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An Unbiased Class of Ratio Type Estimator for Population Mean Using an Attribute and a Variable

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Abstract

In this paper, we have considered a class of exponential type ratio estimator using the auxiliary information in both the form attribute and variable. We propose an improved class of estimator using Jack-Knife technique. Further, it has been shown that the proposed Jack-Knife estimator is unbiased and has lesser minimum mean square error under the optimum value of characterising parameter as compared to some commonly used estimators available in the literature. An empirical study is included as an illustration.

Keywords Unbiased Class of Estimators, Jack-Knife technique, Unbiasedness and Mean Square Error.

1. INTRODUCTION

In many situations auxiliary information is used to improve the precision of an estimator. This auxiliary information may be in the form of a variable or an attribute or both. For example - Height is different for male and female which shows that sex is a helpful attribute while dealing with height and also height is related with the weight which is a variable; amount of milk produced by cow depends on the breed as well as on the diet; yield of wheat crop depends on the variety of wheat and manure as well etc. In such situations, we can take the advantages of the available information on variable and attribute to increase the efficiency of the estimator.

Let y be the study variable, x be the auxiliary variable and ϕ be the auxiliary attribute. Also, let consider a finite population of size N and we denote by

$\bar{Y}=\frac{1}{N}\sum_{i=1}^{N}Y_i$, $\bar{X}=\frac{1}{N}\sum_{i=1}^{N}X_i$ and $P=\frac{1}{N}\sum_{i=1}^{N}\phi_i$ be the population means of y, x and ϕ respectively; $S_Y^2=\frac{1}{N}\sum_{i=1}^{N}(Y_i-\bar{Y})^2$, $S_X^2=\frac{1}{N}\sum_{i=1}^{N}(X_i-\bar{X})^2$ and $S_\phi^2=\frac{1}{N}\sum_{i=1}^{N}(\phi_i-P)^2$ be the population variances. Further, on the basis of a simple random sample of size n drawn without replacement from the population of size N, we denote by $\bar{y}=\frac{1}{n}\sum_{i=1}^{n}Y_i$, $\bar{x}=\frac{1}{n}\sum_{i=1}^{n}X_i$ and $p=\frac{1}{n}\sum_{i=1}^{n}\phi_i$ be the sample means of y, x and ϕ respectively; $s_X^2=\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{x})^2$, $s_Y^2=\frac{1}{n}\sum_{i=1}^{n}(Y_i-\bar{y})^2$ and $s_\phi^2=\frac{1}{n}\sum_{i=1}^{n}(\phi_i-p)^2$ be the sample variances.

Following Naik and Gupta (1996) and Abu Dayyeh (2003), we propose a class of estimators assuming that the auxiliary population mean and auxiliary population proportions are known

$$\hat{\bar{Y}}_p=\bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha_1}\left(\frac{p}{P}\right)^{\alpha_2} \tag{1.1}$$

It may be noted that the sample mean, ratio estimator, Srivastava type generalized ratio estimator (1967), Naik and Gupta (1996) ratio estimator, Naik and Gupta – Srivastava type generalized ratio estimator are the members of the proposed class of estimators.

2. BIAS AND MSE OF THE PROPOSED ESTIMATOR

In order to obtain the bias and mean square error (MSE), let us denote by

$\bar{y}=\bar{Y}(1+e_0),\ \bar{x}=\bar{X}(1+e_1),\ p=P(1+e_2)$, $E(e_0)=E(e_1)=E(e_2)=0$

$E(e_0^2)=f_nC_Y^2$, $E(e_1^2)=f_nC_X^2$, $E(e_2^2)=f_nC_P^2$ $E(e_0e_1)=f_n\rho_{YX}C_YC_X$ $E(e_0e_2)=f_n\rho_{YP}C_YC_P$

$E(e_1e_2)=f_n\rho_{XP}C_XC_P$ (2.1)

Substituting the values from (2.1) in (1.1), we get

$$\hat{\bar{Y}}_p=\bar{Y}\left\{1+e_0+\alpha_1e_1+\alpha_2e_2+\alpha_1e_0e_1+\alpha_2e_0e_2+\frac{\alpha_1(\alpha_1-1)}{2}e_1^2+\frac{\alpha_2(\alpha_2-1)}{2}e_2^2+\alpha_1\alpha_2e_1e_2\right\} \tag{2.2}$$

Thus, the bias and mean square error of the proposed class is given by

$$Bias(\hat{\bar{Y}}_p)=f_n\bar{Y}\left\{\frac{\alpha_1(\alpha_1-1)}{2}C_X^2+\frac{\alpha_2(\alpha_2-1)}{2}C_P^2+\alpha_1\rho_{XY}C_XC_Y+\alpha_2\rho_{PY}C_PC_Y+\alpha_1\alpha_2\rho_{XP}C_XC_P\right\}$$

$$=f_n\bar{Y}A\ (say) \tag{2.3}$$

$$MSE(\hat{\bar{Y}}_p) = f_n\bar{Y}^2\left\{C_Y^2+\alpha_1^2C_X^2+\alpha_2^2C_P^2+2\alpha_1\rho_{XY}C_XC_Y+2\alpha_2\rho_{PY}C_PC_Y+2\alpha_1\alpha_2\rho_{XP}C_XC_P\right\} = f_n\bar{Y}B\ (say) \tag{2.4}$$

where $f_n = (N-n)/Nn$

The optimizing values of α_1 and α_2 minimizing the MSE are

$$\alpha_{1opt} = -\frac{(\rho_{YX}-\rho_{YP}\rho_{PX})}{(1-\rho_{XP}^2)}\frac{C_Y}{C_X} \tag{2.5}$$

$$\alpha_{2opt} = -\frac{(\rho_{YP}-\rho_{YX}\rho_{XP})}{(1-\rho_{XP}^2)}\frac{C_Y}{C_P} \tag{2.6}$$

The minimum MSE within the class of proposed estimators is given by

$$MSE(\hat{\bar{Y}}_p)_{\min} = f_n\bar{Y}^2(1-R_{Y.XP}^2)C_Y^2 = M(say) \tag{2.7}$$

where $R_{Y.XP}^2$ is the multiple correlation coefficient of y on x and p.

3. THE PROPOSED JACK – KNIFE ESTIMATOR

Let a random sample of size $n = 2m$ is drawn from the finite population of size N by SRSWOR and split it into two random sub-sample of size m each.

Define the following estimator

$$\hat{\bar{Y}}_1 = \bar{y}^{(1)}\left(\frac{\bar{x}^{(1)}}{\bar{X}}\right)^{\alpha_1}\left(\frac{p^{(1)}}{P}\right)^{\alpha_2} \quad \hat{\bar{Y}}_2 = \bar{y}^{(2)}\left(\frac{\bar{x}^{(2)}}{\bar{X}}\right)^{\alpha_1}\left(\frac{p^{(2)}}{P}\right)^{\alpha_2} \quad \hat{\bar{Y}}_3 = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha_1}\left(\frac{p}{P}\right)^{\alpha_2} \tag{3.1}$$

where $\bar{y}^{(1)}, \bar{y}^{(2)}, \bar{x}^{(1)}, \bar{x}^{(2)}$ and $p^{(1)}, p^{(2)}$ are the respective sample mean of the study variable, auxiliary variable and auxiliary attribute based on split sample 1 and 2 each of size m and $\bar{y}, \bar{x}$ and p are the respective mean of the study variable, auxiliary variable and auxiliary attribute based on the entire sample. Proceeding similarly, as in (2.3), we have

$$Bias(\hat{\bar{Y}}_1) = \bar{Y}f_mA \qquad Bias(\bar{Y}_2) = \bar{Y}f_mA \qquad Bias(\hat{\bar{Y}}_3) = \bar{Y}f_nA = K_1(say) \tag{3.2}$$

Lets us define $\hat{\bar{Y}}' = \dfrac{\bar{Y}_1+\bar{Y}_2}{2}$ as an alternative estimator of population mean, so that its bias is given by

$$Bias(\hat{\bar{Y}}') = \bar{Y}f_m\left\{\frac{\alpha_1(\alpha_1-1)}{2}C_X^2+\frac{\alpha_2(\alpha_2-1)}{2}C_P^2+\alpha_1\rho_{XY}C_XC_Y+\alpha_2\rho_{PY}C_PC_Y+\alpha_1\alpha_2\rho_{XP}C_XC_P\right\}$$
$$= K_2(say) \tag{3.3}$$

Let us now propose the following Jack-Knife estimator

$$\bar{Y}_s^* = \frac{\bar{Y}_1^{(3)} - R\bar{Y}_1'}{1-R} \quad \text{where} \quad R = \frac{K_1}{K_2} = \frac{f_n}{f_m} = \frac{(N-2m)}{2(N-m)} \tag{3.4}$$

Then it can be easily verified that the proposed Jack-Knife estimator is an unbiased estimator of population mean upto the first order of approximation.Now, mean square error of the proposed estimator is defined as

$$MSE(\bar{Y}_s^*) = E\left(\bar{Y}_s^* - \bar{Y}\right)^2 = \frac{1}{(1-R)^2}\left[E(\hat{\hat{Y}}_3 - \bar{Y})^2 + R^2E(\hat{\hat{Y}}' - \bar{Y})^2 - 2RE(\hat{\hat{Y}}_3 - \bar{Y})(\hat{\hat{Y}}' - \bar{Y})\right] \tag{3.5}$$

By analogy from (2.4), we have

$$E(\hat{\hat{Y}}_3 - \bar{Y})^2 = MSE(\hat{\hat{Y}}_3) = f_n\bar{Y}^2B \tag{3.6}$$

Consider

$$E(\hat{\hat{Y}}' - \bar{Y})^2 = E\left(\frac{\hat{\hat{Y}}_1 + \hat{\hat{Y}}_2}{2} - \bar{Y}\right)^2 = \frac{1}{4}\left[E\left(\hat{\hat{Y}}_1 - \bar{Y}\right)^2 + E\left(\hat{\hat{Y}}_2 - \bar{Y}\right)^2 + 2E\left(\hat{\hat{Y}}_1 - \bar{Y}\right)\left(\hat{\hat{Y}}_2 - \bar{Y}\right)\right] \tag{3.7}$$

From (2.4)

$$E(\hat{\hat{Y}}_i - \bar{Y})^2 = MSE(\hat{\hat{Y}}_i) = f_m\bar{Y}^2B \qquad ; i = 1,2 \tag{3.8}$$

Taking

$$\bar{y}^{(i)} = \bar{Y}(1+e_0^{(i)}), \bar{x}^{(i)} = \bar{X}(1+e_1^{(i)}), p^{(i)} = P(1+e_2^{(i)}); i = 1,2 \quad \text{with}$$

$$E(e_0^{(i)}) = E(e_1^{(i)}) = E(e_2^{(i)}) = 0 \qquad ; i = 1,2 \tag{3.9}$$

using (2.12) we can write

$$\hat{\hat{Y}}_i - \bar{Y} = \bar{Y}\left\{1 + e_0^{(i)} + \alpha_1 e_1^{(i)} + \alpha_2 e_2^{(i)} + \alpha_1 e_0^{(i)} e_1^{(i)} + \alpha_2 e_0^{(i)} e_2^{(i)} + \alpha_1\alpha_2 e_1^{(i)} e_2^{(i)} + \frac{\alpha_1(\alpha_1 - 1)}{2} e_1^{(i)2} + \frac{\alpha_2(\alpha_2 - 1)}{2} e_2^{(i)2}\right\} \tag{3.10}$$

To the first order of approximation, we have

$$E(\hat{\hat{Y}}_1 - \bar{Y})(\hat{\hat{Y}}_2 - \bar{Y}) = \bar{Y}^2E\left[\left(e_0^{(1)} + \alpha_1 e_1^{(1)} + \alpha_2 e_2^{(1)}\right)\left(e_0^{(2)} + \alpha_1 e_1^{(2)} + \alpha_2 e_2^{(2)}\right)\right]$$

$$= \bar{Y}^2\left[E(e_0^{(1)}e_0^{(2)}) + \alpha_1\left\{E(e_0^{(1)}e_1^{(2)}) + E(e_0^{(2)}e_1^{(1)})\right\} + \alpha_2\left\{E(e_0^{(1)}e_2^{(2)}) + E(e_0^{(2)}e_2^{(1)})\right\}\right.$$

$$\left. + \alpha_1^2E(e_1^{(1)}e_1^{(2)}) + \alpha_2^2E(e_2^{(1)}e_2^{(2)}) + \alpha_1\alpha_2\left\{E(e_1^{(1)}e_2^{(2)}) + E(e_1^{(2)}e_2^{(1)})\right\}\right]$$

Now, using the results given in Sukhatme and Sukhatme (1997)

$$E(e_0^{(1)}e_0^{(2)})=-\frac{1}{N}C_Y^2,\quad E(e_1^{(1)}e_1^{(2)})=-\frac{1}{N}C_X^2,\quad E(e_2^{(1)}e_2^{(2)})=-\frac{1}{N}C_P^2,\quad E(e_0^{(1)}e_1^{(2)})=-\frac{1}{N}\rho_{YX}C_YC_X$$

$$E(e_0^{(2)}e_1^{(1)})=-\frac{1}{N}\rho_{YX}C_YC_X\quad E(e_0^{(1)}e_2^{(2)})=-\frac{1}{N}\rho_{YP}C_YC_P\quad E(e_0^{(2)}e_2^{(1)})=-\frac{1}{N}\rho_{YP}C_YC_P$$

$$E(e_1^{(1)}e_2^{(2)})=-\frac{1}{N}\rho_{XP}C_XC_P\quad E(e_1^{(2)}e_2^{(1)})=-\frac{1}{N}\rho_{XP}C_XC_P \tag{3.11}$$

we get

$$E(\hat{\bar{Y}}_1-\bar{Y})(\hat{\bar{Y}}_2-\bar{Y})=-\frac{1}{N}\bar{Y}^2B \tag{3.12}$$

Putting the values from (3.8) and (3.12) in (3.7), we get

$$E(\hat{\bar{Y}}'-\bar{Y})^2=MSE(\hat{\bar{Y}}')=\frac{1}{4}\left[2\left(\frac{1}{m}-\frac{1}{N}\right)\bar{Y}^2B-\frac{2}{N}\bar{Y}^2B\right]=f_n\bar{Y}^2B \tag{3.13}$$

Now, consider

$$E(\hat{\bar{Y}}_3-\bar{Y})(\hat{\bar{Y}}'-\bar{Y})=E(\hat{\bar{Y}}_3-\bar{Y})\left(\frac{\hat{\bar{Y}}_1+\hat{\bar{Y}}_2}{2}-\bar{Y}\right)=\frac{1}{2}\left[E\left(\hat{\bar{Y}}_3-\bar{Y}\right)\left(\hat{\bar{Y}}_1-\bar{Y}\right)+E\left(\hat{\bar{Y}}_3-\bar{Y}\right)\left(\hat{\bar{Y}}_2-\bar{Y}\right)\right] \tag{3.14}$$

To the first order of terms

$$E(\hat{\bar{Y}}_3-\bar{Y})(\hat{\bar{Y}}_i-\bar{Y})=\bar{Y}^2E\left[\left(e_0+\alpha_1e_1+\alpha_2e_2\right)\left(e_0^{(i)}+\alpha_1e_1^{(i)}+\alpha_2e_2^{(i)}\right)\right];i=1,2$$

$$=\bar{Y}^2\left[E(e_0e_0^{(i)})+\alpha_1\left\{E(e_0e_1^{(i)})+E(e_0^{(i)}e_1)\right\}+\alpha_2\left\{E(e_0e_2^{(i)})+E(e_0^{(i)}e_2)\right\}\right.$$
$$\left.+\alpha_1^2E(e_1e_1^{(i)})+\alpha_2^2E(e_2e_2^{(i)})+\alpha_1\alpha_2\left\{E(e_1e_2^{(i)})+E(e_1^{(i)}e_2)\right\}\right]$$

Substituting the following results given by Sukhatme and Sukhatme (1997)

$$E(e_0e_0^{(i)})=f_nC_Y^2\quad E(e_1e_1^{(i)})=f_nC_X^2\quad E(e_2e_2^{(i)})=f_nC_P^2\quad E(e_0e_1^{(i)})=f_n\rho_{YX}C_YC_X$$

$$E(e_0^{(i)}e_1)=f_n\rho_{YX}C_YC_X\quad E(e_0e_2^{(i)})=f_n\rho_{YP}C_YC_P\quad E(e_0^{(i)}e_2)=f_n\rho_{YP}C_YC_P$$

$$E(e_1e_2^{(i)})=f_n\rho_{XP}C_XC_P\quad E(e_1^{(i)}e_2)=f_n\rho_{XP}C_XC_P$$

we get

$$E(\hat{\bar{Y}}_3-\bar{Y})(\hat{\bar{Y}}_i-\bar{Y})=f_n\bar{Y}^2B\qquad ;i=1,2 \tag{3.15}$$

Putting the values from (3.15) in (3.14), we get

$$E(\hat{\hat{Y}}_3 - \bar{Y})(\hat{\hat{Y}}' - \bar{Y}) = f_n \bar{Y}^2 B \tag{3.16}$$

Substituting the values from (3.6), (3.13), and (3.16) in (3.5), we get

$$MSE\left(\hat{\hat{Y}}_s^*\right) = \frac{1}{(1-R)^2} f_n \bar{Y}^2 (1 + R^2 - 2R) B = f_n \bar{Y}^2 B \tag{3.17}$$

which is same as that of (2.4) but this proposed jack – knife estimator is better than the proposed estimator given in section 1 and 2 in the sense of unbiasedness. Also, the optimum values of the parameters are also same as given by (2.4) and (2.5); thereby giving the same minimum MSE as that of (2.6).

4. COMPARATIVE STUDY

Let us consider the following estimators for population mean:

1. Sample mean (SRSWOR) $\hat{Y}_1 = \bar{y}$ vs. $\hat{\hat{Y}}_p$ $\left(\text{or } \hat{\hat{Y}}_s^*\right)$ $MSE(\hat{Y}_1) - M = R^2_{Y.XP} C_Y^2 \geq 0$ (4.1)

2. Ratio estimator using auxiliary variable $\hat{Y}_2 = \frac{\bar{y}}{\bar{x}} \bar{X}$ vs. $\hat{\hat{Y}}_p$ $\left(\text{or } \hat{\hat{Y}}_s^*\right)$

$$MSE(\hat{Y}_2) - M = f_n \bar{Y}^2 \left[(C_X - \rho_{YX} C_Y)^2 + (R^2_{Y.XP} - \rho^2_{YX}) C_Y^2 \right] \geq 0 \tag{4.2}$$

3. Naik and Gupta (1996) Ratio estimator using auxiliary attribute $\hat{Y}_3 = \frac{\bar{y}}{p} P$ vs. $\hat{\hat{Y}}_p$ $\left(\text{or } \hat{\hat{Y}}_s^*\right)$

$$MSE(\hat{Y}_3) - M = f_n \bar{Y}^2 \left[(C_P - \rho_{YP} C_Y)^2 + (R^2_{Y.XP} - \rho^2_{YP}) C_Y^2 \right] \geq 0 \tag{4.3}$$

4. Bahl and Tuteja (1991)Exponential Ratio estimator using auxiliary variable $\hat{Y}_4 = \bar{y} \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right)$ vs. $\hat{\hat{Y}}_p$ $\left(\text{or } \hat{\hat{Y}}_s^*\right)$

$$MSE(\hat{Y}_4) - M = f_n \bar{Y}^2 \left[(\frac{C_X}{2} - \rho_{YX} C_Y)^2 + (R^2_{Y.XP} - \rho^2_{YX}) C_Y^2 \right] \geq 0 \tag{4.4}$$

5. Using Bahl and Tuteja (1991) Exponential Ratio estimator using auxiliary variable,

Sawan (2010) proposed the following estimator using auxiliary attribute $\hat{\bar{Y}}_5 = \bar{y}\exp\left(\frac{P-p}{P+p}\right)$ vs. $\hat{\bar{Y}}_p$ $\left(\text{or } \hat{\bar{Y}}_s^*\right)$

$$MSE(\hat{\bar{Y}}_5) - M = f_n\bar{Y}^2\left[\left(\frac{C_P}{2} - \rho_{YP}C_Y\right)^2 + (R^2_{Y.XP} - \rho^2_{YP})C_Y^2\right] \geq 0 \tag{4.5}$$

5. EMPIRICAL STUDY

Consider the data from the data source: **Advance Data from Vital and Health Statistics, Number 347, October 7, 2004 (CDC)** dealing the height of the people of different age group of the United States.

$N = 36,\ n = 12,\ \bar{Y} = 140.18,\ \bar{X} = 39.63,\ P = 0.50, C_Y^2 = 0.036731,\ C_X^2 = 0.232649,$
$C_P^2 = 1.028196, \rho_{YX}^2 = 0.946729,\ \rho_{YP}^2 = 0.0049,\ \rho_{XP}^2 = 0.005329,\ R_{Y.XP}^2 = 0.94673$

Table 5.1: PRE of various estimators with respect to sample mean

Estimator	PRE
$\hat{\bar{Y}}_1$	100
$\hat{\bar{Y}}_2$	41.05
$\hat{\bar{Y}}_3$	5.43
$\hat{\bar{Y}}_4$	742.39
$\hat{\bar{Y}}_5$	13.11
$\hat{\bar{Y}}_s^*$ $\left(\text{or } \hat{\bar{Y}}_p\right)$	1877.23

6. CONCLUSION

The comparative study of the proposed Jack-Knife estimator establishes its superiority in the sense of unbiased and minimum mean square of error over sample mean, ratio estimator and exponential ratio estimator using auxiliary variable and auxiliary attribute under the optimum conditions.

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