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# A Concise form of Continuity in Fine Topological Space

**P.L. Powar and Pratibha Dubey**
*Department of Mathematics and Computer Science,*
*R.D.V.V., Jabalpur, India.*

**Abstract**

By using the topology on a space X, a wide class of sets called fine open sets have been studied earlier. In the present paper, it has been noticed and verified that the class of fine open sets contains the entire class of A-sets, AC-sets and $\alpha$AB-sets, which are already defined. Further, this observation leads to define a more general continuous function which in tern reduces to the four continuous functions namely A-continuity, AB-continuity, AC-continuity and $\alpha$AB-continuity as special cases.

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**Keywords:** fine $\alpha$AB-sets, fine A-sets, fine topology.

## 1. Introduction

The concept of fine space has been introduced by Powar and Rajak in [1] is the special case of generalized topological space (see[2]). The major difference between the two spaces is that the open sets in the fine space are not the random collection of subsets of $X$ satisfying certain conditions but the open sets (known as fine sets) have been generated with the help of topology already defined over $X$. It has been studied in [1] that this collection of fine open sets is really a magical class of subsets of $X$ containing semi-open sets, pre-open sets, $\alpha$-open sets, $\beta$-open sets etc.

The idea of C-sets, $\eta$-sets, A-sets, AB-sets, AC-sets, $\alpha$AB-sets has been initiated by Noiri et al. (cf. [3]) and Ekici et al. (cf. [4]). By using the concept of these sets they have introduced (cf. [4]) five generalized concepts of continuities viz. A-continuity, AB-continuity, AC-continuity $\alpha$AB-continuity and $\gamma$-continuity respectively. (see also [5])

We have already established in [6] that the concepts of A-set and AB-set coincide and consequently A-continuity and AB-continuity are equivalent.

Section two covers some preliminaries. In section three, we have explored that the concept of $\gamma$-open set and $\beta$-open set coincide. Consequently, the two different continuities introduced on the basis of these sets are also equivalent. Moreover, we covered some properties of A-set, $\alpha$AB-set and $\gamma$-set etc. We have analytically proved that A-set, AC-set $\alpha$AB-set, $\gamma$-sets are also the members of the collection of fine open sets along with the some results in section four. In the last section, we have defined fine A-continuity, fine AC-continuity, fine $\alpha$AB-continuity, which are the generalized forms of continuities like A-continuity, AB-continuity, AC-continuity $\alpha$AB-continuity defined in [4]. We have also studied the relations amongst these fine continuities in this section.

## 2. Pre-requisites

Throughout this paper, $(X, \tau)$ and $(Y, \sigma)$ denote topological spaces on which no separation axioms are assumed. $F_X$ and $F_Y$ denote collection of closed sets corresponding to the topologies on $X$ and $Y$ respectively. For a subset $A \subseteq X$, the closure and the interior of $A$ are denoted by $cl(A)$ and $int(A)$ respectively. A function $f : X \to Y$ denotes a single valued function of a topological space $(X, \tau)$ into topological space $(Y, \sigma)$. $\tau_f$ denotes the collection of fine-open sets generated by the topology $\tau$ on X and $\sigma_f$ denotes the collection of fine-open sets generated by the topology $\sigma$ on $Y$.

The following definitions and the concepts are required for establishing the assertions of the present paper:

**Definition 2.1. [2]** Let $X$ be a nonempty set and $\boldsymbol{g}$ be a collection of subsets of $X$. Then $\boldsymbol{g}$ is called a **generalized topology** (briefly $GT$) on $X$ if $\phi \in \boldsymbol{g}$ and $G_i \in \boldsymbol{g}$ for $i \in I \neq \phi$ implies $G = \cup_{i \in I} G_i \in \boldsymbol{g}$. We say $\boldsymbol{g}$ is **strong** if $X \in \boldsymbol{g}$; and we call the pair $(X;\ \boldsymbol{g})$ a generalized topological space on X. The elements of $\boldsymbol{g}$ are called **$\boldsymbol{g}$-open sets** and their complements are called **$\boldsymbol{g}$-closed sets**.

**Example 2.1.** Let $X = \{a, b, c\}$ then, we may define following generalized topologies on $X$.
$\boldsymbol{g_1} = \{\phi, \{a\}\}$
$\boldsymbol{g_2} = \{\phi, \{a\}, \{b\}, \{a, b\}\}$.
$\boldsymbol{g_3} = \{\phi, X, \{a, c\}, \{a, b\}\}$.

In the Example 2.1 $\boldsymbol{g_1}$, $\boldsymbol{g_2}$ are the generalized topologies but not strong generalized topologies (cf. Definition 2.1) whereas $\boldsymbol{g_3}$ is not a topology but it is a strong generalized topology.

**Definition 2.2. [1]** Let $(X, \tau)$ be a topological space we define, $\tau(A_\alpha) = \tau_\alpha = \{G_\alpha(\neq X) : G_\alpha \cap A_\alpha \neq \phi$, for $A_\alpha \in \tau$ and $A_\alpha \neq X, \phi$ for some $\alpha \in J$, where $J$ is the index set$\}$. Now, define $\tau_f = \{\phi, X\} \cup \{\tau_\alpha\}$. The above collection $\tau_f$ of subsets of $X$ is called the **fine collection** of subsets of $X$ and $(X, \tau, \tau_f)$ is said to be the **fine space** $X$ and generated by the topology $\tau$ on $X$.

**Example 2.2.** Consider a topological space $X = \{a, b, c\}$ with the topology $\tau =$

$\{X, \phi, \{a\}\} \cong \{X, \phi, A_\alpha\}$ where $A_\alpha = \{a\}$. In view of Definition 2.2 we have, $\tau_\alpha = \tau(A_\alpha) = \tau\{a\} = \{\{a\}, \{a, b\}, \{a, c\}\}$ then the **fine collection** is $\tau_f = \{\phi, X\} \cup \{\tau_\alpha\} = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$.

We quote some important properties of fine topological spaces.

**Lemma 2.1. [1]** Let $(X, \tau, \tau_f)$ be a fine space then arbitrary union of fine open set in $X$ is fine-open in $X$.

**Lemma 2.2. [1]** The intersection of two fine-open sets need not be a fine-open set as the following example shows.

**Example 2.3.** Let $X = \{a, b, c\}$ be a topological space with the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$, $\tau_f = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. It is easy to see that, the above collection $\tau_f$ is not a topology. Since, $\{a, c\} \cap \{b, c\} = \{c\} \notin \tau_f$. Hence, the collection of fine open sets in a space $X$ does not form a topology on $X$, but it is a generalized topology on $X$.

**Remark 2.1.** In view of Definition 2.1 of generalized topological space and above Lemmas 2.1 and 2.2 it is apparent that $(X, \tau, \tau_f)$ is a special case of generalized topological space. It may be noted specifically that the topological space plays a key role while defining the fine space as it is based on the topology of $X$ but there is no topology in the back of generalized topological space.

**Definition 2.3. [1]** A subset $U$ of a fine space $X$ is said to be a **fine-open set** of $X$, if $U$ belongs to the collection $\tau_f$ and the complement of every fine open set of $X$ is called the **fine-closed** set of $X$ and we denote the collection by $\mathbf{F_f}$.

**Remark 2.2. [1]** The family of all $\alpha$-open sets respectively ($\beta$-open sets, pre-open sets, semi-open sets) is denoted by $(\alpha O(X), \beta O(X), PO(X), SO(X))$. Moreover, $\tau \subseteq \alpha O(X) \subseteq SO(X)) \subseteq \beta O(X) \subseteq \tau_f$. $\tau \subseteq \alpha O(X) \subseteq PO(X)) \subseteq \beta O(X) \subseteq \tau_f$.

**Definition 2.4. [1]** Let A be a subset of a fine space $X$ the **fine-interior** of $A$, is defined as the union of all fine-open sets contained in the set $A$ ie. the largest fine-open set contained in the set $A$ and it is denoted by $\boldsymbol{f_{int}}$.

**Example 2.4.** Let $X = \{a, b, c\}$ be a topological space with the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ then the fine collection is $\tau_f = \{X, \phi, \{a\}, \{b\}, \{a, sb\}, \{b, c\}, \{a, c\}\}$. We can see that, $int\{a, c\} = \{a\}$ and $f_{int}\{a, c\} = \{a, c\}$.

**Definition 2.5. [1]** Let A be a subset of a fine space $X$ the **fine-closure** of $A$, is defined as the intersection of all fine-closed sets containing the set $A$ and it is denoted by $\boldsymbol{f_{cl}}$.

**Example 2.5.** Let $X = \{a, b, c\}$ be a topological space with the topology $\tau = \{X, \phi, \{a\}\}$, $F_X = \{X, \phi, \{b, c\}\}$ then the fine collection is $\tau_f = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$, $F_f = \{\phi, X, \{b, c\}, \{c\}, \{b\}\}$. We can see that, $cl\{b\} = \{b, c\}$ and $f_{cl}\{b\} = \{b\}$.

**Definition 2.6.** A subset S of a space $(X, \tau)$ is called

- **Pre-open** [2] if $S \subseteq int(cl(S))$.
- **Semi-open** [7] if $S \subseteq cl(int(S))$.
- $\boldsymbol{\alpha}$ **-open** [8] if $S \subseteq int(cl(int(S)))$.
- $\boldsymbol{\beta}$**-open** [9] if $S \subseteq cl(int(cl(S))$.
- **Regular-open** [2] if $S = int(cl(S))$.

The complement of pre-open set (semi-open set, $\alpha$-open set, $\beta$-open set and regular-open set) is called pre-closed set, (semi-closed set, $\alpha$-closed set, $\beta$-closed set and regular-closed set) respectively. The collection of $\alpha$-open sets (semi-open sets, pre-open sets, $\beta$-open sets and regular-open sets) denoted by $\alpha O(X)$, $(SO(X)$, $PO(X)$, $\beta O(X)$, and $RO(X))$ respectively.

**Remark 2.3. [1]** The following inclusions follow directly from the definitions:

- $\tau \subseteq \alpha O(X) \subseteq SO(X) \subseteq \beta O(X)$.
- $\tau \subseteq \alpha O(X) \subseteq PO(X) \subseteq \beta O(X)$.

**Definition 2.7. [4]** A subset $A$ of a space $(X, \tau)$ is said to be $\boldsymbol{\gamma}$**-open** or **b-open** if $A \subseteq int(cl(A)) \cup cl(int(A))$. The collection of $\boldsymbol{\gamma}$**-open** sets of $X$ is denoted by $\boldsymbol{\gamma O(X)}$.

**Example 2.6.** Let $X = \{a, b, c\}$ be a topological space with the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$, $F_X = \{\phi, X, \{b, c\}, \{a, c\}, \{c\}\}$. Taking $A = \{a, c\}$ a subset of $X$. Then, $cl\{a, c\} = \{a, c\}$, $int(cl\{a, c\}) = int\{a, c\} = \{a\}$, and again consider $int\{a, c\} = \{a\}$, $cl(int\{a, c\}) = cl\{a\} = \{a, c\}$. Therefore, we have $\{a, c\} \subseteq int(cl\{a, c\}) \cup cl(int\{a, c\})$ $\Rightarrow \{a, c\} \subseteq \{a\} \cup \{a, c\} = \{a, c\}$. Hence, $A$ is a $\boldsymbol{\gamma}$**-open set**. The complement of $\{a, c\}$ is $\{b\}$ which is $\boldsymbol{\gamma}$**-closed set**.

**Definition 2.8. [4]** A subset $A$ of a space $(X, \tau)$ is said to be $\boldsymbol{\gamma}$**-clopen** if it is both $\gamma$-open and $\gamma$-closed. The collection of $\gamma$-clopen sets is denoted by $\boldsymbol{\gamma co(X)}$.

**Example 2.7.** Consider a topological space $X = \{a, b, c\}$ with the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$, $F_X = \{\phi, X, \{b, c\}, \{a, c\}, \{c\}\}$. Then, $\gamma O(X) = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. The collection of $\gamma$-closed sets of $X$ is $\{\phi, X, \{b, c\}, \{a, c\}, \{c\}, \{b\}, \{a\}\}$. Therefore, the collection of $\gamma$-clopen sets of $X$ is $\gamma co(X) = \{X, \phi, \{a\}, \{b\}, \{a, c\}, \{b, c\}\}$.

**Definition 2.9. [4]** A subset $A$ of a space $(X, \tau)$ is said to be **semi-regular** if $A$ is both semi-open and semi-closed. The collection of all semi-regular sets in $X$ is denoted by $\boldsymbol{Sr(X)}$.

**Example 2.8.** Consider a topological space $X = \{a, b, c\}$ with the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $F_X = \{\phi, X, \{b, c\}, \{a, c\}, \{c\}\}$. Let $A = \{b, c\}$ be a subset of $X$. Then, $int\{b, c\} = \{b\}$, $cl(int\{b, c\}) = cl\{b\} = \{b, c\}$. Therefore, we have $\{b, c\} \subseteq cl(int\{b, c\}) = \{b, c\}$.

Hence, $A$ is a semi-open set. In order to establish that $A$ is semi-closed, it is enough if we show that complement of $A = B(\text{say}) = \{a\}$ is semi-open. $int\{a\} = \{a\}$. $cl(int\{a\}) = cl\{a\} = \{a, c\}$. Therefore, we have $\{a\} \subseteq cl(int\{a\}) = \{a, c\}$. Hence, $B = \{a\}$ is a semi-open set. Therefore, $A$ is a **semi-regular set**.

**Remark 2.4.** Collection of semi-regular sets of $X$ does not form a topology on $X$.

In support of Remark 2.4 we elaborate the following example.

**Example 2.9.** Consider a topological space $X = \{a, b, c\}$ with the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$, $F_X = \{\phi, X, \{b, c\}, \{a, c\}, \{c\}\}$. It is easy to see that the collection of semi-regular sets of $X$ is $Sr(X) = \{X, \phi, \{a\}, \{b\}, \{a, c\}, \{b, c\}\}$. It may be noted that the collection of $Sr(X)$ is neither a topology nor the generalized topology on $X$.

**Definition 2.10.** A subset $H$ of a space $(X, \tau)$ is called

- A **A-set** [4] if $H \in \boldsymbol{A(X)} = \{A \cap B : A \in \tau,\ B = cl(int(B))\}$.
- A **αAB-set** [4] if $H \in \boldsymbol{\alpha AB(X)} = \{A \cap B : A$ is $\alpha$-open, $B$ is semi-regular$\}$.
- A **AC-set**[4] if $H \in \mathbf{AC(X)} = \{A \cap B : A \in \tau,\ B$ is $\gamma$-clopen$\}$.

**Example 2.10.** Consider a topological space $X = \{a, b, c\}$ with the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$, $F_X = \{\phi, X, \{b, c\}, \{a, c\}, \{c\}\}$. We now show that $H = \{a, c\} \subseteq X$ is an A-set.
**Claim.** $H = \{A \cap B : A \in \tau,\ B = cl(int(B))\}$. We now consider $A = X \in \tau$ and $B = \{a, c\}$.
**Claim.** $B = cl(int(B))$. It may be checked easily that $cl(int(B)) = B$.
Thus, $H = A \cap B = X \cap \{a, c\} = \{a, c\} \in$ **A-set**.

**Example 2.11.** Consider a topological space $X = \{a, b, c, d\}$ with the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$, $F_X = \{\phi, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}\}$. We now show that $H = \{b, c\} \subseteq X$ is an $\alpha$AB-set.
**Claim.** $H = \{A \cap B : A$ is $\alpha$-open, $B$ is semi-regular$\}$.
It is easy to see that $\alpha$-open set and semi-regular set of $X$ are $\alpha O(X) = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ and $Sr(X) = \{X, \phi, \{a\}, \{b\}, \{a, c\}, \{b, d\}, \{b, c\}, \{a, d\}, \{a, c, d\}, \{b, c, d\}\}$ respectively.

We now Consider $A = \{a, b, c\} \in \alpha O(X)$ and $B = \{b, c, d\} \in Sr(X)$. Thus, $H = A \cap B = \{a, b, c\} \cap \{b, c, d\} = \{b, c\} \in \boldsymbol{\alpha AB(X)}$ or **αAB-set**.

**Example 2.12.** Consider a topological space $X = \{a, b, c\}$ with the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$, $F_X = \{\phi, X, \{b, c\}, \{a, c\}, \{c\}\}$. We now show that $H = \{a, c\} \subseteq$

$X$ is an AC-set.
**Claim.** $H = \{A \cap B : A \in \tau$ and $B$ is $\gamma$-clopen$\}$. We now consider $A = X \in \tau$ and $B = \{a, c\}$.
**Claim.** $B$ is $\gamma$-clopen. It may be checked easily that $B = \{a, c\}$ is $\gamma$-clopen. Then, $A \cap B = X \cap \{a, c\} = \{a, c\} \in$ **AC(X)** or **AC-set**.

Definition 2.11. A subset S of a space $(X, \tau)$ is called

- **Fine pre-open**[1] if $S \subseteq f_{int}(f_{cl}(S))$.
- **Fine semi-open**[1] if $S \subseteq f_{cl}(f_{int}(S))$.
- **Fine $\alpha$-open**[1] if $S \subseteq f_{int}(f_{cl}(f_{int}(S)))$.
- **Fine $\beta$-open**[1] if $S \subseteq f_{cl}(f_{int}(f_{cl}(S)))$.
- **Fine regular-open**[1] if $S = f_{int}(f_{cl}(S))$.

The complement of fine pre-open set (fine semi-open set, fine $\alpha$-open set, fine $\beta$-open set and fine regular-open set and) is called fine pre-closed set, (fine semi-closed set, fine $\alpha$-closed set, fine $\beta$-closed set, fine regular-closed set) respectively. The collection of fine $\alpha$-open sets (fine semi-open sets, fine pre-open sets, fine $\beta$-open sets and fine regular-open sets) denoted by $f\alpha(X)$, $(fS(X)$, $fP(X)$, $f\beta(X)$ and $fR(X))$ respectively. For the examples of above stated sets, please refer [1].

**Definition 2.12. [1]** A function $f : (X, \tau, \tau_f) \to (Y, \sigma, \tau_f)$ is called **fine-continuous** if $f^{-1}(V)$ is open in $X$ for every fine-open set $V$ of $Y$.

**Example 2.13.** Consider a topological spaces $X = \{a, b, c\}$ and $Y = \{1, 2, 3\}$ with their corresponding topologies $\tau = \{X, \phi, \{a\}, \{b, c\}\}$, $\sigma = \{Y, \phi, \{1, 3\}\}$ respectively. In view of Definition 2.2 we have, $\sigma_f = \{Y, \phi, \{1\}, \{3\}, \{1, 3\}, \{1, 2\}, \{2, 3\}\}$. Define a function $f : X \to Y$ as

$$f(x) = \begin{cases} 1 & for\ x = a; \\ 2 & for\ x = b, c. \end{cases} \tag{2.1}$$

In view of 2.1 for $G \in \sigma$, we get

$$f^{-1}(G) = \begin{cases} X & for\ G = Y; \\ \phi & for\ G = \phi; \\ \{a\} & for\ G = \{1\}; \\ \phi & for\ G = \{3\}; \\ \{a\} & for\ G = \{1, 3\}; \\ X & for\ G = \{1, 2\}; \\ \{b, c\} & for\ G = \{2, 3\}. \end{cases} \tag{2.2}$$

Hence, we notice that $f^{-1}(G) \in \tau$. Therefore $f$ is **fine-continuous**.

**Definition 2.13.** **[4]** A function $f : (X,\tau) \to (Y,\sigma)$ is called **A-continuous** if $f^{-1}(G) \in A(X)$ for each $G \in \sigma$.

**Example 2.14.** Consider the topological spaces $X = \{a, b, c, d\}$, $Y = \{1, 2, 3, 4\}$ with their corresponding topologies $\tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}, \{a, b\}, \{a, b, c\}\}$, $\sigma = \{Y, \phi, \{2, 3\}, \{1, 2, 4\}, \{2\}\}$ respectively. $A(X) = \{X, \phi, \{b\}, \{c\}, \{b, c\}, \{a, b\}, \{a, b, c\}, \{c, d\}, \{a, b, d\}\}$. Define a function $f : X \to Y$ as

$$f(x) = \begin{cases} 1 & for\ x = a; \\ 2 & for\ x = b; \\ 3 & for\ x = c; \\ 4 & for\ x = d. \end{cases} \tag{2.3}$$

In view of 2.3 for $G \in \sigma$, we get

$$f^{-1}(G) = \begin{cases} X & for\ G = Y; \\ \phi & for\ G = \phi; \\ \{b\} & for\ G = \{2\}; \\ \{b, c\} & for\ G = \{2, 3\}; \\ \{a, b, d\} & for\ G = \{1, 2, 4\}. \end{cases} \tag{2.4}$$

Hence, we notice that $f^{-1}(G) \in A(X)$ and conclude that $f$ is **A-continuous**, when we appeal to Definition 2.10 but not continuous.

**Definition 2.14.** **[4]** A function $f : (X,\tau) \to (Y,\sigma)$ is called **αAB-continuous** if $f^{-1}(G) \in \alpha$AB(X) for each $G \in \sigma$.

**Example 2.15.** Consider the topological spaces $X = \{a, b, c\}$ and $Y = \{1, 2, 3\}$ with their corresponding topologies $\tau = \{X, \phi, \{a\}\}$, $\sigma = \{Y, \phi, \{1\}, \{1, 2\}\}$ respectively. $\alpha AB(X) = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$. Define a function $f : X \to Y$ as

$$f(x) = \begin{cases} 1 & for\ x = a, b; \\ 2 & for\ x = c. \end{cases} \tag{2.5}$$

In view of 2.5 for $G \in \sigma$, we get

$$f^{-1}(G) = \begin{cases} X & for\ G = Y; \\ \phi & for\ G = \phi; \\ \{a, b\} & for\ G = \{1\}; \\ X & for\ G = \{1, 2\}. \end{cases} \tag{2.6}$$

Hence, we notice that $f^{-1}(G) \in \alpha AB(X)$ and conclude that $f$ is **αAB-continuous**, when we appeal to Definition 2.10 but not continuous.

**Definition 2.15.** **[4]** A function $f : (X,\tau) \to (Y,\sigma)$ is called **AC-continuous** if $f^{-1}(G) \in$ AC(X) for each $G \in \sigma$.

**Example 2.16.** Consider a topological space $X = \{a, b, c, \}$ and $Y = \{1, 2, 3\}$ with their corresponding topologies $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$, $\sigma = \{Y, \phi, \{2\}, \{1, 2\}, \{2, 3\}\}$ respectively. $F_X = \{\phi, X, \{b, c\}, \{a, c\}, \{c\}\}$, $F_Y = \{\phi, Y, \{1, 3\}, \{3\}, \{1\}\}$. In view of Definition 2.10, $AC(X) = \{X, \phi, \{a\}, \{b\}, \{a, c\}, \{b, c\}, \{a, b\}\}$. Define a function $f : X \to Y$ as

$$f(x) = \begin{cases} 1 & for\ x = a; \\ 2 & for\ x = b; \\ 3 & for\ x = c. \end{cases} \tag{2.7}$$

In view of 2.7 for $G \in \sigma$, we get

$$f^{-1}(G) = \begin{cases} X & for\ G = Y; \\ \phi & for\ G = \phi; \\ \{b\} & for\ G = \{2\}; \\ \{a, b\} & for\ G = \{1, 2\}; \\ \{b, c\} & for\ G = \{2, 3\}. \end{cases} \tag{2.8}$$

Hence, we notice that $f^{-1}(G) \in AC(X)$ and conclude that $f$ is **AC-continuous**, when we appeal to Definition 2.10 but not continuous.

## 3. Certain direct implications

In view of the definitions given in section 2, we notice the following consequences:

- Collection of $\gamma$-open subsets of $X$ does not form a topology on $X$.
- Collection of A-set, $\alpha$AB set, AC-set and $\gamma$-open set respectively do not form a topology on $X$.
- In view of Definition 2.10, it is clear that all semi-regular sets are A-sets as well as $\alpha$ AB-sets.
- Collection of semi-regular sets of $X$ does not form a topology on $X$. (see Example 2.9)

We discuss the following example in support of our above assertions.

**Example 3.1.** Consider a topological space $X = \{a, b, c\}$ with the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$, $F_X = \{\phi, X, \{b, c\}, \{a, c\}, \{c\}\}$. Then, in the view of Definition 2.7 and 2.10 we have, $\gamma O(X) = A(X) = \alpha AB(X) = AC(X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. It may be verified easily that the collection of all these sets does not form the topology on $X$, but form a generalized topology on $X$. We now discuss some obvious implications.

**Lemma 3.1.** Every A-set is an $\alpha$AB-set.

*Proof.* Proof follows directly by the definitions. ■

**Lemma 3.2.** Every $\alpha$AB-set is $\gamma$ open-set.

*Proof.* Let H be an arbitrary $\alpha$AB-set. Then, $H = \{A \cap B; A \in \alpha O(X), B = clint(B)\}$ (cf.Definition 2.10).
**Claim.** $H = A \cap B$ is $\gamma$ open-set.
It is sufficient if we show that $H \subseteq int(cl(H)) \cup cl(int(H))$ (cf. Definition 2.7). Since $H = A \cap B$ is $\alpha$ AB-set; where $A \subseteq int(cl(int(A)))$, $B = cl(int(B))$ (see Definitions 2.6, 2.10). It is enough, if we prove that $A \cap B \subseteq int(cl(A \cap B)) \cup cl(int(A \cap B))$. That is to show:

- Either $A \cap B \subseteq int(cl(A \cap B))$ or
- $A \cap B \subseteq cl(int(A \cap B))$.

Since $A \subseteq int(cl(int(A)))$ and $B = cl(int(B))$. By hypothesis,

$$A \cap B \subseteq int(cl(int(A))) \cap cl(int(B)) \tag{3.9}$$

We wish to show that

$$A \cap B \subseteq cl(int(A \cap B)) \tag{3.10}$$

In view of 3.9 and 3.10, it is enough if we show that

$$int(cl(int(A)) \cap cl(int(B)) \subseteq cl(int(A \cap B))$$

$$\subseteq cl(int(A) \cap cl(int(B))$$

$(\because\ int(A \cap B) = int(A) \cap int(B))$

$$\Rightarrow\ int(cl(int(A))) \subseteq cl(int(A))$$

Which holds trivially by definition. Thus, H is $\gamma$-open set. ■

**Theorem 3.1.** Let $(X, \tau)$ be a topological space and $A$ be any subset of $X$. Then following are equivalent:

**i** *A is $\beta$-open.*

**ii** *A is $\gamma$-open.*

*Proof.* We have to show that **(i)** $\Rightarrow$ **(ii)** Since, A is $\beta$-open $\Rightarrow A \subseteq cl(int(cl(A)))$ (cf. Definition 2.6). We know that every $\beta$-open set is semi-open as well as pre-open set (see Remark 2.3). Therefore, we have $A \subseteq int(cl(A))$ and $A \subseteq cl(int(A)) \Rightarrow A \subseteq int(cl(A)) \cup cl(int(A))$. Hence A is ***$\gamma$*-open** (cf. Definition 2.7).
**(ii)** $\Rightarrow$ **(i)** Since, A is $\gamma$-open $\Rightarrow A \subseteq cl(int(A)) \cup int(cl(A))$
(cf. Definition 2.7). Then two cases arises:

**(a)** $A \subseteq cl(int(A))$

**(b)** $A \subseteq int(cl(A))$.

If (a) holds, then A is semi-open. Since every semi-open set is $\beta$-open (see Remark 2.3). A is $\beta$-open.
If (b) holds, then A is pre-open and again every pre-open set is $\beta$-open (see Remark 2.3). Hence $A$ is $\beta$-open. ■

**Remark 3.1.** In view of Theorem 3.1, it is clear that $\gamma$-open sets and $\beta$-open sets are equivalent. Hence, the family of $\gamma$-open sets belongs to $\tau_f$ as it has been shown that all $\beta$-open sets are the members of $\tau_f$ (cf. [1]). Moreover, it is obvious that the collection of AC-set is the member of $\tau_f$. (see Definitions 2.2, 2.6, 2.7, 2.10).

## 4. Main results

In this section, we introduce fine A-set, fine $\alpha$AB-set, fine AC-set and establish the relationship between fine A-set, fine $\alpha$AB-set, fine AC-set and fine set of a topological space $X$, also verify some results concerning these sets.

**Definition 4.1.** A subset S of a space $(X, \tau, \tau_f)$ is called

- **Fine semi-regular** if $S = f_{cl}(f_{int}(S))$.
- **Fine $\gamma$-open** if $S = f_{cl}(f_{int}(S)) \cup f_{int}(f_{cl}(S))$.

The complement of fine semi-regular set and fine $\gamma$-open set are fine semi-regular closed set and fine $\gamma$-closed set. The collection of fine semi-regular set and fine $\gamma$-open set are denoted by $fSr(X)$ and $f\gamma(X)$ respectively.

**Example 4.1.** Consider a topological space $X = \{a, b, c, d\}$ with the topology $\tau = \{X, \phi, \{a, b\}\}$. In view of Definition 2.2 we have, $\tau_f = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}\}$. $F_f = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b, d\}, \{a, d\}, \{b, c\}, \{a, c\}, \{d\}, \{b\}, \{c\}, \{a\}\}$. Let $A = \{b, c\}$ be a subset of $X$. Then, $f_{int}\{b, c\} = \{b, c\}$, $f_{cl}(f_{int}\{b, c\}) = f_{cl}\{b, c\} = \{b, c\}$. Therefore, we have $\{b, c\} = f_{cl}(f_{int}\{b, c\})$. Hence, $A$ is a fine semi-regular set. Therefore, $A$ is a **fine semi-regular set**.

**Example 4.2.** Consider $X = \{a, b, c, d\}$ with the topology $\tau = \{X, \phi, \{a\}\}$, $F_X = \{\phi, X, \{b, c, d\}, \}$. In view of Definition 2.2, $\tau_f = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$, $F_f = \{X, \phi, \{b, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$. Taking $A = \{a, c, d\}$ a subset of $X$. Then, $f_{cl}\{a, c, d\} = X$, $f_{int}(f_{cl}\ \{a, c, d\}) = f_{int}X = X$. Again consider $f_{int}\{a, c, d\} = \{a, c, d\}$, $f_{cl}(f_{int}\{a, c, d\}) = f_{cl}\{a, c, d\} = X$. Therefore, we have $\Rightarrow \{a, c, d\} \subseteq f_{int}(f_{cl}\{a, c, d\}) \cup f_{cl}(f_{int}\{a, c, d\}) \Rightarrow \{a, c, d\} \subseteq X \cup X = X$. Hence, $A$ is a **fine $\gamma$-open set**. The complement of $\{a, c, d\}$ is $\{b\}$ which is **fine $\gamma$-closed set**.

**Definition 4.2.** A subset $A$ of a space $(X, \tau, \tau_f)$ is said to be **fine $\gamma$-clopen** if it is both **fine $\gamma$-open** and **fine $\gamma$-closed**. The collection of fine $\gamma$-clopen sets is denoted by $\boldsymbol{f\gamma co(X)}$.

**Example 4.3.** Consider $X = \{a, b, c\}$ with the topology $\tau = \{X, \phi, \{a\}, \{a, b\}\}$, $F_X = \{\phi, X, \{b, c\}, \{c\}\}$. In view of Definition 2.2, $\tau_f = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$, $F_f = \{\phi, X, \{b, c\}, \{a, c\}, \{c\}, \{b\}, \{a\}\}$. Then, $f\gamma(X) = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. The collection of fine $\gamma$-closed sets of $X$ is $= \{\phi, X, \{b, c\}, \{a, c\}, \{c\}, \{b\}, \{a\}\}$. Therefore, collection of fine $\gamma$-clopen sets of $X$ is $f\gamma co(X) = \{X, \phi, \{a\}, \{b\}, \{a, c\}, \{b, c\}\}$.

**Definition 4.3.** A subset $H$ of a space $(X, \tau, \tau_f)$ is called

- A **fine AB-set** or **fine A-set** if $H \in \boldsymbol{fA(X)} = \{A \cap B : A \in \tau_f,\ B = f_{cl}(f_{int}(B))\}$.
- A **fine $\alpha$AB-set** if $H \in \boldsymbol{f\alpha AB(X)} = \{A \cap B : A \in f\alpha(X), B$ is fine semi-regular$\}$.
- A **fine AC-set** if $H \in \boldsymbol{fAC(X)} = \{A \cap B : A \in \tau_f,\ B$ is fine $\gamma$-clopen$\}$.

**Definition 4.3.** A subset $H$ of a space $(X, \tau, \tau_f)$ is called

- A **fine AB-set** or **fine A-set** if $H \in \boldsymbol{fA(X)} = \{A \cap B : A \in \tau_f,\ B = f_{cl}(f_{int}(B))\}$.
- A **fine $\alpha$AB-set** if $H \in \boldsymbol{f\alpha AB(X)} = \{A \cap B : A \in f\alpha(X), B$ is fine semi-regular$\}$.
- A **fine AC-set** if $H \in \boldsymbol{fAC(X)} = \{A \cap B : A \in \tau_f,\ B$ is fine $\gamma$-clopen$\}$.

**Example 4.4.** Consider a topological space $X = \{a, b, c, d\}$ with the topology $\tau = \{X, \phi, \{a, b\}\}$, $F_X = \{\phi, X, \{c, d\}\}$. In view of Definition 2.2, $\tau_f = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$, $F_f = \{\phi, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{a, d\}, \{a, c\}, \{d\}, \{c\}, \{b\}, \{a\}\}$. We now show that $H = \{c\} \subseteq X$ is **fine A-set**.
**Claim.** $H = \{A \cap B : A \in \tau_f,\ B = f_{cl}(f_{int}(B))\}$. We now consider $A = \{a, c\} \in \tau_f$ and $B = \{b, c\}$.
**Claim.** $B = f_{cl}(f_{int}(B))$. It may be checked easily that $B = f_{cl}(f_{int}(B))$.
Thus, $H = A \cap B = \{a, c\} \cap \{b, c\} = \{c\} \in \boldsymbol{fA(X)}$ or **fine AB-set** or **fine A-set**.

**Example 4.5.** Consider a topological space $X = \{a, b, c, d\}$ with the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$, $F_X = \{\phi, X, \{b, c\}, \{a, c\}, \{c\}\}$. In view of Definition 2.2, $\tau_f = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$, $F_f = \{\phi, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{a, d\}, \{a, c\}, \{d\}, \{c\}, \{b\}, \{a\}\}$. We now show that $H = \{d\} \subseteq X$ is an **fine $\alpha$AB-set**.
**Claim.** $H = \{A \cap B : A$ is fine $\alpha$-open, $B$ is fine semi-regular set$\}$.
It is easy to see that collection of fine $\alpha$-open sets and fine semi-regular sets of $X$ are $f\alpha(X) = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\},$

$\{b, c, d\}\}$, $fSr(X) = \{X, \phi, \{a\}, \{b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, c, d\}, \{b, c, d\}\}$ respectively. We now Consider $A = \{a, d\} \in f\alpha(X)$ and $B = \{b, c, d\} \in fSr(X)$. Thus, $H = A \cap B = \{a, d\} \cap \{b, c, d\} = \{d\} \in \boldsymbol{f\alpha AB(X)}$ or **fine αAB-set**.

**Example 4.6.** Consider a topological space $X = \{a, b, c, d\}$ with the topology $\tau = \{X, \phi, \{c\}, \{c, d\}\}$, $F_X = \{\phi, X, \{a, b, d\}, \{a, b\}\}$. In view of Definitions 2.2 and 4.1, $\tau_f = f\gamma(X) = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}\}$, $F_f = \{\phi, X, \{a, b, d\}, \{a, b, c\}, \{a, b\}, \{b, d\}, \{b, c\}, \{a, d\}, \{a, c\}, \{d\}, \{b\}, \{c\}, \{a\}\}$. In view of Definition 4.2, $f\gamma co(X) = \{X, \phi, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$. We now show that $H = \{b\} \subseteq X$ is an **fine AC-set**.
**Claim.** $H = \{A \cap B : A \in \tau_f$ and $B$ is fine $\gamma$-clopen$\}$. We now consider $A = \{b, c\} \in \tau_f$ and $B = \{b, d\} \in f\gamma co(X)$. Then, $A \cap B = \{b, c\} \cap \{b, d\} = \{b\} \in \boldsymbol{fAC(X)}$ or **fine AC-set**.

**Lemma 4.1.** Let $X$ be a non empty set and $(X, \tau, \tau_f)$ is a **fine space** of $X$ generated by the topology $\tau$ on $X$. Then every superset of open set is fine-open set.

*Proof.* Proof follows directly by the definition. (cf. Definition 2.2) ■

**Lemma 4.2.** Let $X$ be a non empty set and $(X, \tau, \tau_f)$ is a fine space of $X$ generated by the topology $\tau$ on $X$. Then every subset of closed set is fine-closed set.

*Proof.* Let $A$ and $B$ be subsets of $X$ such that $A \subset B$, where $B$ is a closed set of $X$. It follows that $B \in F_f$ and $CB \in \tau$.
**Claim.** $A \in F_f$.
Since $B \in F_f$. Then by definition of fine space, $CB \in \tau_f$. Given that $A \subset B \Rightarrow CB \subset CA \Rightarrow CB \cap CA \neq \phi$ ($\because CB \in \tau$). Then, It is directly by the definition, $CA \in \tau_f$. Thus, $A \in F_f$. Therefore, $A$ is fine-closed set of $X$. ■

We quote some important note about fine topological space.

**Lemma 4.3.** Let $X$ be a non empty finite set and $(X, \tau, \tau_f)$ is a fine space of $X$ generated by the topology $\tau$ on $X$. If $A$ is a subset of $X$, then either $A \in \tau_f$ or $A \in F_f$ i.e. $\tau_f \cup F_f = P(X)$.

*Proof.* Given that $A \subseteq X$. Let if $A \in \tau_f$, then the result is obvious. Let $A \notin \tau_f$, then by definition, $A \subseteq f_{cl}A$. It is a direct consequence of Lemma 4.2, $A$ is fine closed set i.e. $A \in F_f$. Therefore we have, $\tau_f \cup F_f = P(X)$. ■

**Theorem 4.1.** Every fine semi-regular set is fine-clopen set.

*Proof.* Let A be fine semi-regular set. Let if possible that A is not a fine-clopen set. Then either $A \in \tau_f$ or $A \in F_f$ (see Lemma 4.3). If $A \in \tau_f \Rightarrow f_{int}A = A$ and $A \subseteq f_{cl}(A)$. Then, we have $A \subseteq f_{cl}(f_{int}A)$. This is a contradiction of our hypothesis. Hence, our assumption is wrong. Therefore, A is not a fine semi-regular set.

Again if $A \in F_f$ doing in the same manner as above, we get a contradiction again of our hypothesis. Hence, A is fine-clopen set of $X$. ■

**Remark 4.1.** Let $X$ be a finite non empty set and $(X, \tau, \tau_f)$ is a **fine space** of $X$ generated by the topology $\tau$ on $X$. If the collection of fine semi-regular set contains only $X$ and $\phi$, only when the collection of fine A-sets, fine $\alpha AB$-sets, fine AC-sets and fine $\gamma$-sets are not a discrete set. Otherwise it is a discrete set.

**Theorem 4.2.** Let $(X, \tau)$ be a topological space and $\tau_f$ be the fine topological space generated by $\tau$. If $H \in A(X)$ then, $H \in \tau_f$.

*Proof.* Given that $H \in A(X)$ then, by the Definition of A-set $H = \{A \cap B : A \in \tau, B = cl(int(B))\}$. Then Three Conditions arises.
(i) $H = A$ ie $A \subseteq B$ (ii) $H = B$ ie $B \subseteq A$ (iii) $H = A \cap B \neq \phi$
**Case 1.** If $H = A$ then, $H \in \tau$. Then, by the Definition of fine space, $H$ is an open set of $X$. Hence, $H \in \tau_f$. (cf.[1])
**Case 2.** If $H = B = cl(int(B)) \neq \phi$. i.e. $B \neq \phi$.
**Claim.** $B \in \tau_f$. Let $B \notin \tau_f$ then again by the Definition of fine space, $U_\alpha \cap B = \phi$ for each $\alpha$. It follows that $int(B) = \phi \Rightarrow cl(int(B)) = \phi$. $\Rightarrow B \neq cl(int(B))$, which is a contradiction to the hypothesis. Hence, $B \in \tau_f$.
**Case 3.** $H = A \cap B \neq \phi$. ($\because H \subseteq A$ and $H \subseteq B$). It follows that $A \cap H \neq \phi$. Since, $A \in \tau_f$ Then directly by the definition of fine space $H \in \tau_f$. ■

**Theorem 4.3.** Let $(X, \tau)$ be a topological space and $\tau_f$ be the fine topological space generated by $\tau$. Then, if $H \in \alpha AB$-set implies $H \in \tau_f$.

*Proof.* Given that $H$ be an $\alpha$AB-set, then, By the definition of $\alpha$AB-set $H = A \cap B$, where A is $\alpha$-open set and B is semi-regular set. Then, three cases arises.
(i) $H = A$ ie $A \subseteq B$ (ii) $H = B$ ie $B \subseteq A$ (iii) $H = A \cap B \neq \phi$
**Case 1.** If $H = A$, then H is an $\alpha$-open set of $X$, and $\alpha O(X) \subseteq \tau_f$. (cf.[1]). Hence, $H \in \tau_f$.
**Case 2.** If $H = B = cl(int(B)) \neq \phi$. i.e. $B \neq \phi$.
**Claim.** $B \in \tau_f$. Let $B \notin \tau_f$ then again by the Definition of fine space, $U_\alpha \cap B = \phi$ for each $\alpha$. It follows that $int(B) = \phi \Rightarrow cl(int(B)) = \phi$. $\Rightarrow B \neq cl(int(B))$, which is a contradiction to the hypothesis. Hence, $B \in \tau_f$.
**Case 3.** $H = A \cap B \neq \phi$. ($\because H \subseteq A$ and $H \subseteq B$). It follows that $A \cap H \neq \phi$. Since, $A \in \tau_f$ then directly by the definition of fine space $H \in \tau_f$. ■

## 5. Some General Classes of Continuity

In this section, the author would discuss some wider aspects of continuous functions.

**Definition 5.1.** A function $f : (X, \tau) \to (Y, \sigma)$ is called **fine super-continuous** if $f^{-1}(G)$ is fine-open in $X$ for each $G \in \sigma$.

**Example 5.1.** Consider the topological spaces $X = \{a, b, c, d\}$, $Y = \{1, 2, 3, 4\}$ with their corresponding topologies $\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, b, c\}\}, \sigma = \{Y, \phi, \{1\}, \{2\}, \{1, 2\}\}$ respectively. In view of Definition 2.2 we have, $\tau_f = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. We now define the function $f : X \to Y$ as

$$f(x) = \begin{cases} 1 & for\ x = a; \\ 2 & for\ x = b; \\ 3 & for\ x = c; \\ 4 & for\ x = d. \end{cases} \tag{5.11}$$

In view of 5.11 for $G \in \sigma$, we get

$$f^{-1}(G) = \begin{cases} X & for\ G = Y; \\ \phi & for\ G = \phi; \\ \{a\} & for\ G = \{1\}; \\ \{b\} & for\ G = \{2\}; \\ \{a, b\} & for\ G = \{1, 2\}. \end{cases} \tag{5.12}$$

Hence, we notice that $f^{-1}(G) \in \tau_f$ and conclude that $f$ is **fine super-continuous**, when we appeal to Definition 2.2 but not fine-continuous as well as continuous.

**Remark 5.1.** In view of the above definition and the different continuous functions given in section 2, it may be observed that:
**fine-Continuity ⇒ Continuity ⇒ fine super-continuity**.
**A-continuity ⇒ $\alpha$AB-continuity ⇒ $\gamma$-continuity** (see Lemmas 3.1 and 3.2)

**Definition 5.2.** A function $f : (X, \tau) \to (Y, \sigma)$ is called **fine A-continuous** if $f^{-1}(G) \in A(X)$ for each $G \in \sigma_f$.

**Example 5.2.** Consider the topological spaces $X = \{a, b, c, d\}$ and $Y = \{1, 2, 3, 4\}$ with their corresponding topologies $\tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}, \{a, b\}, \{a, b, c\}\}$, $\sigma = \{Y, \phi, \{1, 4\}, \{1, 2, 4\}, \{2\}\}$ respectively. $F_X = \{\phi, X, \{a, c, d\}, \{a, b, d\}, \{a, d\}, \{c, d\}, \{d\}\}$, $F_Y = \{\phi, Y, \{2, 3\}, \{3\}, \{1, 3, 4\}\}$.In view of Definitions 2.2 and 2.10 we have, $A(X) = \{X, \phi, \{b\}, \{c\}, \{b, c\}, \{a, b\}, \{a, b, c\}, \{c, d\}, \{a, b, d\}\}$ and $\sigma_f = \{Y, \phi, \{2\}, \{1\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3\}\}$. We now define the function $f : X \to Y$ as

$$f(x) = \begin{cases} 2 & for\ x = a, b, d; \\ 3 & for\ x = c. \end{cases} \tag{5.13}$$

In view of 5.13 for $G \in \sigma$, we get

$$f^{-1}(G) = \begin{cases} X & for\ G = Y; \\ \phi & for\ G = \phi; \\ \phi & for\ G = \{1\}; \\ \{a, b, d\} & for\ G = \{2\}; \\ \phi & for\ G = \{4\}; \\ \{a, b, d\} & for\ G = \{1, 2\}; \\ \{c\} & for\ G = \{1, 3\}; \\ \phi & for\ G = \{1, 4\}; \\ X & for\ G = \{2, 3\}; \\ \{a, b, d\} & for\ G = \{2, 4\}; \\ \{c\} & for\ G = \{3, 4\}; \\ X & for\ G = \{1, 2, 3\}; \\ \{a, b, d\} & for\ G = \{1, 2, 4\}; \\ \{c\} & for\ G = \{1, 3, 4\}; \\ X & for\ G = \{2, 3, 4\}. \end{cases} \tag{5.14}$$

Hence, we notice that $f^{-1}(G) \in A(X)$ and conclude that $f$ is **fine A-continuous**, when we appeal to Definition 2.10 but not fine-continuous as well as continuous.

**Definition 5.3.** A function $f : (X, \tau) \to (Y, \sigma)$ is called **fine αAB-continuous** if $f^{-1}(G) \in \alpha AB(X)$ for each $G \in \sigma_f$.

**Example 5.3.** Consider the topological spaces $X = \{a, b, c, d\}$ and $Y = \{1, 2, 3, 4\}$ with their corresponding topologies $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c\}, \{a, c, d\}\}$, $\sigma = \{Y, \phi, \{3, 4\}, \{2, 4\}, \{4\}, \{2, 3, 4\}\}$ respectively. In view of Definitions 2.2 and 2.10 we have, $\alpha AB(X) = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c\}, \{b, d\}, \{b, c\}\}$ and $\sigma_f = \{Y, \phi, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3\}\}$. We now define the function $f : X \to Y$ as

$$f(x) = \begin{cases} 3 & for\ x = a, c; \\ 4 & for\ x = b, d. \end{cases} \tag{5.15}$$

In view of 5.15 for $G \in \sigma$, we get

$$f^{-1}(G) = \begin{cases} X & for\ G = Y; \\ \phi & for\ G = \phi; \\ \phi & for\ G = \{2\}; \\ \{a, c\} & for\ G = \{3\}; \\ \{b, d\} & for\ G = \{4\}; \\ \phi & for\ G = \{1, 2\}; \\ \{a, c\} & for\ G = \{1, 3\}; \\ \{b, d\} & for\ G = \{1, 4\}; \\ \{a, c\} & for\ G = \{2, 3\}; \\ \{b, d\} & for\ G = \{2, 4\}; \\ X & for\ G = \{3, 4\}; \\ \{a, c\} & for\ G = \{1, 2, 3\}; \\ \{b, d\} & for\ G = \{1, 2, 4\}; \\ X & for\ G = \{1, 3, 4\}; \\ X & for\ G = \{2, 3, 4\}. \end{cases} \tag{5.16}$$

Hence, we notice that $f^{-1}(G) \in \alpha AB(X)$ and conclude that $f$ is **fine αAB-continuous**, when we appeal to Definition 2.10 but not fine-continuous as well as continuous.

**Definition 5.4.** A function $f : (X, \tau) \to (Y, \sigma)$ is called **fine AC-continuous** if $f^{-1} \in AC(X)$ for each $G \in \sigma_f$.

**Example 5.4.** Consider the topological spaces $X = \{a, b, c, d\}$ and $Y = \{1, 2, 3, 4\}$ with their corresponding topologies $\tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}, \sigma = \{Y, \phi, \{1, 3\}\}$ respectively. $F = \{\phi, X, \{b, c, d\}, \{a, d\}, \{d\}\}$. In view of Definitions 2.2 and 2.10 we have, $AC(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ and $\sigma_f = \{Y, \phi, \{1\}, \{3\}, \{1, 3\}, \{1, 2\}, \{1, 4\}, \{2, 3\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{2, 3, 4\}, \{1, 3, 4\}\}$. We now define the function $f : X \to Y$ as

$$f(x) = \begin{cases} 1 & for\ x = a, d; \\ 2 & for\ x = b; \\ 3 & for\ x = c. \end{cases} \tag{5.17}$$

In view of 5.17 for $G \in \sigma$, we get

$$f^{-1}(G) = \begin{cases} X & for\ G = Y; \\ \phi & for\ G = \phi; \\ \{a, d\} & for\ G = \{1\}; \\ \{c\} & for\ G = \{3\}; \\ \{a, c, d\} & for\ G = \{1, 3\}; \\ \{a, b, d\} & for\ G = \{1, 2\}; \\ \{a, d\} & for\ G = \{1, 4\}; \\ \{c\} & for\ G = \{3, 4\}; \\ \{b, c\} & for\ G = \{2, 3\}; \\ \{a, b, d\} & for\ G = \{1, 2, 4\}; \\ \{a, c, d\} & for\ G = \{1, 3, 4\}; \\ X & for\ G = \{1, 2, 3\}; \\ \{b, c\} & for\ G = \{2, 3, 4\}. \end{cases} \tag{5.18}$$

Hence, we notice that $f^{-1}(G) \in AC(X)$ and conclude that $f$ is **fine AC-continuous**, when we appeal to Definition 2.10 but not fine-continuous as well as continuous.

**Remark 5.2.** A function f is **fine A-continuous** then it is necessary that it is **A-continuous**. But converse may not true in general.

**Example 5.5.** Consider the topological spaces $X = \{a, b, c, d\}$ and $Y = \{1, 2, 3, 4\}$ with their corresponding topologies $\tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}, \{a, b\}, \{a, b, c\}\}$, $\sigma = \{Y, \phi, \{2, 3\}, \{1, 2, 4\}, \{2\}\}$ respectively. $F_X = \{\phi, X, \{a, c, d\}, \{a, b, d\}, \{a, d\}, \{c, d\}, \{d\}\}$, $F_Y = \{\phi, Y, \{1, 4\}, \{3\}, \{1, 3, 4\}\}$.In view of Definitions 2.2 and 2.10 we have, $A(X) = \{X, \phi, \{b\}, \{c\}, \{b, c\}, \{a, b\}, \{a, b, c\}, \{c, d\}, \{a, b, d\}\}$, $\sigma_f = \{Y, \phi, \{2\}, \{1\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3\}\}$. We now define the function $f : X \to Y$ as

$$f(x) = \begin{cases} 1 & for\ x = a; \\ 2 & for\ x = b; \\ 3 & for\ x = c; \\ 4 & for\ x = d. \end{cases} \tag{5.19}$$

In view of 5.19 for $G \in \sigma$, we get

$$f^{-1}(G) = \begin{cases} X & for\ G = Y; \\ \phi & for\ G = \phi; \\ \{b\} & for\ G = \{2\}; \\ \{b, c\} & for\ G = \{2, 3\}; \\ \{a, b, d\} & for\ G = \{1, 2, 4\}. \end{cases} \tag{5.20}$$

Hence, we notice that $f^{-1}(G) \in A(X)$ and conclude that $f$ is **A-continuous**. Whereas, $f$ is not **fine A-continuous**. Since, $f^{-1}\{1\} = \{a\} \notin A(X)$ (see Definition 5.2) as well as continuous.

**Remark 5.3.** A function is **fine αAB-continuous** then it is necessary that it is **αAB-continuous**. But converse may not be true in general.

**Example 5.6.** Consider the topological spaces $X = \{a, b, c, d\}$ and $Y = \{1, 2, 3, 4\}$ with their corresponding topologies $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c\}, \{a, c, d\}\}$, $\sigma = \{Y, \phi, \{3, 4\}, \{2, 4\}, \{4\}, \{2, 3, 4\}\}$ respectively. $F_X = \{\phi, X, \{a, b, d\}, \{a, b, c\}, \{a, b\}, \{b, d\}, \{b\}\}$, $F_Y = \{\phi, Y, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}, \{1\}\}$. In view of Definitions 2.2 and 2.10 we have, $\alpha AB(X) = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c\}, \{a, c, d\}, \{a, b, c\}, \{b, c, d\}, \{b, d\}, \{b, c\}\}$. $\sigma_f = \{Y, \phi, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{3, 4\}, \{2, 4\}, \{4\}, \{2, 3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}\}$. We now define the function $f : X \to Y$ as

$$f(x) = \begin{cases} 1 & for\ x = a; \\ 2 & for\ x = b; \\ 3 & for\ x = c; \\ 4 & for\ x = d. \end{cases} \tag{5.21}$$

In view of 5.21 for $G \in \sigma$, we get

$$f^{-1}(G) = \begin{cases} X & for\ G = Y; \\ \phi & for\ G = \phi; \\ \{d\} & for\ G = \{4\}; \\ \{b, d\} & for\ G = \{2, 4\}; \\ \{c, d\} & for\ G = \{3, 4\}; \\ \{b, c, d\} & for\ G = \{2, 3, 4\}. \end{cases} \tag{5.22}$$

Hence, we notice that $f^{-1}(G) \in \alpha AB(X)$ and conclude that $f$ is **αAB-continuous**. Whereas, $f$ is not **fine αAB-continuous**. Since, $f^{-1}\{2\} = \{b\} \notin \alpha AB(X)$ (see Definition 5.2) nor $\alpha$-continuous and continuous.

**Definition 5.5.** A function $f : (X, \tau) \to (Y, \sigma)$ is called

- **fine super A-continuous** if $f^{-1}(G)$ is fine A-set in $X$ for each $G \in \sigma$.
- **fine super αAB-continuous** if $f^{-1}(G)$ is fine αAB-set in $X$ for each $G \in \sigma$.
- **fine super AC-continuous** if $f^{-1}(G)$ is fine AC-set in $X$ for each $G \in \sigma$.

**Example 5.7.** Consider the topological spaces $X = \{a, b, c, d\}$ and $Y = \{1, 2, 3, 4\}$ with their corresponding topologies $\tau = \{X, \phi, \{b\}\}$, $\sigma = \{Y, \phi, \{1, 2\}, \{2, 3\}, \{2\}, \{1, 2, 3\}\}$ respectively. Then the fine topology generated by $\tau$ is $\tau_f = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\{b, c, d\}\}$. In view of Definition 4.3 we have, $fA(X) = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$. Define a function $f : X \to Y$ as

$$f(x) = \begin{cases} 1 & for\ x = a; \\ 2 & for\ x = b; \\ 3 & for\ x = c; \\ 4 & for\ x = d. \end{cases} \tag{5.23}$$

In view of 5.23 for $G \in \sigma$, we get

$$f^{-1}(G) = \begin{cases} X & for\ G = Y; \\ \phi & for\ G = \phi; \\ \{b\} & for\ G = \{2\}; \\ \{a, b\} & for\ G = \{1, 2\}; \\ \{b, c\} & for\ G = \{2, 3\}. \end{cases} \tag{5.24}$$

Hence, we notice that $f^{-1}(G) \in fA(X)$ and conclude that $f$ is **fine super A-continuous** when we appeal to Definition 4.3.

**Example 5.8.** Consider the topological spaces $X = \{a, b, c, d\}$ and $Y = \{1, 2, 3, 4\}$ with their corresponding topologies $\tau = \{X, \phi, \{c\}\}$, $\sigma = \{Y, \phi, \{3, 4\}, \{2, 4\}, \{4\}, \{2, 3, 4\}\}$ respectively. Then the fine topology generated by X is $\tau_f = \{X, \phi, \{c\}, \{c, d\}, \{a, c\}, \{b, c\}, \{b, c, d\}, \{a, c, d\}, \{a, b, c\}\}$. Then, in view of Definition 4.3 we have, $f\alpha AB(X) = \{X, \phi, \{c\}, \{c, d\}, \{a, c\}, \{b, c\}, \{b, c, d\}, \{a, c, d\}, \{a, b, c\}\}$. Define a function $f : X \rightarrow Y$ as

$$f(x) = \begin{cases} 4 & for\ x = a, c; \\ 1 & for\ x = b; \\ 3 & for\ x = d; \end{cases} \tag{5.25}$$

In view of 5.25 for $G \in \sigma$, we get

$$f^{-1}(G) = \begin{cases} X & for\ G = Y; \\ \phi & for\ G = \phi; \\ \{a, c\} & for\ G = \{4\}; \\ \{a, c\} & for\ G = \{2, 4\}; \\ \{a, c, d\} & for\ G = \{3, 4\}; \\ \{a, c, d\} & for\ G = \{2, 3, 4\}. \end{cases} \tag{5.26}$$

Hence, we notice that $f^{-1}(G) \in f\alpha AB(X)$ and conclude that $f$ is **fine super $\alpha AB$-continuous** when we appeal to Definition 4.3.

**Example 5.9.** Consider the topological spaces $X = \{a, b, c, d\}$ and $Y = \{1, 2, 3, 4\}$ with their corresponding topologies $\tau = \{X, \phi, \{d\}\}$, $\sigma = \{Y, \phi, \{3, 4\}, \{2, 4\}, \{1, 2, 4\}, \{4\}, \{2, 3, 4\}\}$ respectively. Then the fine topology generated by X is $\tau_f = \{X, \phi, \{d\}, \{c, d\}, \{a, d\}, \{b, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}\}$ . Then, in view of Definition 5.3 we have, $fAC(X) = \{X, \phi, \{d\}, \{c, d\}, \{a, d\}, \{b, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}\}$. Define a function $f : X \rightarrow Y$ as

$$f(x) = \begin{cases} 1 & for\ x = a, c; \\ 2 & for\ x = b; \\ 4 & for\ x = d; \end{cases} \tag{5.27}$$

In view of 5.27 for $G \in \sigma$, we get

$$f^{-1}(G) = \begin{cases} X & for\ G = Y; \\ \phi & for\ G = \phi; \\ \{d\} & for\ G = \{4\}; \\ \{b, d\} & for\ G = \{2, 4\}; \\ \{d\} & for\ G = \{3, 4\}; \\ \{b, d\} & for\ G = \{2, 3, 4\}; \\ X & for\ G = \{1, 2, 4\}. \end{cases} \tag{5.28}$$

Hence, we notice that $f^{-1}(G) \in AC(X)$ and conclude that $f$ is **fine super AC-continuous** when we appeal to Definition 4.3.

## 6. Applications

The concept of Homeomorphism plays a key role in many branches of science and technology. viz. quantum physics, DNA-structure studies in biological sciences etc. The trace of the Schrodinger stress tensor $v(r)$ (where $v(r)$ denotes coulomb potential of a molecular system) is defined by using the local statement of quantum mechanical virial theorem and it has been established that it is homeomorphic to another space $\rho(r)$ (electronic charge density) which may be explored easily. We have defined the super fine-continuity and this concept may be extended to define super homeomorphism which can generate a very close homeomorphic image of a given space and can be studied better in comparison to the classical homeomorphic image. (See also [10], [11])

## 7. Conclusion

The collection of fine open sets is the class which contains almost all generalized open sets required for defining the generalized concept of continuity. There is a tremendous scope of development of the subject with this new class of fine open sets.

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