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Slight Adjustment of Dijkstra's Algorithm to Solve Shortest Path Problem in an Interval Graph

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Abstract

In this paper the author introduces the notion of interval graph (I-graph) in Graph Theory, considers the Shortest Path Problem (SPP) in an interval graph. The classical Dijkstra's algorithm to find the shortest path in graphs is not applicable to interval graph. In this work the author proposes a new algorithm called by I-Dijkstra's Algorithm with the philosophy of the classical Dijkstra's Algorithm to solve the SPP in an interval graph.

Keywords: I-graph ; I-Dijkstra's.

MSC Code (2000): 05C85

1. INTRODUCTION

In graph theory, the Shortest Path Problem (SPP) is one of the most famous problems studied and being studied by researchers. The SPP is the problem of finding a path between two vertices (or nodes) in a digraph such that the sum of the weights of its constituent edges is minimized. Thus the core problem is to find the shortest path from a source vertex S to a single destination vertex D in a directed graph and to compute the corresponding min cost.

Shortest Path problems (SPP) are among the fundamental problems studied in Computational Geometry, Graph Algorithms, Geographical Information Systems (GIS), Network Optimization etc. to list a few only out of many. Sometimes the network of a real life communication or transportation system can not be modeled into a graph but into a multigraph [2-13]. In such a case the standard algorithms of graph theory cannot be applied in SPP too. Biswas et. el. [2-13] has done rigorous analysis of SPP in multigraphs. However, in this work we consider those networks which can be modeled into graphs. In this work the authors introduces interval graph or I-graph and solves the SPP in an I-graph by developing a new algorithm in the style of the famous Dijkstra's Algorithm to make it applicable in much wider domains.

2. INTERVAL GRAPH (I-GRAPH)

Interval arithmetic is a method developed by mathematicians since the 1950s as an approach to putting bounds on rounding-errors and measurement-errors in mathematical computation and thus developing numerical methods that yield reliable results. It represents each value as a range of possibilities. For example, instead of estimating the height of someone using standard arithmetic as 1.9 metres, using interval arithmetic we might be certain that that person is somewhere between 1.75 and 2.05 metres. For various operations like addition ( ), subtraction, multiplication, etc and also for ranking operation on interval numbers, one could see any good book (viz. [15], [18], [19]).

Decisions are based on information. To be useful, information must be reliable. In this section we define I-graph. An interval graph or I-graph is basically a generalized concept of crisp graph. Consider the following graph (see Figure 1) where at least one of the weights is an interval number. This type of graph is called a interval-weighted graph or I-graph (in short).

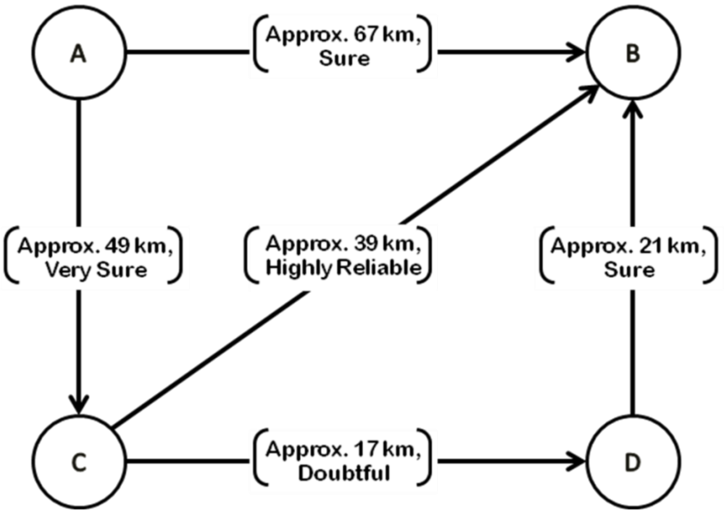

Fig. 1. A transportation network as an interval graph G.

In this I-graph in Figure 1, the edge AB has the i-weight the i-number (approx. 67 km, sure), the edge AC has the i-weight the i-number (Approx. 49 km, very sure), etc.

The i-number weight of an edge is also called the i-distance between the corresponding two nodes.

3. I-Dijkstra's Algorithm for SPP in an I-graph

The Classical Dijkstra's algorithm [16] solves the single-source shortest path problems in a simple graph. In this section we develop a new algorithm called by I-Dijkstra's Algorithm (of the style of the classical famous Dijkstra's algorithm) to solve a SPP in an I-graph.

3.1. *Shortest path estimate of a vertex in a directed I-graph*

Consider a i-weighted directed graph G = (V, E). The i-shortest path estimate d[v] of any vertex v, where vertex v is one of the neighboring vertices of the currently traversed vertex u , is the i-distance between the vertex v and vertex u , added with the shortest i-distance between the starting vertex s and vertex u, where s, u, v ∈ V[G].

∴ d [v] = (shortest i-distance between s and u) $\oplus$ (i-weight of arc between v and u)

This is shown below in Figure 2.

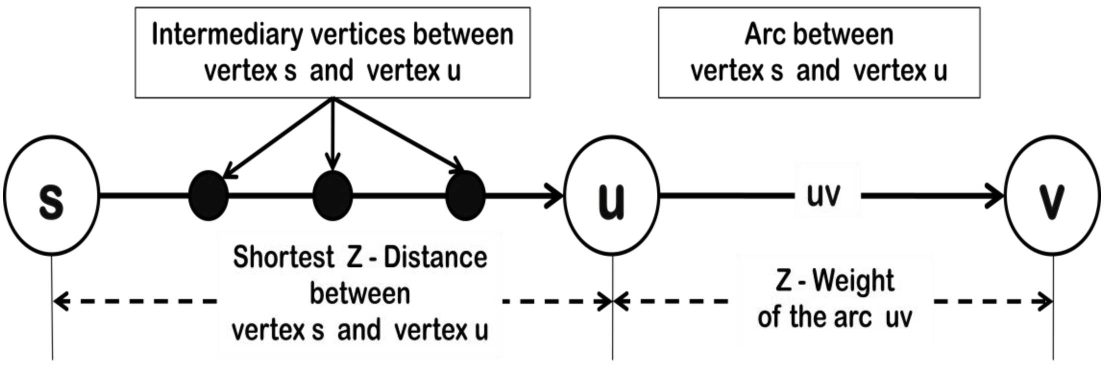

Fig. 2 d[v] in an I-graph.

3.2. *Relaxation of an arc in our proposed I-Dijkstra's Algorithm*

For the relaxation process of an arc to happen, we must first initialize the graph along with its starting vertex s and i-shortest path estimate for each vertices of the graph G.

INITIALIZE-SINGLE-SOURCE(G, s)

1. FOR each vertex v ∈ V[G]

2. d[v] = [∞,∞]

3. v.π = NIL

4. d[s] = 0i

Note : We store <u>all predecessor nodes</u> of u in the attribute u.π. Thus s.π is always Nil, because s is the source node.

Now on the basis of this initialization process, I-Dijkstra's algorithm proceeds further and the process of relaxation of each arc begins. The sub-algorithm RELAX, plays the vital role to update d[v] i.e. the i-shortest distance value between the starting vertex s and the vertex v *(which is neighbor of the current traversed vertex u, ∀ u, v ∈ V[G]).* The RELAX algorithm runs as shown below :

RELAX(u, v, w)

1. IF d[v] > d[u] ⊕ w(u,v)
2. THEN d[v] ← d[u] ⊕ w(u, v)
3. v.π ← u

where, w(u, v) is the Z-weight of the arc from vertex u and vertex v, and v.π denotes the parent node of a vertex v, ∀ u, v ∈ V[G]. This is shown below in the Figure 3.

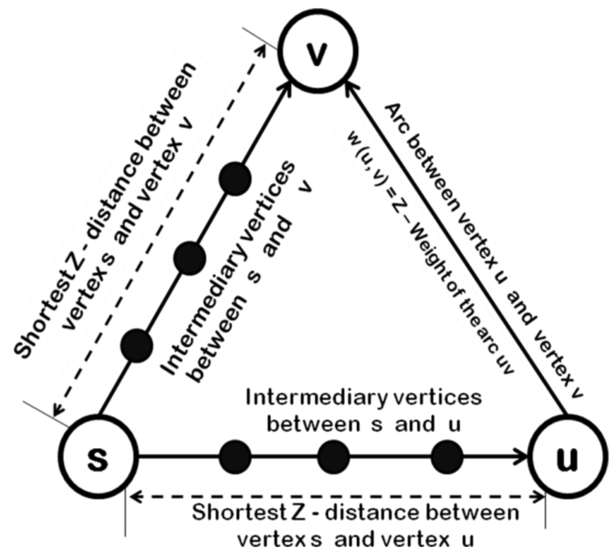

Fig. 3. Diagram showing how RELAX algorithm works.

I-Dijkstra's algorithm solves the single-source shortest-path on a i-weighted directed I-graph G = (V, E) for the case in which all edge i-weights are non-negative. The I-Dijkstra's algorithm maintains a set S of vertices whose final i-shortest path weights from the source vertex s has already been determined.

The algorithm repeatedly selects the vertex $u \in V - S$ with the minimum i-shortest-path estimate, adds u to S, and relaxes all edges leaving u. Our proposed algorithm is as follows:

I-DIJKSTRA (G, w, s)

1. INITIALIZE-SINGLE-SOURCE (G, s)
2. $S \leftarrow \emptyset$
3. $Q \leftarrow V[G]$
4. *WHILE* $Q \neq \emptyset$
5. *DO* $u \leftarrow$ EXTRACT-MIN(Q)
6. $S \leftarrow S \cup \{u\}$
7. *FOR* each vertex $v \in Adj[u]$
8. *DO* RELAX (u, v, w)

4. CONCLUSION

The I-graph is a generalization of crisp graph. There are many real life problems of networks in communication systems, transportation, circuit systems, etc. which cannot be modeled into graphs but into I-graphs only. The classical Dijkstra's algorithm to find the shortest path in graphs is not applicable to I-graphs.

In this paper we have done slight adjustment in the classical famous Dijkstra's algorithm to make it applicable to I-graphs to find the i-shortest path from a source vertex to a destination vertex. The modified algorithm is called as I-Dijkstra's algorithm. The networks where the classical Dijkstra's algorithm fails can now be dealt with the I-Dijkstra's algorithm for performing efficient and optimal communication/transportation in many cases.

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