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Compressed Sensing and Applications by using Dictionaries in Image Processing

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Abstract

Compressed Sensing is a rapidly growing field which is based on sparse and redundant representations. It becomes a centre of attraction for research among the people working in the applied mathematics, statistics and various engineering fields. For the representation of the compressible signal, the compressed sensing or compressive sensing theory is useful with the Nyquist rate. The research work on the compressive sensing theory with its new strategies or facts, algorithms, and its applications which are used in Image processing with the efficiently better results than the existing ones is the new trend of mathematical applications. Also the other way to deal with this type of problems is to use the concept of compression by which the storage capacity, calculations and time are more used than the concept of compressed sensing in image processing. So to overcome this limitations, we have to move ahead to understand and derive another technique which takes less timing, less storage capacity, less calculations than Compression technique. This can be done by Compressed Sensing technique. But how it is better than the other techniques or methods to get the required or approximate result of signal is discussed in this research work.

Keywords: Compressed sensing, Image processing, Dictionary learning

1. INTRODUCTION AND PROBLEM STATEMENT

Image processing is the processing of any given image or video (as input) with the help of mathematical operations or transformations to produce the image itself or characteristics of the image (as output). Various reconstruction algorithms are already surveyed with their results for compressed sensing field. These algorithms can be modified for optimum results and will be utilized for various image processing applications. Based on complexity and calculations, from Greedy Iterative Algorithm the Orthogonal Matching Pursuit (OMP) algorithm is more preferable than other. Based on some conditions it takes an atom (column) of the measurement dictionary at the each iteration. By the least square or projecting the original function in the each iteration the column that is mostly correlated with the residual is identified and least square error is minimized.

As the mathematical point of view existing algorithms with some mathematical operations or transformations can be changed by other mathematical transformations/operations/terms are useful to get betterment in the result. Also one can identify the complications of the output by changing the mathematical forms. Implementation of reconstruction algorithms can also be programmed by MATLAB toolbox and their comparison ca be carried out based on necessary conditions (parameters).

The application of Compressed Sensing theory with mathematical forms on image/video in real life like low resolution to high resolution of image [1] or high resolution to low resolution of image and from the sketch to photo is the main problem which needs to be focused. By which kinds of mathematical operations this task will give better result needs to be found. The changes in the mathematical forms are affected or not and if yes then how much it affects on the results of the problems is the big question of the work.

There are two ways to represent the signals. One of it, is Compression or Traditional Sampling and another is Compressed Sensing or Compressive Sensing or it is called as Sparse Sampling or Compressive Sampling. Sparsity means the number of nonzero entries in the given any vector, which provides the less time consuming, less calculations and less storage capacity to gain a reconstruction of the signal. Compressed Sensing theory has many strategies, algorithms, applications to help us out with the current scenario like feature extraction, denoising, restoration etc.

Compressed Sensing is a swiftly emerging area among various fields like mathematics, signal and image processing, information theory, communication engineering, biomedical etc., which has enticed a lot of attention recently [2], [3]. The motivation behind compressed sensing is to perform the task of Sampling and Compression at the same time. Compressed Sensing has a wide range of applications in various fields that include error correction, imaging, radar and many more. Given a sparse signal in a high dimensional space, one wishes to reconstruct that signal accurately and efficiently from a much less number of linear measurements than its actual dimension. Reconstruction and acquiring of the signal/ image with the help of primary levels' data and current scenario for this is based on Nyquist rate and compression of the image. But here we are emphasize on the acquiring and reconstruction of the signal/image at a rate less than the Nyquist rate and with less calculation than the compression. According to Nyquist rate [2], [3], [4] also called as sampling theorem, the sampling rate means the number of samples per second must be at least twice of the highest analog frequency to reconstruct the signal from digital (discrete) to analog (continuous). Compressed Sensing provides the representation of the signal at a rate less than the Nyquist rate it means it provides the relation between sparsity and sampling. The acquisition of the signal is based on Nyquist- Shanon sampling theorem [5], [6], which depends on the collecting the discrete data (samples) from the continuous signal. The question arise when this theorem can be applied? The answer is that when the signal should be the band limited with its highest frequency f(max) and samples are collected by sampling the signal with twice of it, i.e. 2f(max). Continuous signal is recovered without any loss by using this sampling procedure. By this sampling procedure more storage capacity, more time, more calculations are used due to its all samples are covered approximately by 2f(max). But we all need to approach that procedure in which everything should be less used to recover the signal without any loss.

Due to this reason there is a need to introduce another technique, which is called Compressed Sensing. In Compressed Sensing the sparsity is applied in which the number of samples of signal are 'n' and after compression and sampling it turns to 'k' samples where $k << n$ and other (n - k) samples are omitted. It means in Compressed Sensing the compression of signal is done first and then it sampled to collect the required samples only. So we are emphasized on the acquiring and reconstruction of the signal at a rate less than the Nyquist rate without any loss and getting the better result. So question is can we get the reasonable result by using the sparsity?

Moreover by application of compressed sensing this task is possible in lower calculations and lower burdens and getting the approximately near result (reconstruction) of the signal/ image. It is the interesting topic of research to search the mathematical relation with the technical point of view with the help of compressed sensing and its applications. Instead of working in the conventional way, compressed sensing guarantees to recover the high dimensional signals exactly or accurately, by using very less number of non adaptive linear samplings or measurements than conventional sampling method [7]. In general form, signals in this context are represented by vectors from linear spaces, many of which in the applications will represent images or other objects. The paramount theorem of linear algebra, as many equations as unknowns, mention that it is not possible to reconstruct a unique signal from an incomplete set of measurements. Depends on the type of signal, a number of representation techniques can used to provide sparse approximations. There are many technical terms and figures of signals are required, but how the mathematics works with them that is the important and interesting part of the proposed research work.

2. LITERATURE REVIEW

The field of Compressed Sensing has existed for around from last four decades. It was first used in Seismology in 1970 when Claerbout and Muir gave attractive alternative of Least Square Solutions [8], Kashin [9] and Gluskin [10] gave norms for random matrices. In mid-eighties, Santosa and Symes [11] suggested l1-norm to recover sparse spike trains. In 1990s, Rudin, Osher and Fatemi [12] used total variation minimization in image processing which is very close to minimization.

The idea of Compressed Sensing got a new beginning in 2004 when David Donoho, Emmanuel Candes, Justin Romberg and Terence Tao presented important results regarding the mathematical foundation of compressive sensing. Lots of papers have published in recent years and the field is witnessing momentous growth almost on a regular basis. Applications of compressed sensing like convert the image in different style domain(lower to higher or higher to lower resolution) and sketch form to the required photo form are most attractive and interesting applications in the current scenario. From the M. Elad and M.Aharon[16], and from J. Yang, J. Wright, T. Huang, Y. Ma.[17] any one can find the better conclusion for the dictionaries and the resolutions of the image.

3. PROPOSED METHOD

This section surveys the theory of Compressed Sensing (CS) also known as Compressive Sensing or Compressed Sampling, an innovative sensing/sampling mechanism in detail. CS theory claims that one can recover certain signals and images from far fewer samples or measurements that conventional sampling methods requires [13]. For this to take place, CS depends mainly on two basic principles: Sparsity, which refers to the properties of natural signals of interest, and Incoherence, which involves how information signal is sensed/ sampled.

The information rate of a continuous time signal may be much smaller than that recommended by its bandwidth. This is the main principle used to express the notion of sparsity. Similarly, in discrete time signals, the number of degrees of freedom of the signal is comparably much smaller than its length can also be stated. General natural signals are sparse or compressible and when expressed in an appropriate basis form have condensed representations. This is the principle which exploited by Compressed Sensing.

Depends on the signal many reconstruction algorithms are there but from all of them Orthogonal Matching Pursuits [14] of Greedy Iterative method is more suitable for the reconstruction of the image. This is an iterative process in which the solution is calculated after every iteration by calculating the atom/column vector in measurement matrix which mostly correlated to a residual vector. Initially this residual is same as the required approximated vector than in each iteration it is adjusted to take into account the vector previously chosen. OMP is more suitable algorithm for reconstruction of image/signal than other algorithms because it removes the selected atom/ column vector from the residual vector at each iteration.

The application of Compressed Sensing like high resolution of image to low resolution of the image or vice versa and from the sketch of any object find the required image of that object, are most useful in our real life. So this type of applications can be solved with the help of mathematics which is the center of the attraction to the all mathematicians. To solve this type of problems we need here the multiple regression from linear algebra in terms of optimization with some dictionaries (means collection or combination of the mathematical transformations).

4. DISCUSSION & CONCLUSION

There are some facts, algorithms, applications of compressed sensing theory based on some primary data and also their results are known but how they work better than before by other methods, are emphasized in this research work. Also how its applications with mathematical terms in real world are helpful is discussed. Always there are some limitations and advantages of any method in any problem are occurred, so what are those and how to overcome that particular problem need to be focused. Also it is emphasized that the error correction for the betterment of the result is possible by using the mathematical methods in compressed sensing.

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