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Stability Analysis of Turning Using a Non-Linear Force-Feed Model

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Abstract

This paper presents an analytical stability analysis of turning using a non-linear force-feed model in two dimensions. Most of the existing analytical models ignored the effect of static feed term on the regeneration phenomenon. In practice there is a marked effect of feed on stability due to force variation .The modified analytical equations for cutting insert using three-dimensional tool geometry are obtained by considering relative motion of tool with respect to a two-dimensional elastic model of the work piece. The critical stability limits obtained as a function of feed are conformed by time domain analysis. Experiments are conducted on a flexible work-piece at varying feed conditions .The measured cutting forces show a marked effect of feed on stability.

Keywords: Dynamic modeling, Tool Geometry, Force feed, Stability

1. INTRODUCTION

Many engineering components manufactured using casting, forming and other processes often require machining as their end operation. Machining or metal cutting is an important manufacturing process. To achieve higher accuracy and productivity, it requires consideration of dynamic instability of cutting process. When there is a relative motion present between the tool and work piece, the performance of the operations may not be satisfactory. The machine tool vibrations have detrimental effect on tool life which in turn lowers the productivity and increases cost of production. A dynamically stable system has masses oscillating with decreasing amplitudes towards original state of equilibrium, while an unstable system oscillates with increasing amplitude deviating more and more from the original position of equilibrium. In order to obtain chatter-free machining conditions, conservative cutting parameters are to be selected based on the stability limits. Machine tool operators often select these process parameters from the stability-lobe diagrams, which are conveniently obtained from the analytical cutting models. In practice, accurate cutting dynamic models are needed to get the realistic representation of stability states. Practically, the regeneration is a nonlinear phenomenon. The various forms of nonlinearity are considered in the analytical models. The popular way is to treat the cutting force as a nonlinear function of time varying chip width or feed. This nonlinear force-feed relation leads to a different stability states. Most nonlinear chatter models consider either nonlinearity in the structure and cutting force or nonlinearity due to friction depending on the cutting speed Hanna and Tobias[1-2] found the dynamic response of the machine structure became nonlinear with increasing force amplitude and proposed a nonlinear structural stiffness in polynomial form. Fabris and Souza [3] also presented an approach to obtain stability chart for a fixed value of chatter amplitude by considering the nonlinear relationship between cutting force and uncut chip thickness. After these pioneering works, several researchers have attempted to consider nonlinearity in chatter phenomena by either analytically approximating nonlinear elements. Lin and Wang[4] developed a strong non-linear dynamic model to investigate the dynamic characteristics of cutting process. Stepan and Nagy [5] have shown that the nonlinear relationship between the cutting force and the uncut chip thickness has a stronger effect on the global dynamics than the nonlinearity of the structural stiffness and damping. Nosyreva and Molinari [6] solution of the nonlinear problem. They studied the influence of different parameters on the linear and nonlinear stability and results were presented. Rao and Shin [7] presented a model of the dynamic cutting force process for the three-dimensional or oblique turning operation. To obtain dynamic force predictions, the mechanistic force model is linked to a tool–workpiece vibration model. Gradisek et al. [8] analyzed a non-linear model of non-regenerative cutting. As chip thickness is increased the system becomes unstable via a subcritical Hopf bifurcation. Deshpande and Fofana [9] presented a single degree of freedom, delay differential equation model and the central idea of the model is the study of regenerative effect which qualitatively explains the nonlinear dynamics in machining. Warminski et al. [10] analyzed vibrations generated in orthogonal cutting process using one degree of freedom model and concluded that nonlinear interaction between the tool and work-piece leads to periodic, Quasi periodic or chaotic vibrations depending on the system parameters. Chandiramani and Pothala [11] analyzed a two-degree-of-freedom (2-dof) model comprising nonlinear delay differential equations (DDEs) for self-excited oscillations during orthogonal turning. Their present model and dynamics would be useful for state estimator design in active control of tool chatter. Dassanayake and Suh [12] presented an approach for simultaneous workpiece-tool deflections in response to the exertion of nonlinear regenerative force and also studied the stability for the workpiece and the tool when subject to the same cutting conditions. They used one-Dimensional models to obtain stability charts. They also investigated the turning dynamics using a three-dimensional model. Insperger et al. [13] analysed non-linear dynamics of a state-dependent delay model of the turning process. The size of the regenerative delay is determined not only by the rotation of the workpiece, but also by the vibrations of the tool. The numerical analysis of their model revealed that Hopf bifurcations depend on the feed rate. Landers and Ulsoy [14] presented a nonlinear force feed model to illustrate the practical machining simulations in turning and milling operations. Here machining chatter analysis techniques are mainly examine the stability of the closed-loop model of the machining operation to determine the stable process parameter space.

2. NONLINEAR FORCE-FEED MODEL

Stability lobe diagrams indicate the critical operating parameters necessary to avoid unstable cutting conditions. For every cutting operation, there is one such diagram which takes into account several features such as varying tool wear, work-piece supporting conditions, range of operating speeds, tool overhang lengths and so on. To draw the stability-lobe diagram involving many cutting states, it is quite tedious to perform the experiments several times. Analytical models on the other hand help to obtain the status of stability by incorporating several features in dynamic equations. However, care should be taken during modeling to incorporate all practical cutting constraints to maximum possible extent. Most of the chatter analysis models assume a linear force-feed variation and develop stability lobe diagrams for a specific value of feed. It is well known that machining force inherently contains nonlinear relationship between the force and the feed, which is typically described by a power law. Simulations and experimental results demonstrate the significant effect of force-feed nonlinearity on chatter. Often a linearization analysis is utilized to explore this effect. In this section the linear chatter analysis technique developed by Budak and Altintas is extended to account for the force feed nonlinearity. The analysis provides insight into the effect feed has on chatter in machining operations. Also, by directly including the force-feed nonlinearity in the chatter analysis the need to calibrate the force process model at different feeds is alleviated. In the present case an oblique cutting operation is considered.

The cutting parameters in turning process are chip thickness h, the depth of cut b and the cutting angles. The side cutting edge angle (ψ) and the inclination angle (i) are measured on the rake face of the tool as shown in Fig.1.

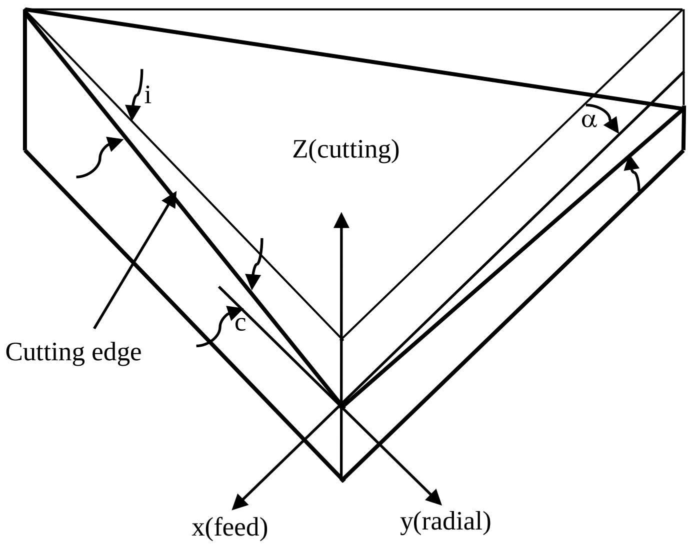


Figure 1. Three dimensional view of insert

2.1. Dynamic Modeling

Proposed analytical model for prediction of stability limit in multi-dimensional dynamic turning systems with real cutting insert is shown in Fig.2.

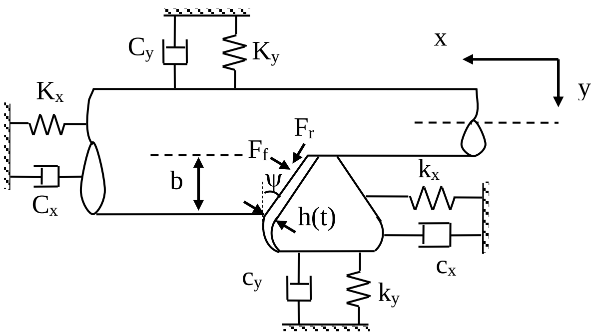


Figure 2. Model under consideration

(b: depth of cut, h (t): chip-thickness,ψ: side cutting edge angle)

When the dynamic displacements in cutting directions are not affecting the dynamic chip thickness, the problem reduces to a 2D model and can be written in terms of modulated chip thickness h (t) as follows:

$$h(t)=f\cos\psi+\Delta x\cos\psi-\Delta y\sin\psi=h_s+\Delta x\cos\psi-\Delta y\sin\psi \qquad (1)$$

where $\Delta x=[x_c(t)-x_c(t-\tau)]-[x_w(t)-x_w(t-\tau)]$ (2)

$$\Delta y = [y_c(t) - y_c(t-\tau)] - [y_w(t) - y_w(t-\tau)] \tag{3}$$

Here $f=v\tau$ is the nominal feed per revolution, $h_s = f\cos\psi$ is static chip thickness, $x_c(t)$, $y_c(t)$, $x_w(t)$ and $y_w(t)$ are the cutter and workpiece displacements in the current pass and $x_c(t-\tau)$, $y_c(t-\tau)$, $x_w(t-\tau)$, $y_w(t-\tau)$ are that in the previous pass along x and y directions respectively. And τ is the time delay required for one spindle revolution.

Referring to Fig.2. the cutting forces along x and y directions can be resolved into:

$$F_x = -F_f\cos\psi + F_r\sin\psi \tag{4}$$

$$F_y = F_f\sin\psi + F_r\cos\psi \tag{5}$$

where F_r and F_f are components of cutting forces in directions perpendicular to cutting plane. These are given by:

$$F_f = \frac{K_f}{\cos\psi} b\{h(t)\}^{\alpha} \tag{6}$$

$$F_r = \frac{K_r}{\cos\psi} b\{h(t)\}^{\beta} \tag{7}$$

Here K_f, α and K_r, β are the empirically determined cutting coefficients in feed and radial directions. Substituting h(t) and linearizing the forces about h_s the following equations are obtained:

$$F_f = \frac{K_f}{\cos\psi} b[h_s^{\alpha} + \alpha\, h_s^{\alpha-1}(\Delta x\cos\psi - \Delta y\sin\psi)] \tag{8}$$

$$F_r = \frac{K_r}{\cos\psi} b[h_s^{\beta} + \beta h_s^{\beta-1}(\Delta x\cos\psi - \Delta y\sin\psi)] \tag{9}$$

Since static chip-thickness does not contribute to the regeneration, the first terms in equations (8) and (9) can be eliminated.

Now resolving the forces F_f and F_r along x and y directions:

$$F_x = -F_f\cos\psi + F_r\sin\psi = \frac{b}{\cos\psi}\left[(p\Delta y + q\Delta x)\sin\psi\cos\psi - \left(p\Delta x\cos^2\psi + q\Delta y\sin^2\psi\right)\right] \tag{10}$$

$$F_y = F_f\sin\psi + F_r\cos\psi = \frac{b}{\cos\psi}\left[(p\Delta x - q\Delta y)\sin\psi\cos\psi - \left(p\Delta y\sin^2\psi + q\Delta x\cos^2\psi\right)\right] \tag{11}$$

where $p = K_f\,\alpha\, h_s^{\alpha-1}$ and $q = K_r\,\beta h_s^{\beta-1}$ are considered for convenience.

These equations can be written in matrix form as follows:

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = b\frac{1}{\cos\psi}\begin{bmatrix} -\cos\psi & \sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}\begin{Bmatrix} K_f\,\alpha h_s^{\alpha-1} \\ K_r\beta h_s^{\beta-1} \end{Bmatrix}\begin{bmatrix} \cos\psi & -\sin\psi \end{bmatrix}\begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix} \tag{12}$$

This can also be written compactly as:

$$\{F\} = b\,[A]\,\{\Delta d\} \tag{13}$$

where $\{\Delta d\}=[\Delta x \quad \Delta y]^T$ is dynamic displacement vector and [A] is directional coefficient matrix defined as:

$$[A] = \frac{1}{\cos\psi}\begin{bmatrix} -\cos\psi & \sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}\begin{Bmatrix} K_f\,\alpha h_s^{\alpha-1} \\ K_r\beta h_s^{\beta-1} \end{Bmatrix}\begin{bmatrix} \cos\psi & -\sin\psi \end{bmatrix} \tag{14}$$

2.2. Equations of motion of the system

With reference to the notation in Fig.2.5, the dynamic equations of motion of the cutting tool and workpiece in x and y directions can be written as:

$$m_c\ddot{x}_c + c_x\dot{x}_c + k_x x_c = -F_x \tag{15}$$

$$m_w\ddot{y}_c + c_y\dot{y}_c + k_y y_c = -F_y \tag{16}$$

$$m_w\ddot{x}_w + c_x\dot{x}_w + k_x x_w = -F_x \tag{17}$$

$$m_w\ddot{y}_w + c_y\dot{y}_w + k_y y_w = -F_y \tag{18}$$

In terms of natural frequencies, damping ratios and stiffness coefficients, they can be arranged as

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{Bmatrix} \ddot{x}_c \\ \ddot{y}_c \\ \ddot{x}_w \\ \ddot{y}_w \end{Bmatrix} + \begin{bmatrix} 2\xi_{cx}\omega_{ncx} & 0 & 0 & 0 \\ 0 & 2\xi_{cy}\omega_{ncy} & 0 & 0 \\ 0 & 0 & 2\xi_{wx}\omega_{nwx} & 0 \\ 0 & 0 & 0 & 2\xi_{wy}\omega_{nwy} \end{bmatrix}\begin{Bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{x}_w \\ \dot{y}_w \end{Bmatrix}$$

$$+\begin{bmatrix} \omega_{ncx}{}^2 & 0 & 0 & 0 \\ 0 & \omega_{ncy}{}^2 & 0 & 0 \\ 0 & 0 & \omega_{nwx}{}^2 & 0 \\ 0 & 0 & 0 & \omega_{nwy}{}^2 \end{bmatrix}\begin{Bmatrix} x_c \\ y_c \\ x_w \\ y_w \end{Bmatrix} = \begin{Bmatrix} -\omega_{ncx}^2 F_x / k_x \\ -\omega_{ncy}^2 F_y / k_y \\ -\omega_{nwx}^2 F_x / K_x \\ -\omega_{nwy}^2 F_y / K_y \end{Bmatrix} \tag{19}$$

Time-domain solution is obtained by solving these set of coupled delay-differential equations using revised Runge-Kutta fourth order method with all initial conditions taken as zero.

2.3. Stability analysis

The stability states are obtained in frequency-domain using oriented transfer approach. By taking Laplace transforms on both sides of eq. (13), we get:

$$F(s) = b\,[A]\,(1–e^{-s\tau})\,D(s) \tag{20}$$

Considering the transfer function matrix [G(s)] of the system, eq. (20) can be written as:

$$F(s) = b\,[A]\,(1–e^{-s\tau})\,[G(s)]\,F(s) \tag{21}$$

Substituting $s = j\omega$, and simplifying:

$$\{[I] – b\,[A]\,(1\ e^{-\,j\omega\tau})\,[G(j\omega)]\}F(j\omega) = 0 \tag{22}$$

This equation has non-trivial solution if and only if its determinant is zero, yielding:

$$\det\{[I] – \lambda[A][G(j\omega)]\}=0 \tag{23}$$

where the complex eigenvalue

$$\lambda = b(1\ e^{-\,j\omega\tau}) \tag{24}$$

By ignoring the cross terms $G_{xy}(j\omega)$ and $G_{yx}(j\omega)$, the transfer matrix is given by $[G(j\omega)] = \begin{bmatrix} G_{xx}(j\omega) & 0 \\ 0 & G_{yy}(j\omega) \end{bmatrix}$ and the eigenvalue is obtained as

$$\lambda = \frac{-\cos\psi}{(p\cos^2\psi - q\sin\psi\cos\psi)G_{xx} + (p\sin^2\psi + q\sin\psi\cos\psi)G_{yy}} \tag{25}$$

where $p = K_f\,\alpha\,h_s^{\alpha-1}$ and $q = K_r\,\beta h_s^{\beta-1}$ (26)

Writing λ as:

$$\lambda = \lambda_R + \lambda_I = b(1\ e^{-\,j\omega\tau}) = b(1-\cos\omega\tau) + j\,b\sin\omega\tau \tag{27}$$

Then phase shift $\kappa = \dfrac{\lambda_I}{\lambda_R}$ can be written as

$$\frac{\lambda_I}{\lambda_R} = \frac{\sin\omega\tau}{1-\cos\omega\tau} = 1/\tan\frac{\omega\tau}{2} = \tan\left(\frac{\pi}{2} + n\pi - \frac{\omega\tau}{2}\right) \tag{28}$$

This gives the critical values of τ as

$$\tau=\frac{2}{\omega}\left\{\left(n+\frac{1}{2}\right)\pi-a\tan\left[\frac{\lambda_I}{\lambda_R}\right]\right\}\text{ where } n = 0, 1, 2\ldots \tag{29}$$

The corresponding critical values of b is given by

$$b^* = \frac{\lambda_R}{1-\cos\omega\tau} \tag{30}$$

By writing spindle speed in rpm as N = 60/τ, stability lobe diagram can be plotted between the values of b* and N for different values of n. The second order delay differential equation in time-domain are solved at a specific testing depth of cut and feed conditions using a revised fourth order Runge-Kutta method. To determine each time-domain solution point, the closed-loop machining system is simulated with a fixed set of process parameters for several revolutions.

3. EXPERIMENTAL ANALYSIS

In order to verify the analytical models, several tests are conducted to collect the required data through measurements which are done before, during and after the cutting tests. In the first two cases of experiments dynamic cutting forces are only measured in order to know the effects of workpiece flexibility and nonlinear force feed respectively on the cutting stability. In the third case, influence of secondary parameters like tool overhang length and flank wear on the cutting dynamics are studied by measuring the static cutting force data, workpiece surface roughness, tool wear as well as critical chatter lengths.

3.1. Effect of Feed on Stability

Feed influences cutting process to a great extent at higher speeds. In order to study this effect, experiments are conducted on the same 7.5 KW engine lathe by varying feed conditions also. However in this case the work-piece is not supported by tailstock and the same carbide inserts are selected but it is laid at an angle to simulate the oblique cutting condition. Along with feed, the cutting parameters are speed and depth of cut. Fig.3 shows the experimental set-up used to know the feed effect on the stability of the turning process. The selected ranges of these parameters are depicted in Table 1.

Table 1. Cutting conditions in experiment

Speed(RPM)	360	770	1200
DOC(mm)	0.25	0.75	1.5
Feed(mm/rev)	0.05	0.1	0.2

Fig.3 Experimental set-up for finding feed effect on stability

The dynamic cutting forces are measured with dynoware software similar to earlier case. Care is taken to avoid variations in set value of depth of cut, by first giving a prior cut on the surface. The X, Y, Z force sensors are very sensitive and must be calibrated prior to the operation. In this experiment speed and feed are set by adjusting mechanisms to the preset values.

4. RESULTS AND DISCUSSION

4.1. Nonlinear Force Feed in Oblique Cutting Conditions

In this section the results of nonlinear force feed analysis in turning operation are explained. First the results of the proposed analytical models are presented in frequency and time domain.

4.2. Analytical solutions

Simulation studies are conducted to investigate the nonlinear force feed effects on chatter. The cutting and material parameters used in the simulation are given in Table 2.

Table 2. Cutting and material data

Cutting tool	Work-piece
Natural frequency $\omega_{nxc} = \omega_{nyc} = 1100$ Hz	Modulus of elasticity $E = 2.1 \times 10^7$ N/m^2
Stiffness $k_x = k_y = 1.2 \times 10^7$ N/m^2	Density $\rho = 7800$ Kg/m^3
Damping ratio = $\zeta_x = \zeta_y = 0.015$	Diameter d = 39 mm
Cutting coefficients $K_f = 800$ MPa, $K_r = 128$ MPa	Length L = 75 mm
Side cutting edge angle $\psi = 10°$ Normal rake angle = 5° Inclination angle 5° Nose radius = 0.4 mm	Average damping ratio $\zeta_{xw} = \zeta_{yw} = 0.025$

The axial(x) and bending(y) stiffness and corresponding natural frequencies (ω_{nxc}, ω_{nyc}) of the work are computed by considering cantilever boundary conditions. That is: K_x=EA/L and K_y=3EI/L^3 and ω_{nwx} or $\omega_{nwy}=\sqrt{\frac{K_x \text{ or } K_y}{\rho AL}}$. Figure 4 shows the stability lobe diagram at three different values of feed rates. It can be seen that the stable depths of cut increase with feed at all the spindle speeds. Here the nonlinear force feed indices in radial and feed directions (α and β) are both selected as 0.75. That is $F_r = 128bf^{0.75}$ and $F_f = 800bf^{0.75}$.

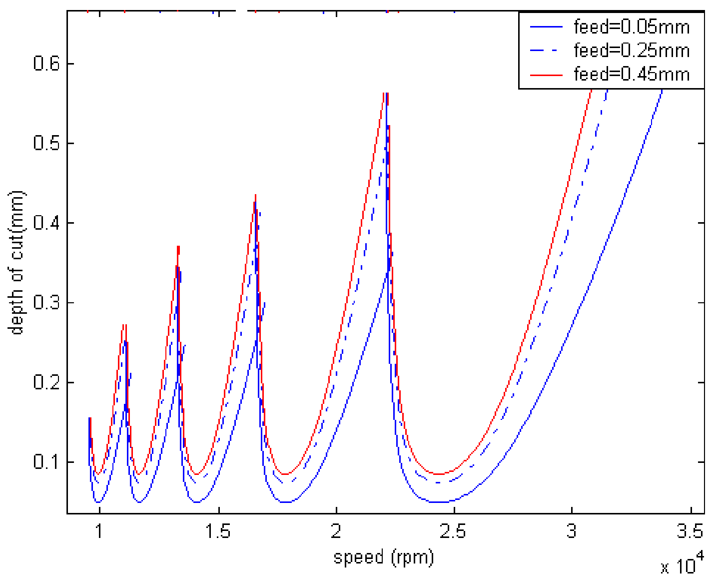


Fig.4 Stability lobe diagram at different feed values

The same lobe diagram at lower spindle speeds (Larger lobe index n) for two different feed values is shown in Figure 5.

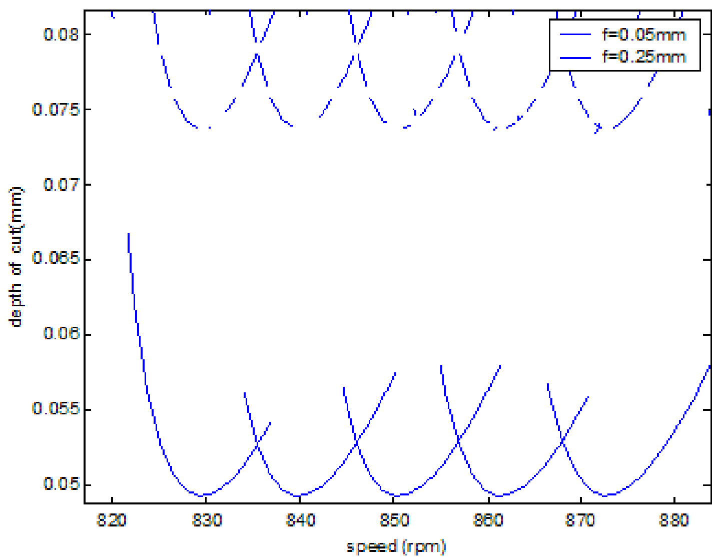


Fig. 5 Stability lobe diagram at different feeds at lower spindle speeds

Fig.6 shows the variations of depth of cut for different feed values as the value of ω (chatter frequency) is swept in each lobe. At lower values of ω, very high values of depths of cuts are noticed. These values of ω correspond to the natural frequencies

(≅1100 Hz) of the tool. A computer program is implemented in MATALB for obtaining time-domain solutions of the dynamic equations of motion using revised Runge-Kuttas' method. Here, in addition to the cutting and material data, the program requires some initial conditions of previous (y) and next previous (z) values of displacements and velocities of the cutting tool and work-piece.

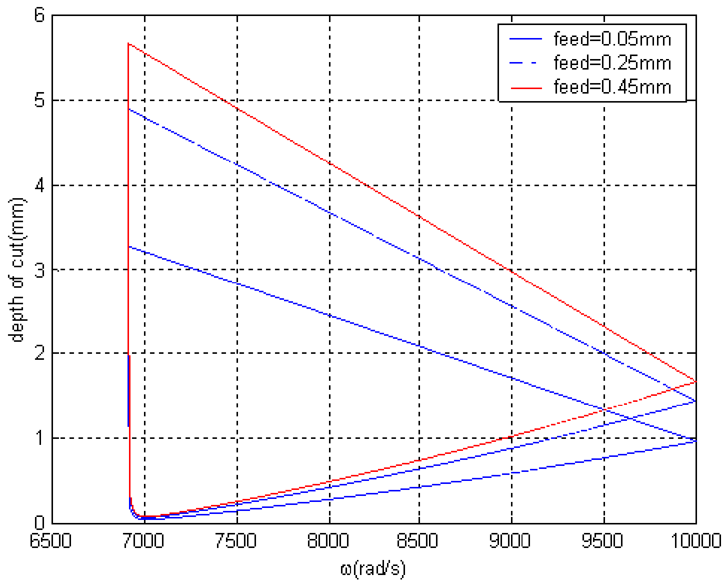
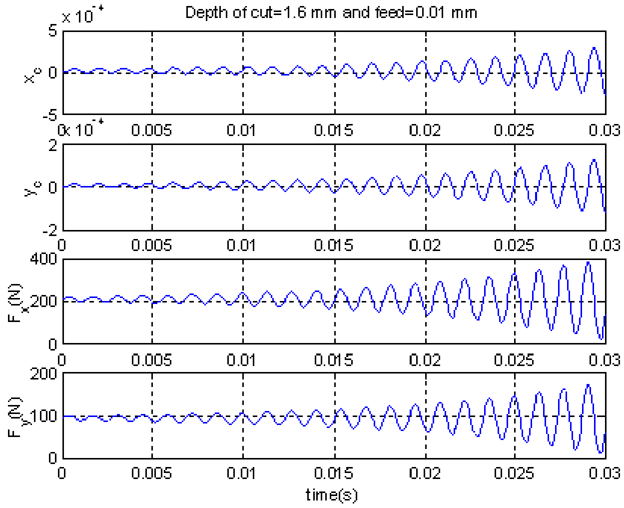


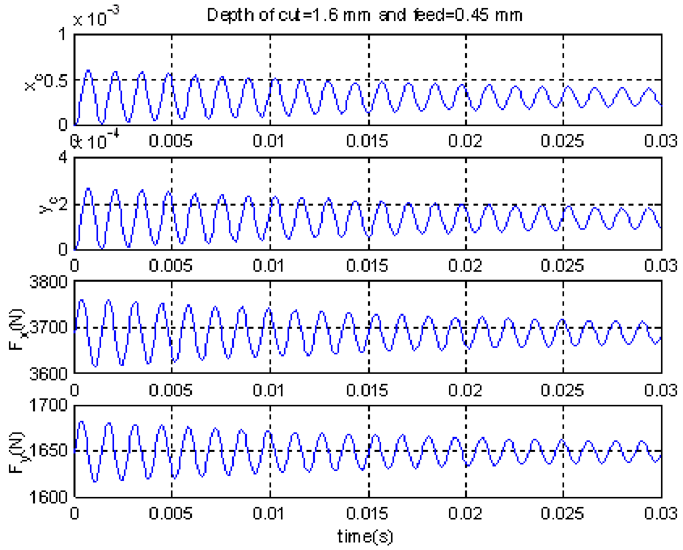
Fig.6. Depth of cut Vs chatter frequency ω

The width of lobe has increased at higher speeds due to this nonlinear force-feed effect.

Altogether, 8 first order coupled delay-differential equations are to be solved simultaneously. The outputs of the program at different cutting states are depicted in Fig.7. The results show that even the depth of cut has more influence on the stability, the feed also affects considerably at higher depths of cut. Figures also show the cutting forces F_x and F_y directions.



(a) Unstable Cut



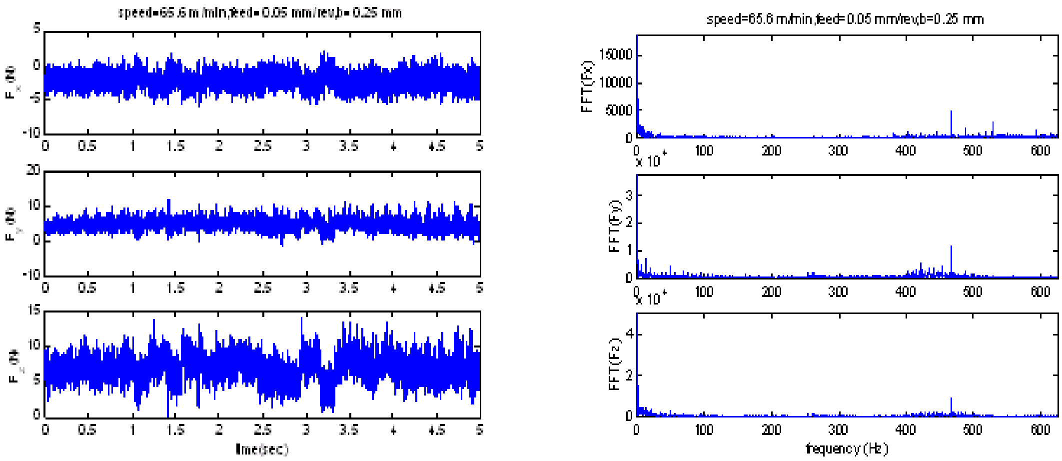
(b) Stable cut

Fig.7. Time-domain solutions

4.3 Experimental analysis

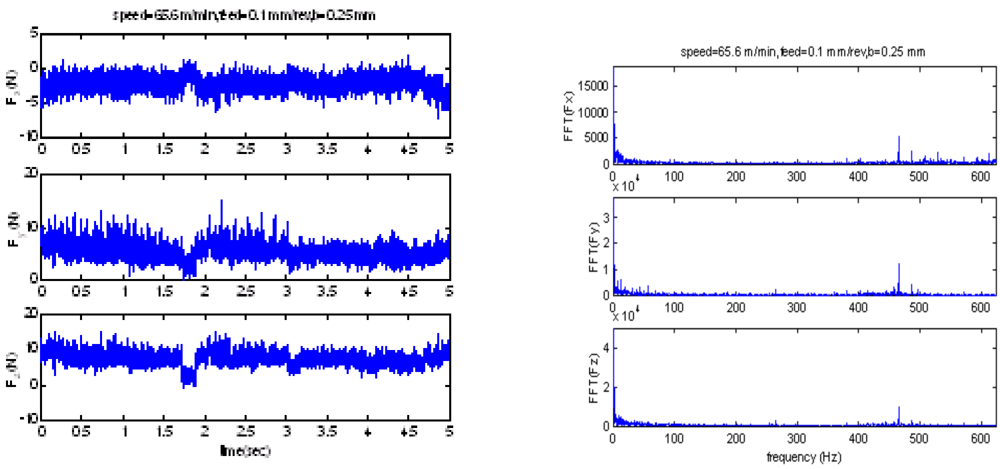
In order verify the influence of feed in practical conditions, experiments are carried-out at different feed values as described in Table.1. Three levels of spindle speeds (360,770 and 1200 rpm), feed rates (0.05, 0.1 and 0.2mm/rev) and depth of cuts (0.25, 0.75 and 1.5mm) are utilized resulting in a combination of 27 experiments. AISI 1045 work pieces of 60 mm diameter are employed and tungsten carbide (WC) tool insert is utilized for oblique cutting. Chatter instability is determined by examining the force histories and corresponding force spectra. Force spectra are obtained from the conventional Fast Fourier Transformations with a sampling frequency of 10 kHz.

Two cases of the stable turning operation are shown in Fig.8 (a) and (b). From these figures it can be seen that there is no high frequency variations in the force plots. For example in the force spectrum plot shown in Fig.4.8 (a), a significant energy occurs at 465 Hz and 520 Hz in the F_x signal, 490 Hz in F_y signal and 490 Hz in F_z signal. The frequency-domain plots show constant energy through out all the three force signals. The operation can be considered as stable.



(i) Time-domain (ii) Frequency domain

(a) spindle speed= 360 rpm, feed = 0.05mm/rev and depth of cut= 0.25 mm

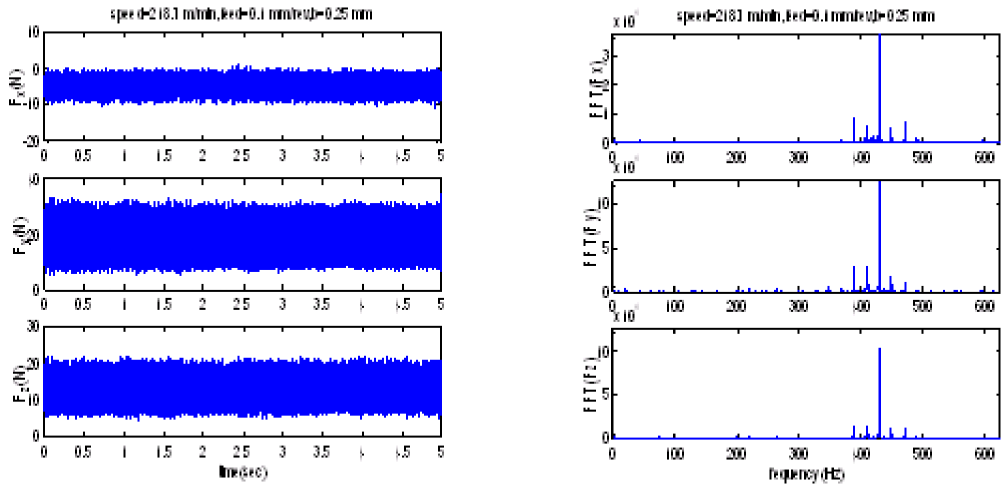


(i) Time-domain (ii) Frequency domain

(b) Spindle speed= 360 rpm, feed = 0.1mm/rev and depth of cut= 0.25 mm

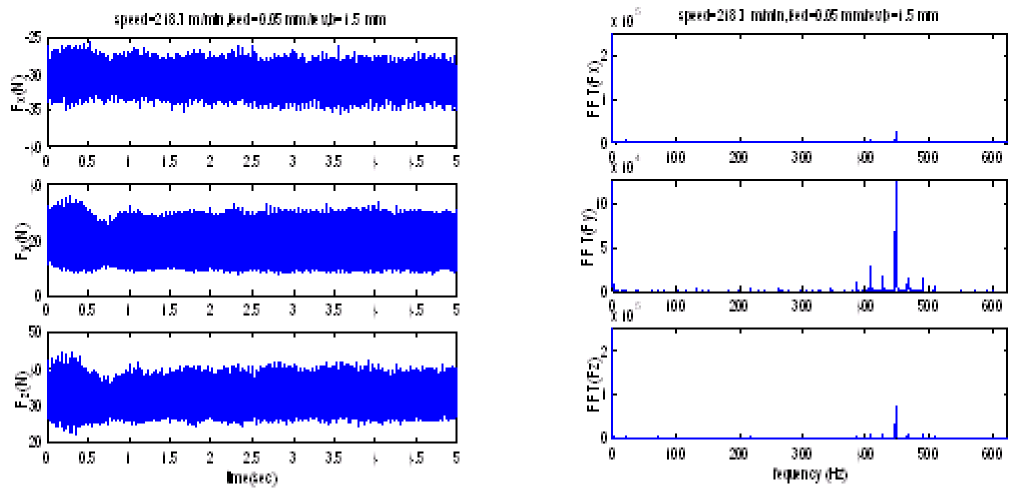
Fig. 8. Stable cutting operations

Two cases of the unstable turning operation are shown in Fig.9 (a) and (b). Here high frequency variations are evident. The frequency spectra show a significant energy in all the three force signals at 430 Hz. In these cases chatter instability is present. In addition, they have smaller peaks surrounding the extreme one.



(i) Time-domain (ii) Frequency domain

(a) spindle speed= 1200 rpm, feed = 0.1mm/rev and depth of cut= 0.25 mm



(i) Time-domain (ii) Frequency domain

(b) Spindle speed= 1200 rpm, feed = 0.05mm/rev and depth of cut= 1.5 mm

Fig. 9 Unstable cutting operations

The results of all 27 experiments are shown in Table.3. It can be seen that chatter is present at all the depths of cut under lower feeds. As the feed increases at higher depths of cut only chatter is noticed. This shows the feed affects dramatically the presence of chatter.

Table 3. Experimental results of oblique turning.

Spindle speed (RPM)	Depth of cut (mm)	Chatter condition (Yes/No)		
		f=0.05	f=0.1	f=0.2
360	0.25	No	No	No
770	0.25	No	Yes	No
1200	0.25	Yes	Yes	No
360	0.75	No	No	No
770	0.75	Yes	No	No
1200	0.75	Yes	No	No
360	1.5	Yes	Yes	No
770	1.5	Yes	Yes	Yes
1200	1.5	Yes	Yes	Yes

5. CONCLUSIONS

In this paper Effects of nonlinear force-feed conditions in chatter stability of turning operation have been presented as a second case study. Cutting forces over the turning insert in radial and feed directions are expressed in terms of deformations of work and tool in current and previous time steps. Both time domain and frequency domain studies were conducted. Conditions of instability have been reported in terms of feed variations. It is observed that in all the cases the critical depth of cut increase with feed. Practically an increase in feed minimizes the cycle time, but machining forces also increase with feed. So this cannot be used to suppress chatter. Due to the inclusion of feed in chatter relations, it is possible to calibrate the force models at any feed. In practical conditions, the cutting coefficients also change with feed. The influence is ignored in the analytical model.

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