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Comparative Study of Various Algorithms dealing with Computational Aspects of One-Dimensional Cutting Stock Problem

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Abstract

In this paper, we compare the computational results of cutting pattern of Category II in One Dimensional Cutting Stock Problem in view of first order sustainable trim introduced by Powar et al, with the cutting pattern developed by Gradisar. The comparison has been carried over on the data taken by Gradisar by using the upper limit of the stock length as mentioned in the constraint and observed that the results of trim loss was less on Category II and further it was observed that the number of the stock length consumed is also less in Category II.

Keywords: Cutting stock problem, sustainable trim, leftovers, CUT, LCUT

1. INTRODUCTION

1.1 General

The cutting stock problem (CSP) is a process of producing small items (order lengths) from available large items (stock length) by minimizing the trim loss. It is a business problem that arises in many industries such as textile, leather, paper, wood, glass etc. In all these industries cutting of shapes from a piece of stock material is a recurring activity that require proper scheduling so that the overall material needed by all cuts is minimized, ie., minimum trim loss. Cutting stock problem was solved initially using integer linear programming but this generated large numbers of cutting patterns, resulted in large number of combinations which made the problem complex. Various exact or heuristic methods based on item oriented Gradisar et al. (1997), Gradisar et al. (1999a), pattern oriented Gau and Washer (1995) or mixed approach Gradisar et al. (1999b) have been developed to solve the cutting stock problem.

1.2 Review of Literature

The sequential heuristic procedure for optimization of roll cutting in the clothing industry was examined by Gradisar et al. (1997). They defined the issue of roll cutting as a bi-criterial multidimensional knapsack problem with side constraints and proposed lexicographic approach to handle the bi-criterial objective function.

Cherri et al. (2013) considered a one-dimensional cutting stock problem in which the stock not used in the cutting patterns, if large enough, is kept for use in the future. They also assumed that leftovers should not remain in stock for a long time; hence, such leftovers have priority-in-use compared to standard objects (objects bought by the industry) in stock. Further, they proposed a heuristic method for the problem, and also its performance was analyzed by solving randomly generated dynamic instances under consideration.

Wascher et al. (2007) introduced typology in cutting and packing which was implemented over the basic typology of Cutting and Packing problems introduced by Dyckhoff (1990). He introduced these new categorization criteria, which define problem categories different from those of Dyckhoff.

Powar et al. (2013) introduced the concept of computing total trim by using the idea of pre-defined sustainable trim. On the given data they computed corresponding to two different sustainable trims of order one and two viz. t_s^1 and t_s^2 respectively. Then they introduced various values of sustainable trims as knots between t_s^1 and t_s^2 and computed the total trim to each knot, then constructed the linear approximation which predict the total trim loss at any point say T which lies between t_s^1 and t_s^2.

Nozarian et al. (2013) introduced an approach of trim-loss concentration as a solution to trim-loss problem based on simulated annealing algorithm using a new kind of virtual cost. They also studied the Trim-loss concentration problem and credibility of the virtual cost theory and introduced a solution based on Imperialist Competitive Algorithm (ICA) that reduces the wastage as well as the use of minimum number of stocks to be used.

Gradisar et al. (1997) examined a hybrid approach for optimizing one-dimensional stock cutting. They combined two methods viz. the *item-oriented* sequential heuristic procedure *and the pattern-oriented* LP-based methods with the purpose to cut order lengths in exactly required number of pieces and to accumulate consecutive remaining lengths that could be used later.

Nuno Braga et al. (2016) explored an exact and compact assignment formulation for the combined cutting stock and scheduling which can be applied to instances with any level of demand per item by using knapsack-based inequalities by dual-feasible functions.

Arbib et al. (2016) addressed a one-dimensional cutting stock problem in view of cutting patterns sequenced so that no more than *s* different part types are in production at any time and proposed a new integer linear programming formulation whose constraints grow quadratically with the number of distinct part types and whose linear relaxation can be solved by a standard column generation procedure.

1.3 Motivation and contribution

The motivation for the study of the comparative study of two algorithms is the observation in advancements of algorithm been designed by Gradisar and its application in One-dimensional Cutting Stock problem. The approach to application of low order length to stock length has motivated to study this algorithm with large constraints been used in the utility of the stock length which resulted in trim loss of 11.98% (LCUT) Powar et al. (2013.

The algorithm of Gradisar et al. (2011) has introduced the model of calculating the first order sustainable trim loss, i.e., to check the feasibility of the trim loss that an industry can accept as a scrap.

In the paper the authors studied the algorithm developed by Powar et al. (2013) and Gradisar et al. (2011). The study was made on the algorithm developed by Powar et al. (2013) and the data, constraints taken by the Gradisar et al. (2011). The algorithm developed by Powar et al. (2013) was implemented on the data taken by Gradisar with upper limit of stock length as per the given be less than equal to 99, and it was noticed that the results of trim loss obtained was less on constraint i.e. the number of stock length should Category II Powar et al. (2013) and also it was observed that the number of stock length consumed is also less to cut the desired order lengths.

It has been noticed that in general for any arbitrary data, the trim loss computed by our algorithm will not exceed the trim loss obtained by using Gradisar’s Algorithm.

1.4 Structure of the paper

The paper is organized in the following manner. Initially the algorithm of Gradisar and Powar has been discussed, it is been supported by a flowchart Then the analysis of data taken by Gradisar, but the stock used is taken into consideration of the constraints as defined by Gradisar. Then the sustainable trim loss is calculated based on the given stock length by the Gradisar et al. (2011). This is followed by the analysis of the cutting plan with the final output of trim loss and the maximum number of stock length used. Finally a comparison of the result of Category II and LCUT and CUT is depicted in the form of a table. It is then concluded focusing on the detailed outcome on the work of Gradisar and the present work.

2. NOTATIONS AND PRELIMINARIES

In this section we shall be using the following notation for the algorithm designed by Gradisar and Powar.

Used by Gradisar

o_i - order lengths: $i = 1, \ldots, m$ (sorted in descending order: $(o_1 \geq o_2 \geq \quad o_3 \ldots)$,

d_i - the demand of each order length o_i

LS_k - length of stock; $k = 1, \ldots, p$,

NSL_k - number of pieces of stock length L_k

Used by Powar's

$BLK(n)$ − Block of integers $0,1, \ldots, n$ (index set), $j \in BLK(n)$ means j can be any number from the set $\{0,1,2, \ldots, n\}$

o_i - Order lengths $i = 0,1,2, \ldots, n$ arranged in ascending order with respect to length and $o_0 = 0$ by convention.

rn_i − Required number of pieces of order length $l_i, rn_0 = 0$.

SL_j − Stock lengths $(j = 1,2, \ldots, m)$ arranged in ascending order with respect to length.

3. ALGORITHM DUE TO GRADISAR [6]

Gradisar et al. [6] defined the problem for every customer order, with sufficient stock length of same lengths or different standard lengths. Non-standard stock lengths are also available which the leftovers of the previous orders are. The cutting pattern is the number of pieces being cut from a given stock to satisfy the demand of the order lengths. Therefore the pattern j cut from stock length k has been expressed by a vector

$$x_{1jk}, x_{2jk}, \ldots, x_{mjk} \tag{3.1}$$

that satisfies

$$\sum_{i=1}^{m} o_i \, . \, x_{ijk} \leq L_k \tag{3.2}$$

$$x_{ijk} \geq 0 \text{ and integer} \tag{3.3}$$

x_{ijk} is the number of times the order length o_i that appears in the pattern.
They denoted
fcp_{jk} as frequency of cutting pattern j from stock length k.
cp_k the total number of cutting patterns cut from the stock length k.
The integer programming model formulated as

$$\min \sum_{k=1}^{p} . \sum_{j=1}^{cp_k} fcp_{jk} . LS_k \tag{3.4}$$

(minimize the sum of stock lengths to be cut)
s.t.

$$\sum_{j=1}^{cp_k} fcp_{jk} \leq NSL_k \quad \forall\, k \qquad \text{(stock constraint)} \tag{3.5}$$

$$\sum_{k=1}^{p} \sum_{j=1}^{cp_k} x_{ijk} . \, fcp_{jk} = d_i \quad \forall\, i \quad \text{(demand constraints)} \tag{3.6}$$

$$fcp_{jk} \geq 0 \text{ and integer} \quad \forall\, j, k \tag{3.7}$$

The above function and constraint is a General One-Dimensional CSP.

3.1 The Problem

A sequential heuristic procedure which has iteration of four basic steps is proposed in [6]. At each step a part of demand is satisfied and the process is continued till all the demand is fulfilled. They assumed all unprocessed stock length USL_k of stock length SL_k is initially equal to NSL_k and unprocessed order length uol_i of each order length o_i is initiallyol_i.

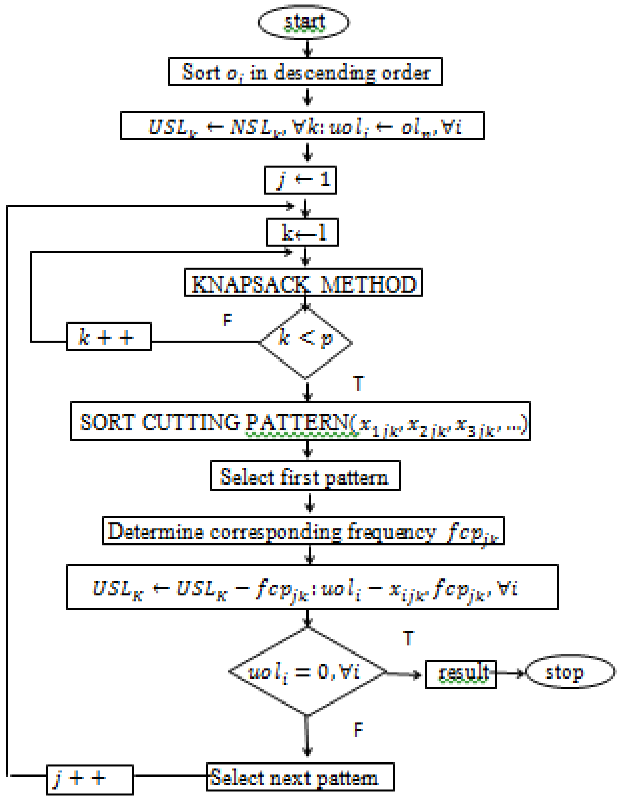

Figure 3.1

The processed stock length and order length is initially empty. After every iteration the unprocessed pieces of stock length reduces and also the number of demand of order length get processed from the stock length when all uol_i equals 0.
Step 1: Solve the problem by taking the unprocessed pieces of order length using the given stock length and generate the cutting patterns to find the optimal solution.

Step 2: The cutting patterns obtained from the Step 1 is sorted in descending order.

Step 3: From the sorted list of patterns the frequency of unprocessed pieces of order lengths and stock lengths are selected. The frequency should not be more than the unprocessed pieces to prevent the overproduction of any order length. After the selection of a particular frequency the corresponding number of unprocessed pieces of stock and order lengths is reduced.

Step 4: If the demand of the orders are not satisfied then repeat step 1, else stop.

The flowchart is depicted in Figure 3.1.

The algorithm has the following assumptions:

i. The optimal solution of the cutting problem is when there is lower ratio between the average stock and order lengths.

ii. It is feasible to find the good solution if the cutting patterns of largest order lengths is processed earlier.

Also they commented that it is necessary to solve a series of knapsack problems in order to obtain cutting patterns and for each of them to then select the corresponding frequency for every iteration.

LCUT was designed for cases with small ratio between stock and order length and can be used for independent solution or as add-in for existing applications.

4. ALGORITHM DUE TO POWAR ET AL. [10]

Authors' observed that in Transmission tower industry for required number of order lengths viz. $rn_i's$ have common integral factor. In view of this observation, the authors have classified the order lengths in the following two categories in accordance with their required number of pieces:

Category I:(C-I) All the order lengths with required number of pieces which have common integral factors greater than 1.

Category II:(C-II) All those order lengths whose required number of pieces have common factor 1.

Remark: Without classifying the order lengths in two different categories, the cutting plan may be executed with only Category II criterion.

In order to cut the linear combination x_{ij} of the two order lengths of o_i and o_i from the given stock lengths $L_1, SL_2, \dots, SL_m$, the authors have to decide up to what extent, the scrape can be allowed from the raw material. Throughout the cutting process, the authors have followed the restriction that $0 \leq SL_k - x_{ij} \leq t_s^1, k = 1,2, \dots, m$ and t_s^1 is the sustainable trim which is defined as follows.

$$U = \frac{o_o rn_0 + o_1 rn_1 + \cdots + o_n rn_n}{rn_0 + rn_1 + \cdots + rn_n} = \frac{\sum_{i=0}^{n} o_i rn_i}{\sum_{j=0}^{n} rn_j}$$

Further

$U_k = |SL_k - iU|$ ($k = 1,2, \dots, m$ and i is an appropriate positive integer ≥ 1, for which U_k is minimum)

where $SL_1, SL_2, \dots, SL_m$ are the stock lengths. Finally t_s^1 is defined as

$$t_s^1 = \frac{\sum_{k=1}^{m} U_k}{m} \quad (4.1)$$

which is the desired sustainable trim.

It has been remarked by the authors that average value covers the acceptable, over all original values, therefore the weighted mean of total required lengths have taken.

The authors have considered two cases to solve the One-Dimensional cutting stock problem, Category-I and Category-II out of which we have analyzed that applying only Category-II the results were found to be plausible. So in this paper we have considered only the algorithm of

Category-II.

Consider order lengths $o_i, o_2, \ldots, o_N$ with the required number of pieces $rn_1, rn_2, \ldots, rn_N$.

For $i \neq j, i, j \in BLK(N)$, define:

$$rn_i = n_i\theta_{i1} + rn_{i1} \tag{4.2}$$

$$rn_j = n_i\xi_{j1} - rn_{j1} \tag{4.3}$$

$0 \leq SLK_k - \left(o_i\,\theta_{i1} + o_j\xi_{j1}\right) = (wst_k\ say) \leq t_s^1$

(for at least one value of k ($k = 1,2\ \ldots, m$))

The number of n_i has been chosen in such a way that wst_k attains a minimum value lying between 0 and t_s^1.

Similarly, choose a number n_2 satisfying the following conditions:

$$rn_{i1} = n_2\ \theta_{i2} + rn_{i2} \tag{4.4}$$

$$rn_{j1} = n_2\ \xi_{j2} + rn_{j2} \tag{4.5}$$

It can be finally defined as

$$rn_{i,s-1} = n_s\theta_{is} + rn_{is} \tag{4.6}$$

$$rn_{j,s-1} = n_s\ \xi_{js} + rn_{js} \tag{4.7}$$

The process would be continued till either $rn_{is} = 0$ or $rn_{js} = 0$ and in view of (4.2) - (4.7), we have

$$rn_i = \sum_{k=1}^{s} n_k\theta_{ik} + rn_{is} \qquad \theta_{ik} > \theta_{i,k+1} \tag{4.8}$$

$$rn_j = \sum_{k=1}^{s} n_k\ \xi_{jk} + rn_{js} \qquad \xi_{jk} > \xi_{j,k+1} \tag{4.9}$$

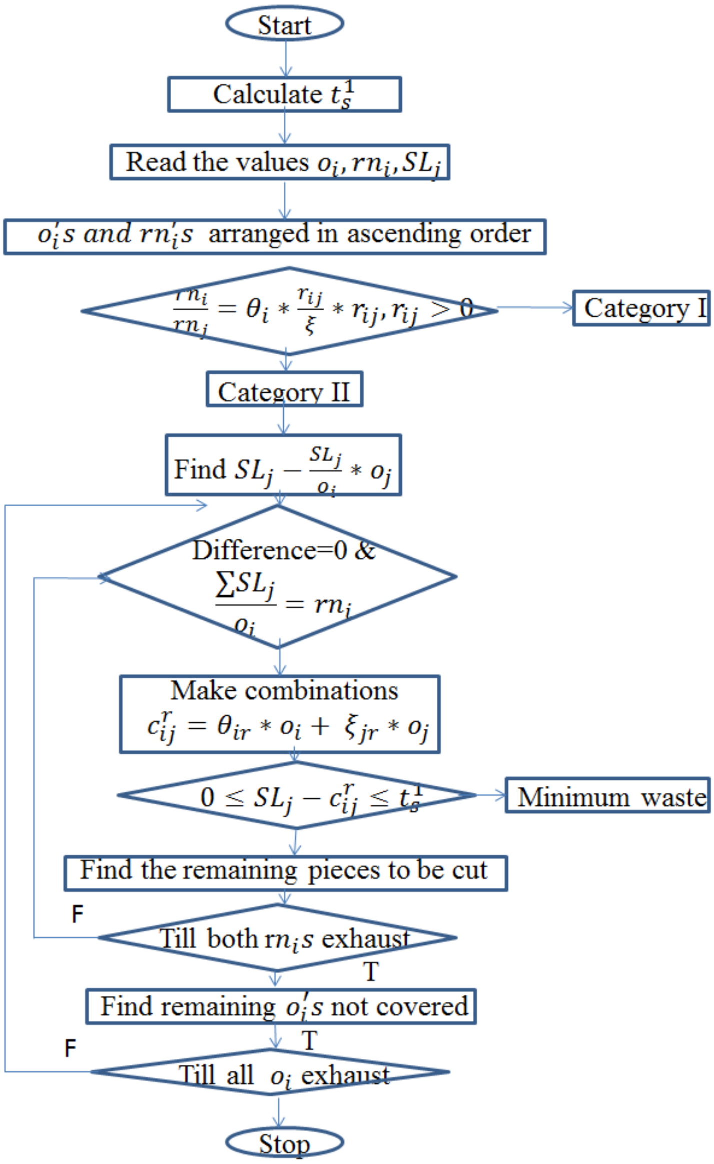

Figure 4.1

$$rn_{us} = n_{s+1}\delta_{u,s+1} + rn_{u,s+1} \quad (rn_{u,s+1} < \delta_{u,s+1}) \tag{4.10}$$

Referring relation (4.8)-(4.10), the authors have defined the set as

$C = \{c_{ij}^r, c_u^{s+2}, c_u^{s+2} \; ; c_{ij}^r = \theta_{ir}o_i + \xi_{jr}o_j, c_u^{s+1} = \delta_{u,s+1} \; o_u, c_u^{s+2} = d_{u,s+1} \, o_u \; where \; c_{ij}^r, c_u^{s+1}, \quad c_u^{s+1} \leq SL_m, u = i \; or \; j, \delta = \theta \; or \; \xi$ according as $u = i \; or \; j$ respectively, $r = 1,2, \dots, s. \quad i, j = 0,1, \dots, N\}$ (4.11)

Define $|e_i| = \max\ e_{ij},\ \ where\ e = a\ or\ b$ for fixed i arbitrary $j\ \ e_{ij} \leq SL_m$ (4.12)

In view of relation (4.11) it is defined as

$$C_k^r = \{\ c_{ij}^r, c_u^{s+1}, c_u^{s+2} : 0\ \leq \left|SL_k - \left(c_{ij}^r \middle| c_u^{s+1} \middle| c_u^{s+2}\right)\right| \leq t_s^1,\ r = 1,2,\dots,s\} \quad (4.13)$$

It has been observed by the authors that if the cutting plan of the order length is taken from largest order length to smallest order length then the smaller order lengths can be adjusted easily amongst the remaining stock lengths thereby resulting in less trim loss. The procedure has been shown in Figure 4.1

5. DATA ANALYSIS DUE TO POWAR ET AL.

In this section, we extract the data used in [6] for our comparative analysis.(cf. Table 5.1, Table 5.2). Throughout our analysis the lengths have been considered in centimeters.

(Data sample supplied by a leading retailer of technical products in South-east Europe, cf.[6])

Table 5.1: Required Order lengths

S.No	Order lengths	Required No. of pieces
1.	965	12
2.	780	7
3.	538	10
4.	430	5

Table 5.2: Available Stocks Lengths

S.No	Stock lengths	Pieces	S.No	Stock lengths	Pieces
1.	1200	12	9.	1400	1
2.	750	10	10.	640	1
3.	685	10	11.	800	1
4.	590	10	12.	765	1
5.	865	4	13.	670	1
6.	1600	2	14.	690	1
7.	600	1	15.	820	1
8.	500	1			

The constraints on the above example are (cf[6]):

- the ratio between the average stock length and average order length should be less and equal to 10
- the number of order lengths should be less than 7
- the number of pieces for each order length should be less than 99
- the number of different stock lengths should be less and equal to 20
- the number of pieces for each stock length should be less than 99

In the above data the order contains four different order lengths with the sum of the required pieces is 34, the total stock given is 57 pieces of different lengths. Standard and non-standard lengths are not treated separately.

Based on the above constraints when the algorithm [10] was applied on the data the following results was obtained. The problem instance of sustainable trim loss is calculated finding ratio between the sum of the product of the individual order lengths and the demand. Therefore using the eq (3.1) we have calculated the sustainable trim loss t_s^1 as : $t_s^1 = 101.01$

Cutting Plan

In the cutting plan, Category II is used in which first we take the order lengths l_i and l_j (say) corresponding to the demands in which the number of pieces have common factor 1. Therefore the order lengths 965 and 430 are cut from stock length 1400, a pair of above mentioned order lengths are catered and the number of stock length used is 5 (constraint of stock length is that it should not exceed 99 in number). In the similar manner the other order lengths are cut from the respective suitable stock lengths as given in Table 3.

Table 5.3: Category II

Sno.	Order Lengths	Pieces to cut	Trim loss	Used Stock Lengths
1.	965 430	1 1	5 x 5=25	1400 x 5=7000
2.	965 538	1 1	97 x 7=679	1600 x 7=11200
3.	965 538	0 1	52 x 3=156	590 x 3=1770
4.	780	1	20 x 7=140	800 x 7=5600
Total			1000	25570
Total Trim Loss(%)			3.91%	

The improved result is obtained (c.f. Table 5.3) with the increase of the stock length with the limit of constraint of stock length [6] which should be less than and equal to 99. The trim loss obtained is 3.91%. The number of stock length used is only 22 numbers in comparison to [6] where the objective function of [6] is to minimize the sum of stock lengths to be cut. It is a general observation that it is difficult to supply various stock lengths by the supplier as mentioned in (c.f. Table 5.2)

Table 5.4: Result of LCUT and CUT due to Gradisar

LCUT				CUT			
Stock Length	Pieces	Pattern	Trim loss	Stock Length	Pieces	Pattern	Trim loss
1200	11	1x965	19.58	1200	12	1x965	19.58
590	10	1x538	8.81	590	9	1x538	8.81
·	2	2x430	0.58	865	2	2x430	0.58
865	1	1x780	9.83	1600	2	2x780	2.50
1600	2	2x780	2.50	500	1	1x430	14.00
1400	1	1x965	.36	1400	1	1x780	5.86
	1	1x430			1	1x538	
800	1	1x780	2.50	800	1	1x780	2.50
820	1	1x780	4.88	820	1	1x780	4.88
Total trim loss/Total Stock used: 3345/27915			11.98%	Total trim loss/Total Stock used: 3590/28160			12.75%

The trim loss calculated by Category II is comparatively less when the stock length is increased relatively and at the same time the number of stock length used is less.

6. COMPARATIVE STUDY

i. In Gradisar's work use of stock length is restricted. According to author the result of LCUT is better only when the ratio between the order length and stock length is low, in the given problem the ratio between the largest stock and the shortest order length is calculated as 3.7.

   The present work is not limited to this constraint, it can be used with any ratio between order length to stock length

ii. It is also observed that in Gradisar's work, trim loss calculated for individual pattern is more than the sustainable trim calculated which was been calculated to decide up to what extent the Industry can allow the raw material to convert into the scrape. Trim loss calculated was 11.98% which is higher for any industry (loss permissibility is 2 – 3%) for smooth running.

The individual pattern trim loss generated in the present work is within the sustainable trim and also the total trim loss calculated is within the restriction of the Industry.

iii. Supply of such a small number of stock length as mentioned in Gradisar's data is not practically relevant for the supplier.

iv. The objective function of LCUT is to minimize the stock length which it does not satisfy when compared with our work. The number of stock quantities used in the present work is 22 where as in LCUT and CUT the number of stock pieces used is 30 and 29 respectively.

By increasing the number of individual stock pieces (within the limit of the constraint specified by Gradisar) in the present work the trim loss obtained is 3.91% which is affordable when compared to LCUT and CUT.

7. CONCLUSION

The paper compares the problem of reducing the trim loss in One-Dimensional Cutting Stock Problem (1DCSP) between Category II, LCUT and CUT. It has been observed by the comparative study that the trim loss calculated by Category II is much less than LCUT and CUT when the number of stock length is increased within the limit of the constraint specified by Gradisar. Also the number of stock length consumed in Category II is less than the stock used by LCUT and CUT.

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