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An Inventory Model for Deteriorating Items with Lead Time price Dependent Demand and Shortages

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Abstract

In the past few decade researchers have developed their economic order quantity (EOQ) model under constant deterioration and constant demand. However, it is not true in actual practice. In this paper deterministic inventory model is developed, in which deterioration rate is time proportional , demand rate is a function of selling price and inventory holding cost, ordering cost and deterioration rate are all of function of time. The optimum replanishment policy rule, which minimizes the total cost is determined. In this study inventory model for deteriorating items with single warehouse is considered. In this model shortages are allowed during lead time and it is completely backlogged. The derived model is illustrated by a numerical example.

Keywords: Lead time, deteriorating items, price dependent demand, shortages and time varying holding cost.

1. INTRODUCTION

It is usually that a large quantity of goods on self in a superstore will lead the customer to buy more goods and that situation creates the greater demand of the goods. This situation motivates the retailer to increase their order quantity. Deterioration of physical goods is one of the important factors in any inventory system.

In recent years, mathematical ideas have been used in different areas in real life problems, particularly for controlling inventory. One of the most important concerns of the management is to decide when and how much to order or to manufacture so that the total cost associated with the inventory system should be minimum. This is somewhat more important, when the inventory undergo decay or deterioration. Deterioration is defined as change, damage, decay, spoilage obsolescence and loss of utility or loss of original value in a commodity that results in the decreasing usefulness from the original one. It is well known that certain products such as vegetable, medicine, gasoline, blood and radioactive chemicals decrease under deterioration during their normal storage period. As a result, while determining the optimal inventory policy of that type of products, the loss due to deterioration cannot be ignored.

Inventory is a part of each and every organization, whether it is manufacturing or service organization. All organizations have to keep some inventory for smooth running of their business. If any organization claims that they are not keeping any inventory it's an absurd. The Economic Order quantity model is one of the oldest known model developed by Wilson [18], Rekha, Vikas, Urvashi. [16] and a lot of work has been done on this model. In classical inventory models the demand rate is assumed to be a constant. In reality demand for physical goods may be time dependent, stock dependent and price dependent. Selling price plays an important role in inventory system Burwell [2], developed economic lot size model for price-dependent demand under quantity and freight discounts. An inventory system of ameliorating items for price dependent demand rate was considered by Mondal, et. al [11]. You, [17] developed an inventory model with price and time dependent demand.

In the last few decades the study of perishable items has gained enormous importance. In present scenario the wastage of resources is considered as a sin. Even most of the companies are facing cut throat competition and deterioration of resources would reduce their profit margins drastically. Therefore, in most of the present models the items considered are deteriorating items and inventory cost compromises of the deterioration cost.

Ghare and Schrader were the pioneer to use the concept of deterioration, they developed an inventory model with a constant rate of deterioration Ghare and Schrader [8], followed by Covert, and G. C. Philip [4] who formulated a model considering a variable rate of deterioration with two parameter Weibull distribution, which was further extended by Pal, S. [15], Nahmais [13] provided the relevant literature on the problem of determining suitable ordering policies for both fixed life perishable inventory, and inventory subject to continuous exponential decay.

Lead time has been a topic of interest for many authors Ben-daya [1], Das [5], Foote [6], Magson [9], Naddor [12], Chung, , and Ting, [3], Fujiwara [7]. Almost all authors assume lead time as prescribed in all cases, i.e. deterministic as well as probabilistic. However, in many practical situations lead time can be reduced at an added cost. By reducing the lead time, customer service and responsiveness to production schedule changes can be improved and reduction in safety stocks can be achieved. The added cost of reducing lead time consists mainly of administrative costs, transportation cost as the item's transit time from the supplier is a major component of lead time, and supplier's speed-up cost. This work was extended by M. Maragatham and R. Palani [10]. Subsequent contributions in this direction came from researchers like Nirmal Kumar Duari and Tripti Chakraborti [14].

In the present paper, we assume a generalized EOQ model for deteriorating items in a single Warehouse system and demand rate is a function of selling price. Shortages are allowed during the lead time and completely backlogged. An analytical solution of the model is discussed and it is illustrated with the help of a numerical example.

II. NOTATIONS AND ASSUMPTIONS

The following notations are used in developing the model.

A_D : Amount of material deterioration during a cycle time.

$\theta(t)$: Time dependent Deterioration rate.

c : The unit cost per item

A : The ordering cost of inventory/order

$D(p)$: Demand rate

t_1 : Replenishment cycle time

L : Lead time

P_C : Purchase cost

S_C : Shortage cost

C_H : The total cost of holding inventory per cycle

C_D : Total deterioration cost per cycle

Q : Maximum Inventory Level

h : The inventory holding cost per unit item per unit time.

I(t) : The inventory level at time t

Assumptions

We adopt the following assumptions and notations for the model to be discussed.

1. The demand rate $D(p)$ is price depending and is of the form $D(p) =$

$ap^{-b}, a, b > 0, p$ is selling price.

2. The item cost remains constant irrespective of the order size.
3. Shortages are allowed.
4. Replenishment rate is infinite and the lead time is constant.
5. Holding cost is a function of time.
6. The items considered are deteriorating items but deterioration is not instantaneous.
7. The rate of deterioration at any time $t > 0$ follow the two parameter Weibull distribution as $\theta = \alpha\beta t^{(\beta-1)}$, where $\alpha(0 < \alpha < 1)$ is the scale parameter and $\beta(> 0)$ is the shape parameter.
8. There is no repair or replenishment of the deteriorated items during the inventory cycle.
9. The inventory is replenished only once in each cycle.
10. During lead time shortages are allowed
11. Ordering quantity is $Q + LD(p)$ when $t = L$.

III. MATHEMATICAL MODEL AND ANALYSIS

In this model deterministic demand is price dependent and depletion of the inventory occurs due to demand (supply) as well as due to deterioration in each cycle. The objective of the inventory problem is to determine the optimal order quantity and the length of ordering cycle so as to keep the total relevant cost as low as possible, where the holding cost is a function time and shortages are allowed. Now shortages occurred and accumulate to the level S_1, at $t = T$. The behavior of inventory system at any time is shown in figure 1**.**

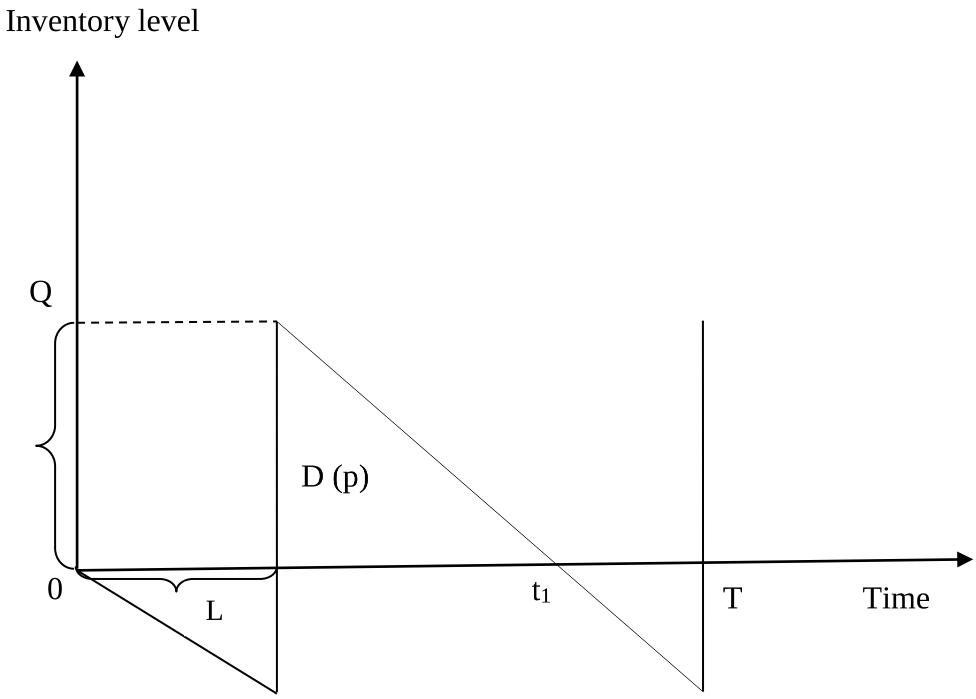

Figure 1. Graphical representation of the inventory system.

The states of $I(t)$ over the cycle time T is given by the following first order differential equation.

$$\frac{d}{dt}I(t) + \alpha\beta(t)^{\beta-1}I(t) = -ap^{-b} \qquad L \le t \le t_1 \qquad (1)$$

$$\frac{d}{dt}I(t) = -ap^{-b} \qquad t_1 \le t \le T \qquad (2)$$

With the boundary condition $I(t_1) = 0, I(T) = -S_1$

Solving above differential equations using boundary conditions we get

$$I(t).e^{\alpha t^{\beta}} = -ap^{-b}\left[t + \alpha\frac{t^{\beta+1}}{\beta+1}\right] + K \qquad (3)$$

Where K is constant of integration

Solving equation (1), we obtain $I\ (t)$ during the time period $(L \le t \le t_1)$

$$I(t) = ap^{-b}\left[(t_1 - t) + \frac{\alpha}{\beta+1}\left(t_1^{\beta+1} - t^{\beta+1}\right)\right]\left[1 - \alpha t^{\beta}\right] \qquad L \le t \le t_1 \qquad (4)$$

At time $t = L,\ I(L) = Q$ i.e. when the items are received, the level at which the organization is having a maximum inventory and equation (4) give the value of Q where $Q + LD(p)$ is the quantity ordered at the start of the cycle. As it is assumed there is lead time, i.e. items are received non-instantaneously.

$$Q = ap^{-b}\left[(t_1 - L) + \frac{\alpha}{\beta+1}\left(t_1^{\beta+1} - L^{\beta+1}\right)\right]\left[1 - \alpha L^{\beta}\right] \quad (5)$$

At $t = T$,

$$S_1 = a(T - t_1)p^{-b}, \text{ since } I(t) = -S_1 \quad (6)$$

The amount of materials which deteriorates during one cycle is

$$A_D\,(t) = ap^{-b}\left[(t_1 - L) + \frac{\alpha}{\beta+1}\left(t_1^{\beta+1} - L^{\beta+1}\right)\right]\left[1 - \alpha L^{\beta}\right] - D(t_1 - L) \quad (7)$$

The total variable cost will consist of the following costs

(a) The ordering cost of the materials, which is fixed per order for the present financial year.

(b) The deterioration cost is given by $c.A_D$ which comes out to be

$$D_c = c\left\{ap^{-b}\left[(t_1 - L) + \frac{\alpha}{\beta+1}\left(t_1^{\beta+1} - L^{\beta+1}\right)\right]\left[1 - \alpha L^{\beta}\right] - D(t_1 - L)\right\} \quad (8)$$

(c) The holding cost is the function of average inventory cost and it is given by

$$\int_L^{t_1} h\, I(t)dt$$

Which, upon simplification, yields

$$C_H = hap^{-b}\left\{\left[\left(\frac{t_1^2}{2} - \alpha\frac{t_1^{\beta+2}}{\beta+1} + \alpha\frac{t_1^{\beta+2}}{\beta+2}\right) - \left(Lt_1 - \frac{L^2}{2} - \alpha\frac{t_1 L^{\beta+1}}{\beta+1} + \alpha\frac{L^{\beta+2}}{\beta+2}\right)\right] + \frac{\alpha}{\beta+1}\left[\left(t_1^{\beta+2} - \frac{t_1^{\beta+2}}{\beta+2} - \alpha\frac{t_1^{2\beta+2}}{\beta+1} + \alpha\frac{t_1^{2\beta+2}}{\beta+2}\right) - \left(Lt_1^{\beta+1} - \frac{L^{\beta+2}}{\beta+2} - \alpha\frac{t_1^{\beta+1}L^{\beta+1}}{\beta+1} + \alpha\frac{L^{2\beta+2}}{2\beta+2}\right)\right]\right\} \quad (9)$$

(d) The purchase cost is given by

$$P_c = c[Q + LD(p)]$$

$$P_c = c\left\{ap^{-b}\left[(t_1 - L) + \frac{\alpha}{\beta+1}\left(t_1^{\beta+1} - L^{\beta+1}\right)\right]\left[1 - \alpha L^{\beta}\right] + Lap^{-b}\right\} \quad (10)$$

(e) Shortage cost is given by $S_c = S\left[-\int_{t_1}^{T} QI(t)dt\right]$

$$S_c = \left[Sap^{-b}\left(\frac{T^2 + t_1^2}{2} - Tt_1\right)\right] \quad (11)$$

Total variable cost function for one cycle is given by

$TC = OC + P_c + H_c + D_c + S_c$

$$TC = A + c\left\{ap^{-b}\left[(t_1 - L) + \frac{\alpha}{\beta+1}\left(t_1^{\beta+1} - L^{\beta+1}\right)\right]\left[1-\alpha L^{\beta}\right] + Lap^{-b}\right\} + hap^{-b}\left\{\left[\left(\frac{t_1^2}{2} - \alpha\frac{t_1^{\beta+2}}{\beta+1} + \alpha\frac{t_1^{\beta+2}}{\beta+2}\right) - \left(Lt_1 - \frac{L^2}{2} - \alpha\frac{t_1L^{\beta+1}}{\beta+1} + \alpha\frac{L^{\beta+2}}{\beta+2}\right)\right] + \frac{\alpha}{\beta+1}\left[\left(t_1^{\beta+2} - \frac{t_1^{\beta+2}}{\beta+2} - \alpha\frac{t_1^{2\beta+2}}{\beta+1} + \alpha\frac{t_1^{2\beta+2}}{\beta+2}\right) - \left(Lt_1^{\beta+1} - \frac{L^{\beta+2}}{\beta+2} - \alpha\frac{t_1^{\beta+1}L^{\beta+1}}{\beta+1} + \alpha\frac{L^{2\beta+2}}{2\beta+2}\right)\right]\right\} + c\left\{ap^{-b}\left[(t_1 - L) + \frac{\alpha}{\beta+1}\left(t_1^{\beta+1} - L^{\beta+1}\right)\right]\left[1-\alpha L^{\beta}\right] - D(t_1 - L)\right\} Sap^{-b}\left[\frac{T^2+t_1^2}{2} - Tt_1\right] \qquad (12)$$

Our objective is to determine optimum value of Q to minimize TC. The values of t_1 for which

$\frac{\partial TC}{\partial t_1} = 0$ satisfying the condition

$\left(\frac{\partial^2 TC}{\partial t_1^2}\right) > 0$

The optimal solution of the equation (12) is obtained by using Mathematica software. This has been illustrated by the following numerical example.

IV. NUMERICAL EXAMPLE

We consider the following parametric values for $A = 300, a = 6$, $b = 1$, $c = 9, \alpha = 0.005$, $\beta = 0.4$, $L = 7$, $h = 5$, $S = 7, p = 12, D = 50, T = 1$ year.

We obtain the optimal value of $t_1 = 72.635\ days$, $Q = 33.1455$ and minimum total cost $(TC) = 3389.72$ / year.

V. CONCLUSION

This paper presents deterministic inventory model for deteriorating items in single warehouse and consider lead time as constant. The demand rate is assumed to be a function of selling price. Selling price is the main criterion of the consumer goes to the market to buy a particular item. Shortages are allowed during the lead time and completely backlogged in the present model. In many practical situations, stock out is unavoidable due to various uncertainties. Consideration of shortages is economically desirable in these cases, deterioration is a natural future in inventory system. There are many items like perfumes, photographic films etc. In future researchers can do more work about several type of demand variable costs etc.

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