C_x^2-2\theta_1\rho C_x C_y)$$

$$MSE(\bar{y}_{pd})=\frac{f}{n}\bar{Y}^2(C_y^2+\theta_1^2C_x^2+2\theta_1\rho C_x C_y)\,,\qquad MSE(\bar{y}_{us1})=\frac{f}{n}\bar{Y}^2(C_y^2+\theta_2^2C_x^2+2\theta_2\rho C_x C_y)$$

$$MSE(\bar{y}_{us2})=\frac{f}{n}\bar{Y}^2(C_y^2+\theta_3^2C_x^2+2\theta_3\rho C_x C_y)\,,\qquad MSE(\bar{y}_{GN1})=\frac{f}{n}\bar{Y}^2(C_y^2+\theta_4^2C_x^2+2\theta_4\rho C_x C_y)$$

$$MSE(\bar{y}_{GN2})=\frac{f}{n}\bar{Y}^2(C_y^2+\theta_5^2C_x^2+2\theta_5\rho C_x C_y)\,,\qquad MSE(\bar{y}_{GN3})=\frac{f}{n}\bar{Y}^2(C_y^2+\theta_6^2C_x^2+2\theta_6\rho C_x C_y)$$

$$MSE(\bar{y}_{TL1})=\frac{f}{n}\bar{Y}^2(C_y^2+\theta_7^2C_x^2-2\theta_7\rho C_x C_y)\,,\qquad MSE(\bar{y}_{TL2})=\frac{f}{n}\bar{Y}^2(C_y^2+\theta_7^2C_x^2+2\theta_7\rho C_x C_y)$$

$$MSE(\bar{y}_{\sin gh1})=\frac{f}{n}\bar{Y}^2(C_y^2+\theta_8^2C_x^2-2\theta_8\rho C_x C_y)\,,$$

$$MSE(\bar{y}_{\sin gh2})=\frac{f}{n}\bar{Y}^2(C_y^2+\theta_8^2C_x^2+2\theta_8\rho C_x C_y)$$

where $\theta_1=\frac{\bar{X}}{\bar{X}+C_x}$, $\theta_2=\frac{\beta_2(x)\bar{X}}{\beta_2(x)+C_x}$, $\theta_3=\frac{C_x\bar{X}}{C_x\bar{X}+C_x}$, $\theta_4=\frac{\bar{X}}{\bar{X}+C_x}$

$\theta_5=\frac{\beta_1(x)\bar{X}}{\beta_1(x)+\sigma_x}$, $\theta_6=\frac{\beta_2(x)\bar{X}}{\beta_2(x)\bar{X}+\sigma_x}$, $\theta_7=\frac{\bar{X}}{\bar{X}+\rho}$, $\theta_8=\frac{\bar{X}}{\bar{X}+\beta_2(x)}$

On comparing the efficiency of the proposed jack-knife estimator with above mentioned estimators, we have

$MSE(\bar{y}_R)-MSE(\bar{y}_{\alpha j})>0$, $MSE(\bar{y}_P)-MSE(\bar{y}_{\alpha j})>0$, $MSE(\bar{y}_{sd})-MSE(\bar{y}_{\alpha j})>0$

$MSE(\bar{y}_{pd})-MSE(\bar{y}_{\alpha j})>0$, $MSE(\bar{y}_{us1})-MSE(\bar{y}_{\alpha j})>0$, $MSE(\bar{y}_{us2})-MSE(\bar{y}_{\alpha j})>0$

$MSE(\bar{y}_{GN1})-MSE(\bar{y}_{\alpha j})>0$, $MSE(\bar{y}_{GN2})-MSE(\bar{y}_{\alpha j})>0$, $MSE(\bar{y}_{GN3})-MSE(\bar{y}_{\alpha j})>0$

$MSE(\bar{y}_{TL1})-MSE(\bar{y}_{\alpha j})>0$, $MSE(\bar{y}_{TL2})-MSE(\bar{y}_{\alpha j})>0$, $MSE(\bar{y}_{\sin gh1})-MSE(\bar{y}_{\alpha j})>0$

$MSE(\bar{y}_{\sin gh2})-MSE(\bar{y}_{\alpha j})>0$

Thus, the proposed jack-knife estimators $\bar{y}_{\alpha j}$ is unbiased and has got lesser mean square error as compared with the above estimators.

4. NUMERICAL ILLUSTRATION

We consider the data used by Pandey and Dubey(1988) to demonstrate what we have discussed in the above sections.

The following values were obtained using the whole data set:

N=20, n=8 $\bar{Y}$ =19.55, $\bar{X}$ =18.8, $C_x^2=0.1555, C_y^2=0.1262$

$\rho_{xy}=-0.9199$, $\beta_2(x)=3.0613$, $\beta_1(x)=0.5473$, $\theta_4=0.7172$

Using the above results we calculated the percent relative efficiency (PRE) of different estimators in Table1,

Table1: PRE of estimators under study based on population data

Estimator	PRE
$\bar{y}$	100
$\bar{y}_R$	23.39
$\bar{y}_P$	526.45
$\bar{y}_{sd}$	23.91
$\bar{y}_{pd}$	550.05
$\bar{y}_{\text{us}1}$	534.49
$\bar{y}_{\text{us}2}$	582.17
$\bar{y}_{GN1}$	591.37
$\bar{y}_{GN2}$	436.19
$\bar{y}_{GN3}$	633.64
$\bar{y}_{TL1}$	22.17
$\bar{y}_{TL2}$	465.25
$\bar{y}_{\sin gh1}$	27.21
$\bar{y}_{\sin gh2}$	644.17
$\bar{y}_{kosh}$	650.26
$\bar{y}_{\alpha j}$	650.26

CONCLUSION

It may be easily observed from the preceding sections, we observe that the proposed jack-knife estimator is preferable over all the considered estimators as it is unbiased and efficient.

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