

## Construction of Normal Imprecise Functions

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### Abstract

To study normal imprecise functions we identify their basic properties and some of the functions are multiplied by sine and cosine function to convert them into normal imprecise functions. Also we study the conversion points and the diversion points of the linear and quadratic polynomial functions and are presented with the graphical examples. Further we define rate of effect of the impreciseness value of different imprecise functions so that it can be identify whether the function is controllable or not.

**Keywords:** Imprecise number, normal imprecise function, diversion point, conversion point, rate effect of impreciseness, controllable function, uncontrollable function, designed, undesigned.

### I. INTRODUCTION

Generally function is a very important in the field of mathematics. In the certain time bound, if we represent graphically the path of activities of any object, then will have a mathematical function. Some of them may be either controllable or uncontrollable. Those activities which form a controllable function are usable for our practical purpose but there are many other activities of the objects which are uncontrollable till today, so we require to investigate about the character of such objects. As for example, the path covered by a man or any insect walking on the ground from one place to another place with respect to the time bound, form a mathematical function. Similarly if we draw the graph of the changing of temperature for a particular day from morning to night is also

form a function. Area covered during the light travelling from one place to another can be represented by a function, which can be seen and available now a day in an auditorium for dance competition with mix up of glamour color of lights are spray to bring out surprise to audience.

The graphs, which are floating freely without meeting the ground or the real line repeatedly can be controlled or can be meet the ground level repeatedly with the help of multiplication of some multiplication factor. The resulting function is known as normal imprecise function. It has many applications in the field of Design technology, Instrumentation Engineering related to signal and sound system etc. For example ultrasonic sound is a controllable sound whose effect of wave form a function that can be controlled by our ears since it has the frequency range up to 20,000 Hz such type of sounds are generally known as audible sound or ultrasonic sound. Such sounds are controllable due to the presence of conversion point obtained by the multiplication of small integral multiple of the angles in the sine and cosine functions. Supersonic sounds are uncontrollable sound whose effect of the sound wave form a function which are uncontrollable by our ears since the frequency range of such sounds are more than 20,000 Hz so we called such sounds are inaudible sound. It is due to the presence of conversion point obtained by the large integral multiple of the angles in the sine and cosine functions. So our aim in this article is to control the uncontrollable or un-designed function into controllable or designed function with the multiplication of suitable multiplication factors of the functions like sine and cosine so that it can be controlled to use in our practical purposes. These activities are presently applied in some common equipment like the regulator of a volume of speaker, which controls the loudness of sound. Here the supersonic sound can be minimized to audible range as per our needs. Lights coming from a device having high range, which are not saturated for eye, can be converted into eye-saturated level with the help of regulators. In both the phenomena we obtain controllable level by changing conversion point from very immediate or short distance to slowly or longer distance.

Normal imprecise function is a controllable function where the sections or the area bounded by all the half periods of the function always forms a normal imprecise number having maximum and minimum values within this period. Where the imprecise number is an interval definable number having special characters are discussed in the preliminary section.

## **II. PRELIMINARIES**

Before starting this article it is necessary to recall the definition of imprecise number, partial presence, membership value etc. over the real line which are discussed as in the article of Baruah [3],[4], [5], [6]. Borgoyary [9] also defined the same definitions in their article to apply in the some other fields of mathematics.

**A. Imprecise Number:** Imprecise number  $N = [\alpha, \beta, \gamma]$  is divided into closed sub-intervals with the partial presence of element  $\beta$  in both the intervals.

**2.2 Partial presence:** Partial presence of an element in an imprecise number  $N = [\alpha, \beta, \gamma]$  is described by the present level indicator function  $p(x)$  which is counted from the reference function  $r(x)$  such that present level indicator for any  $x$ ,  $\alpha \leq x \leq \gamma$ , is  $(p(x) - r(x))$ , where  $0 \leq r(x) \leq p(x) \leq 1$

Here  $p(x)$  is the highest level reachable function obtained by the impreciseness of object which is discussed in the article [4], [5], [6]. Where the impreciseness is the membership function obtained by an object in the extension definition of fuzzy set. Impreciseness of the object is always formed an imprecise number in a certain interval.

**B. Membership value:** If an imprecise number  $N = [\alpha, \beta, \gamma]$  is associated with a presence level indicator function  $\mu_N(x)$ , where

$$\mu_N(x) = \begin{cases} \psi_1(x), & \text{when } \alpha \leq x \leq \beta \\ \psi_2(x), & \text{when } \beta \leq x \leq \gamma \\ 0, & \text{otherwise} \end{cases}$$

With a constant reference function 0 in the entire real line. Where  $\psi_1(x)$  is continuous and non-decreasing function or the membership function in the interval  $[\alpha, \beta]$ , and  $\psi_2(x)$  is a continuous and non-increasing or the reference function in the interval  $[\beta, \gamma]$  with

$$\begin{aligned} \psi_1(\beta) &= \psi_2(\beta) \\ \psi_1(\alpha) &= \psi_2(\gamma) = 0, \end{aligned}$$

then  $(\psi_1(x) - \psi_2(x))$  is called membership value for any  $x$ , which is discussed in the article [5] section 2.

**C. Normal Imprecise Number:** A normal imprecise number  $N = [\alpha, \beta, \gamma]$  is associated with a presence level indicator function  $\mu_N(x)$ , where

$$\mu_N(x) = \begin{cases} \psi_1(x), & \text{when } \alpha \leq x \leq \beta \\ \psi_2(x), & \text{when } \beta \leq x \leq \gamma \\ 0, & \text{otherwise} \end{cases}$$

With a constant reference function 0 in the entire real line. Where  $\psi_1(x)$  is continuous and non-decreasing or membership function in the interval  $[\alpha, \beta]$  and  $\psi_2(x)$  is a continuous and non-increasing or reference function in the interval  $[\beta, \gamma]$  with

$$\psi_1(\alpha) = \psi_2(\gamma) = 0$$

$$\psi_1(\beta) = \psi_2(\beta) = 1$$

Here, the imprecise number would be characterized by  $\{x, \mu_N(x), 0 : x \in R\}$ ,  $R$  being the real line.

For any real line,  $0 \leq \psi_1(x) \leq \psi_2(x) \leq 1$  normal and subnormal imprecise number will be characterized in common,  $\{x, \psi_1(x), \psi_2(x) : x \in R\}$ , where  $\psi_1(x)$  is called membership function measured from the reference function  $\psi_2(x)$  and  $\psi_1(x) - \psi_2(x)$  is called the membership value of the indicator function.

Here, the number is normal imprecise number when membership value of indicator function  $\mu_N(x)$  is equal to 1 otherwise subnormal if it is less than 1. Moreover it can be the universal set if the membership value of  $\mu_N(x)$  is equal to 1, and null or empty set if  $\mu_N(x)$  is equal to 0.

### III. NORMAL IMPRECISE FUNCTION

**A Definition:** If the function is sectionally dividable into finite number of imprecise numbers having for each interval is obtainable of distinct or similar maximum value point of the function, we name it normal imprecise functions. If the maximum value of the function is unique we call it periodic and if the maximum value is not unique then we call it semi periodic. However both the functions are having normal imprecise number properties for many intervals. From this point of view it can be introduced some conditions for normal imprecise functions as follows:

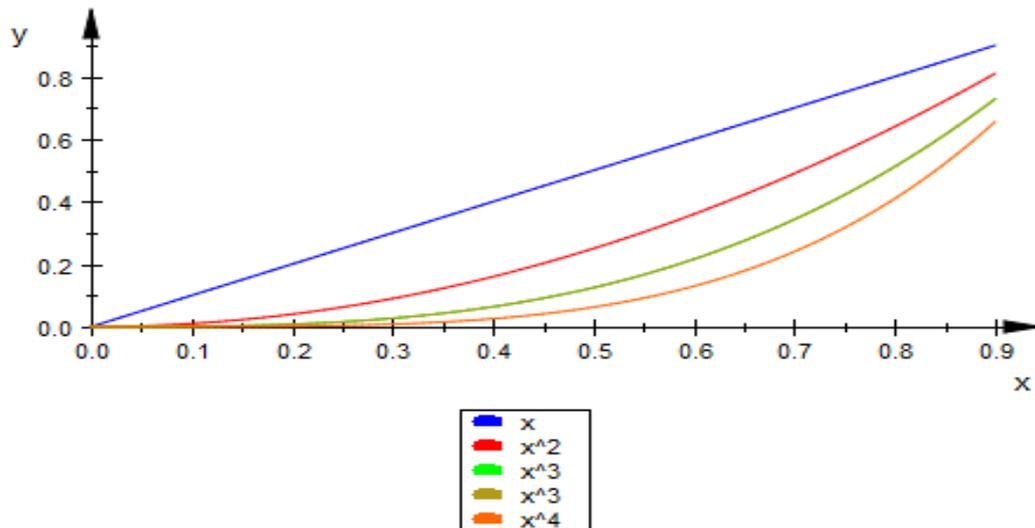
- (i) Function must be continuous
- (ii) Function must be oscillation.
- (iii) Function must have finite number of maximum and minimum values
- (iv) Function must be semi-periodic/periodic in nature

For example formation of the effect of impreciseness in the form of sine and cosine functions is interval definable numbers. As the multiplication of sine and cosine function with any other function has obtained finite number of intervals having each interval contains maximum and minimum value within in it. Here impreciseness is the membership function of an object obtained by extension definition of the Complement of fuzzy umber defined in Baruah [3]. Thus effect of the function defined for any interval can be converted into normal imprecise function with the help of multiplication factors. Here, in our article multiplication factors are the function of sine and cosine.

#### **B. Normal Imprecise functions formed by multiplication of sine function**

As we have mentioned above, any function can be obtained into normal imprecise

function with the help of sine function. For this purpose, let us consider  $f(x) = x^n, n \geq 0; n \in N$  is a continuous function formed by the effect of any light. Graph of this function has and area bounded by the regions as follows:

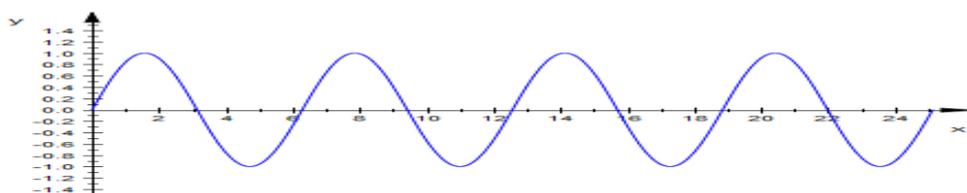


Which is not an imprecise function since it does not have maximum and minimum point. So, we call it an uncontrollable as the function does not meet the x-axis for several times.

To convert this function into imprecise function let us multiply by sine function so that it becomes as  $f(x) = x^n \sin(x), n \geq 1; n \in N$

If  $n=1$ , then for the following data we will get a graph of the normal imprecise function  $y = x \sin(x)$ ,

X	0	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
Y	0	$\frac{\pi}{2}$	0	$-3\frac{\pi}{2}$	0



Here for every imprecise numbers,  $[0, \frac{\pi}{2}, \pi], [\pi, \frac{3\pi}{2}, 2\pi], [2\pi, \frac{5\pi}{2}, 3\pi], \dots, [(n -$

$1)\pi, \frac{(2n-1)\pi}{2}, n\pi]$  are imprecise numbers whose indicator function may be defined as,

$$\mu_N(x) = \begin{cases} \psi_1(x) \text{ when } (n-1)\pi \leq x \leq \frac{(2n-1)\pi}{2} \\ \psi_2(x) \text{ when } \frac{(2n-1)\pi}{2} \leq x \leq n\pi \end{cases}; n \in Z$$

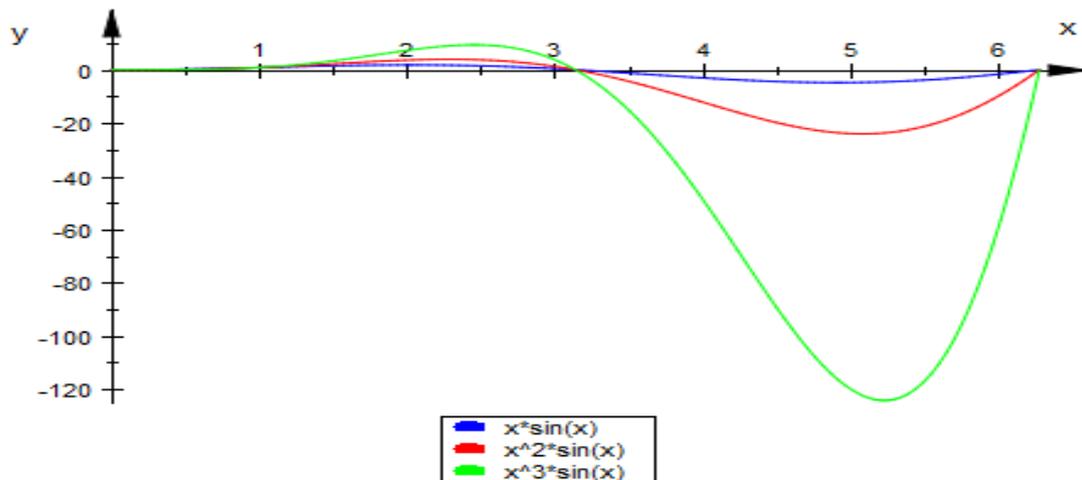
Here  $\psi_1(x)$  is a non-decreasing or membership function and  $\psi_2(x)$  is a non-increasing or reference function in the defined intervals such that

$$\psi_1(x) = 0 \text{ at } x = (n-1)\pi, \psi_2(x) = 0 \text{ at } x = n\pi$$

$$\text{and } \psi_1(x) = \psi_2(x) = \frac{(2n-1)\pi}{2} \text{ at } x = \frac{(2n-1)\pi}{2}.$$

Here the maximum value of the indicator function is  $\frac{(2n-1)\pi}{2}$  and the minimum value is 0. Where the maximum value varies for different values of n. Thus  $y = x \sin(x)$  is a normal imprecise function of oscillation.

In general  $f(x) = x^n \sin(x), n \geq 1; n \in N$  is also an oscillation function having indicator function defined above with maximum values  $\left(\frac{(2n-1)\pi}{2}\right)^n$  and minimum value as 0 for  $n \in Z$ . Thus we call it normal imprecise function having graph repeatedly cuts the x-axis for several times and is shown below.



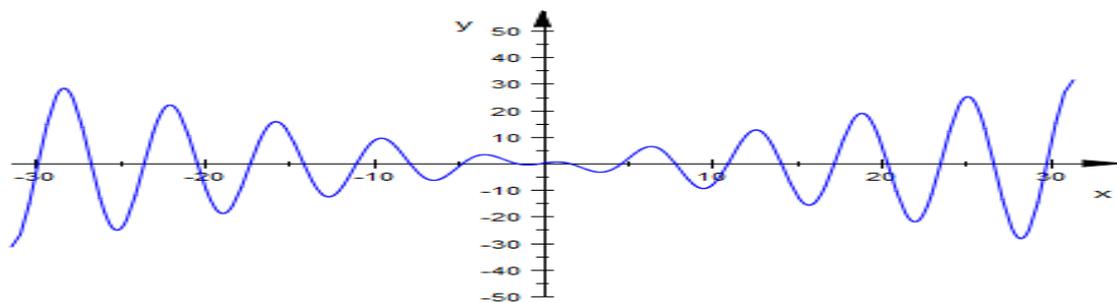
Here the curves are showing oscillation to have the imprecise functions having many imprecise numbers for the different intervals within functions.

**C. Normal Imprecise functions formed by multiplication of cosine function**

As we have mentioned above, any function can be obtained into normal imprecise function with the help of cosine function. For this purpose, let us multiply  $f(x) = x^n, n \geq 0; n \in N$  by cosine function to have  $y=x^n \cos(x), n \geq 1; n \in N$  normal imprecise function. As the function will have semi periodic in nature and is discussed below.

Let  $n=1$ , then  $y = x \cos(x)$  has a graph,

x	$-2\pi$	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
y	$-2\pi$	0	$\pi$	0	0	0	$-\pi$	0	$2\pi$



Here, every imprecise numbers,  $[\frac{\pi}{2}, \pi, \frac{3\pi}{2}], [\frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}], [\frac{5\pi}{2}, 3\pi, \frac{7\pi}{2}], \dots, [ \frac{(2n-1)\pi}{2}, n\pi, \frac{(2n+1)\pi}{2} ]$  are imprecise numbers whose indicator function may be defined as,

$$\mu_N(x) = \begin{cases} \psi_1(x) & \text{when } \frac{(2n-1)\pi}{2} \leq x \leq n\pi \\ \psi_2(x) & \text{when } n\pi \leq x \leq \frac{(2n+1)\pi}{2} \end{cases}; n \in Z$$

Here  $\psi_1(x)$  is a non-decreasing or membership function and  $\psi_2(x)$  is a non-increasing or reference function in the defined intervals such that

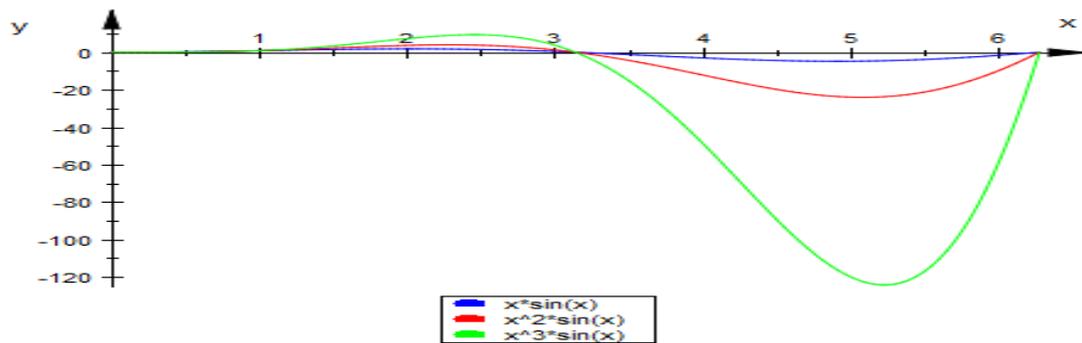
$$\psi_1(x) = 0 \text{ at } x = \frac{(2n-1)\pi}{2}, \quad \psi_2(x) = 0 \text{ at } x = \frac{(2n+1)\pi}{2}$$

$$\text{and } \psi_1(x) = \psi_2(x) = n\pi \text{ at } x = n\pi .$$

Here the maximum value of the indicator function is  $n\pi$  and the minimum value is 0 for  $n \in \mathbb{Z}$ . Where the maximum value varies for different values of  $n$ . Thus  $y = x \cos(x)$  is a normal imprecise function of oscillation.

In general  $f(x) = x^n \cos(x)$ ,  $n \geq 1$ ;  $n \in \mathbb{N}$  is also an oscillation function having indicator function defined above with maximum value as  $(n\pi)^n$  and minimum value as 0 for  $n \in \mathbb{Z}$ . Thus we call it normal imprecise function.

Thus we call it normal imprecise function having graph repeatedly cuts the x-axis for several times and is shown below.



Here the curves are showing oscillation to have the imprecise functions having many imprecise numbers for the different intervals within functions.

#### IV. CONVERSION POINT

The main vision of our article is to control the un-controllable functions. However the transformation of controllable form of the function is started from a certain point which we call conversion point. So, it is a point from where non- imprecise function starts to convert into imprecise function. If we can identify this point for an object, then we will get easily their applications by converting the functions into different shapes according to our needs. As for example loudness of sound causes brain and heart diseases because of non-imprecise function character produces from a device. So, it may be possible to control by converting this into human ear audible level. Further ultraviolet ray reach in the ground is non-imprecise form of function due to lake of friction of Ozone layer cause by the present environmental pollutions. This phenomenon is creating problems

in the life of living being causing skin disease etc. Thus if can invent a new device having Ozone layer character, then it can be saved us form such problems and it may be possible by using a multiplication factor which can be converted into imprecise function. Normally these phenomena are already applied in the some common equipments and other experiments like volume regulator and in other types of regulators, which are used for different purposes in different fields.

**A. Conversion points obtained by sine function**

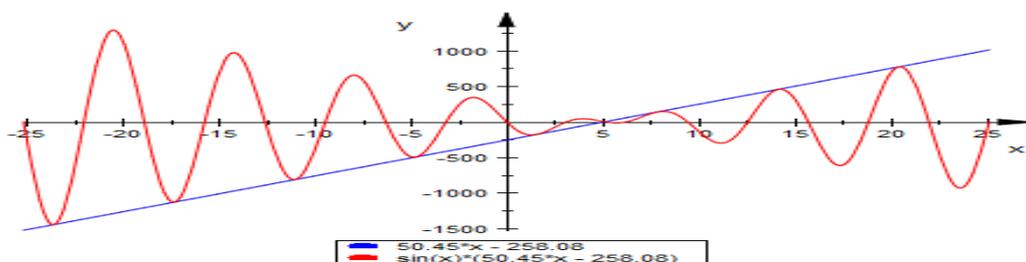
From the figure 1 observe that conversion point during the normal imprecise function obtained by the multiplication of sine function have different conversion points for different values of n. Thus we have, If  $n = 1, y = x$  start to oscillate by multiplication of  $\sin(x)$  to form an imprecise function  $y = x \sin(x)$  from the point  $(\frac{\pi}{2}, \frac{\pi}{2})$  along the positive x-axis.

If  $n = 2, y = x^2$  start to oscillate by multiplication of  $\sin(x)$  to form an imprecise function  $y = x^2 \sin(x)$  from the point  $(\frac{\pi}{2}, (\frac{\pi}{2})^2)$  along the positive x-axis.

If  $n = 3, y = x^3$  start to oscillate by multiplication of  $\sin(x)$  to form an imprecise function  $y = x^3 \sin(x)$  from the point  $(\frac{\pi}{2}, (\frac{\pi}{2})^3)$ , along the positive x-axis and so on.

Thus in general it can be obtained  $(\frac{\pi}{2}, (\frac{\pi}{2})^n)$  is a conversion point of the function  $y = x^n$  with respect to  $\sin(x)$  along the positive x-axis.

In particular, for a linear polynomial normal imprecise function  $y = (-258.08 + 50.45x) \sin(x)$  has the conversion points as shown in the below figure.



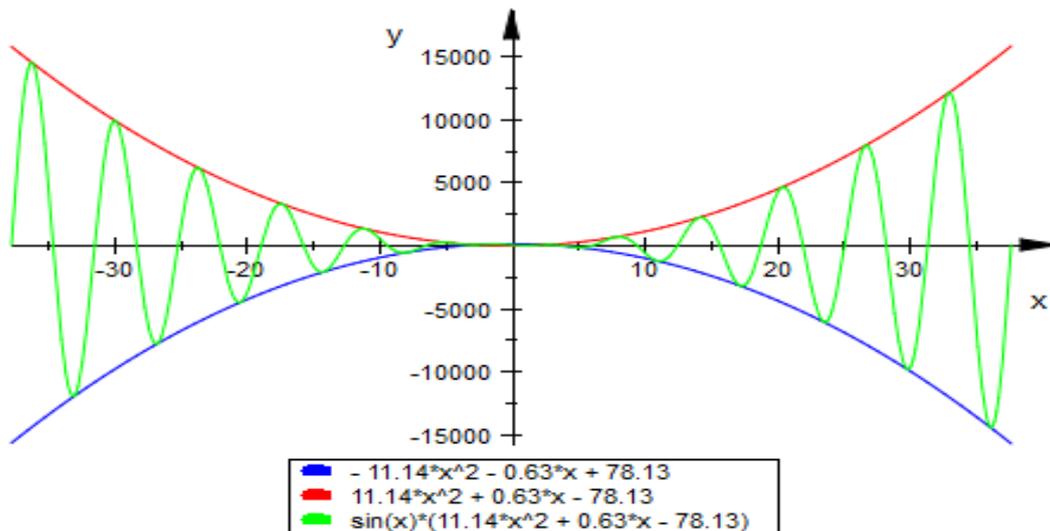
**Fig.1.** Graph of  $y = -258.08 + 50.45x$  and  $y = (-258.08 + 50.45x)\sin(x)$

To defined mathematically we consider ,  $-258.08 + 50.45x = 0$ . Here,  $x = \frac{258.08}{50.45} = 5.115$  and  $\pi < 5.115 < 2\pi$ , for which  $x = \pi$  and  $2\pi$  are the nearest two values such that  $\sin(x) = 0$

So, the graph starts to oscillate from the point,  $x = \frac{5.115+2\pi}{2} \approx 5.699$ (*approx.*) along the positive x-axis and  $x = \frac{5.115+\pi}{2} \approx 4.128$ (*approx.*) along the negative x-axis which is shown in the above Fig1.

So,  $\left(\frac{5.115+2\pi}{2}, \left(-258.08 + 50.45 \left(\frac{5.115+2\pi}{2}\right)\right) \sin\left(\frac{5.115+2\pi}{2}\right)\right)$  is called conversion point along the positive x-axis.

Similarly for a quadratic curve or polynomial normal imprecise function  $y = (-78.13 + 0.63x + 11.14x^2)\sin(x)$  has the conversion points shown in the below figure,



Thus the solution is  $x = \frac{0.63 \pm \sqrt{0.63^2 + 4 \times 78.13 \times 11.14}}{2 \times 78.13} = 0.38$ (*approx.*),  $-0.37$ (*approx.*)

and  $-\pi < -0.37 < 0$  and  $0 < x < \pi$

Here, the graph starts to oscillate from the point,  $x = \frac{0.38+\pi}{2} \approx 1.76$ (*approx.*) along the positive x-axis, and  $x = \frac{-0.37-\pi}{2} \approx -1.75$ (*approx.*) along the negative x-axis which is shown in the Fig 4. So,  $\left(\frac{0.38+\pi}{2}, \left(-78.13 + 0.63 \left(\frac{0.38+\pi}{2}\right)\right) \sin\left(\frac{0.38+\pi}{2}\right)\right)$

$11.14 \left(\frac{0.38+\pi}{2}\right)^2 \sin\left(\frac{0.38+\pi}{2}\right)$  is called conversion point along the positive x-axis.

**B. Conversion points obtained by cosine function:**

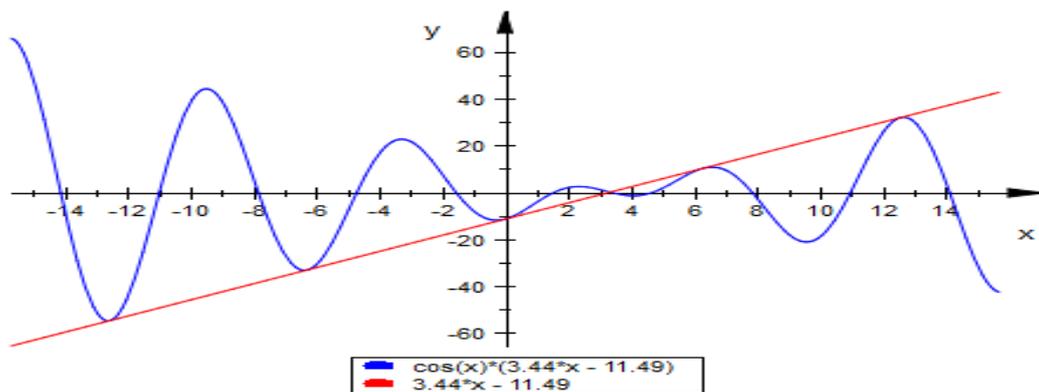
If  $n = 1, y = x$  start to oscillate by multiplication of  $\sin(x)$  to form an imprecise function  $y = x \cos(x)$  from the point  $\left(\frac{\pi}{4}, \frac{\pi}{4} \cos\left(\frac{\pi}{4}\right)\right)$  along the positive x-axis.

If  $n = 2, y = x^2$  start to oscillate by multiplication of  $\sin(x)$  to form an imprecise function  $y = x^2 \cos(x)$  from the point  $\left(\frac{\pi}{4}, \left(\frac{\pi}{4}\right)^2 \cos\left(\left(\frac{\pi}{4}\right)^2\right)\right)$  along the positive x-axis.

If  $n = 3, y = x^3$  start to oscillate by multiplication of  $\sin(x)$  to form an imprecise function  $y = x^3 \cos(x)$  from the point  $\left(\frac{\pi}{4}, \left(\frac{\pi}{4}\right)^3 \cos\left(\left(\frac{\pi}{4}\right)^3\right)\right)$ , along the positive x-axis and so on.

Thus in general it can be obtained  $\left(\frac{\pi}{4}, \left(\frac{\pi}{4}\right)^n \cos\left(\left(\frac{\pi}{4}\right)^n\right)\right)$  is a conversion point of the function  $y = x^n$  with respect to  $\sin(x)$  along the positive x-axis.

In particular, for a linear polynomial normal imprecise function  $y = (-11.49 + 3.44x)\cos(x)$  has the conversion points as shown in the below figure.



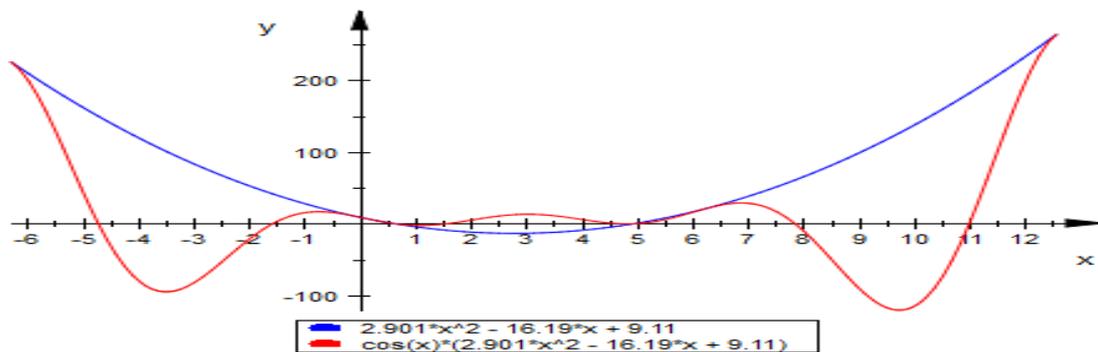
Here, the given cosine imprecise function meets the x-axis only when  $y = 0$ . For this condition we take,  $x = \frac{11.49}{3.44} = 3.340$  and  $\frac{\pi}{2} < 3.340 < \frac{3\pi}{2}$ , for which  $x = \frac{\pi}{2}$  and  $\frac{3\pi}{2}$  are the nearest two values such that  $\cos(x) = 0$

So, the graph starts to oscillate from the point,  $x = \frac{3.340 \times 2 + 3\pi}{4} \approx 4.067(\text{approx.})$  along the positive x-axis and  $x = \frac{3.340 \times 2 + \pi}{4} \approx 2,456(\text{approx})$ . along the negative x-axis which is shown in the Fig1.

So,  $\left(\frac{3.340 \times 2 + 3\pi}{4}, \left(-11.49 + 3.44 \left(\frac{3.340 \times 2 + 3\pi}{4}\right)\right) \cos\left(\frac{3.340 \times 2 + 3\pi}{4}\right)\right)$  is called conversion point along the positive x-axis.

Thus, the given function  $y = (-11.49 + 3.44x)\cos(x)$  cuts the real line X-axis repeatedly to meet the ground level again and again. So, it is a cosine imprecise function.

Similarly for a quadratic curve or polynomial normal imprecise function  $y = (9.11 - 16.19x + 2.901x^2)\cos(x)$  has the conversion points as shown in the following,



Here, the given cosine imprecise function meets the x-axis only when  $y = 0$ . For this condition we take the solution of the given polynomial as follows-

$$x = \frac{-(-16.19) \pm \sqrt{(-16.19)^2 + 4 \times 9.11 \times 2.901}}{2 \times 2.901} = 4.94(\text{approx.}), 0.63(\text{approx.})$$

We observe that  $-\frac{\pi}{2} < 0.63$  and  $4.94 < \frac{5\pi}{2}$  for which  $x = -\frac{\pi}{2}$  and  $\frac{5\pi}{2}$  are the nearest two values of the given solution of the above polynomial such that  $\cos(x) = 0$

Here the given polynomial will start to oscillate from the average value of the respective interval along the part of the axes. So, the graph starts to oscillate from the point,

$x = \frac{4.94 \times 2 + 5\pi}{4} \approx 6.40(\text{approx})$  along the positive x-axis and  $x = \frac{0.63 \times 2 - \pi}{4} \approx -0.468(\text{approx})$  along the negative x-axis which is shown in the Fig 4.

So,  $\left(\frac{4.94 \times 2 + 5\pi}{4}, \left(9.11 - 16.19 \left(\frac{4.94 \times 2 + 5\pi}{4}\right) + 2.901 \left(\frac{4.94 \times 2 + 5\pi}{4}\right)^2\right) \cos\left(\frac{4.94 \times 2 + 5\pi}{4}\right)\right)$

is called conversion point along the positive x-axis.

Thus, the given function  $y = (9.11 - 16.19x + 2.901x^2)\cos(x)$  cuts the real line X-axis repeatedly to meet the ground level again and again. So, it is a cosine imprecise function.

**V. RATE EFFECT OF IMPRECISENESS OF IMPRECISE FUNCTION**

Rate effect of impreciseness of imprecise function is the ratio of the volume occupied by the objects and the place where the object is used. This value of the ratio will give us information for any place whether the designed object or equipment is properly working or not. If it is not properly designed then we will require further modification. Normally 1(One) is the value of the rate effect of impreciseness for which an experiment is well designed. So to know the effectiveness of the objects in the medium it is very much necessary to know area occupied by a particular object.

To obtain the area of the imprecise function some of the intervals are not allowed to take as a limit of the integration. For example-

$$\int_0^{2\pi} \sin(x) dx = 0, \int_0^{\pi} \sin(2x) dx = 0$$

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos(x) dx = 0, \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \cos(2x) dx = 0$$

etc. For this reason to obtain the area of an imprecise function, imprecise numbers will be taken as a limit of the integration. After defining area obtained by the imprecise number we will obtain area of the imprecise function for any interval in the summation form. Here the imprecise numbers are obtained half period of the sine function.

Thus the area bounded by the imprecise function  $y = x \sin(x)$  along the right part of the x-axis is obtained by the imprecise numbers,  $[0, \frac{\pi}{2}, \pi], [\pi, \frac{3\pi}{2}, 2\pi], [2\pi, \frac{5\pi}{2}, 3\pi] \dots \dots \dots [(n - 1)\pi, \frac{(2n-1)\pi}{2}, n\pi]$  for  $n \in N$ . Here angle of sine is the integral multiple of 1(one), so to have half period of this function we take integral multiple 1(one) with  $\pi$  as the limit of the integration, which is shown below as,

$$\int_0^{\pi} x \sin(x) dx = \pi, \text{ sq. unit}, \int_{\pi}^{2\pi} x \sin(x) dx = -3 \pi \text{ sq. unit}, \int_{2\pi}^{3\pi} x \sin(x) dx = 5 \pi \text{ sq. unit}, \dots \dots \dots \text{so on.}$$

In general, Area bounded by the imprecise function  $y = x \sin(x)$  in the imprecise number or the interval  $\left[(n-1)\pi, \frac{(2n-1)\pi}{2}, n\pi\right]$  is

$$\begin{aligned} \int_{(n-1)\pi}^{n\pi} x \sin(x) dx &= (-x \cos x)_{(n-1)\pi}^{n\pi} + \int_{(n-1)\pi}^{n\pi} \cos x dx \\ &= (-1)^{n+1}(2n-1)\pi + 0 \\ &= (-1)^{n+1}(2n-1)\pi \end{aligned}$$

Where negative sign and positive sign showing the area bounded by the above and below the axis

Here, the total area bounded by  $y = x \sin(x)$  along the positive axis is

$$\pi + 3\pi + 5\pi + 7\pi + \dots \dots \dots \text{to } \infty = \pi(1 + 3 + 5 + 7 + \dots \dots \dots \text{to } \infty)$$

Thus the rate effect of impreciseness over the axis is given by

$$\frac{(2n-1)\pi}{\text{Total area of the experiment}} ; n \in I^+$$

And the total rate of effect of impreciseness over the axis is given by

$$\frac{2 \sum (2n-1)\pi}{\text{Total area of the experiment}} ; n \in Z^+$$

Thus it can be observed that imprecise function formed by multiplication of sine function is forming a regular imprecise function, as every oscillation there is always form a section of normal imprecise number having indicator function,

$$\mu_N(x) = \begin{cases} \mu_1(x), & \alpha \leq x \leq \beta \\ \mu_2(x), & \beta \leq x \leq \gamma \end{cases}$$

$$\text{Such that } \mu_1(\alpha) = \mu_2(\beta) = 0$$

$$\text{and } \mu_1(\beta) = \mu_2(\beta) = 1$$

## VI. SOME IMPORTANT PROPOSITIONS AS A FINDING OF THE WORK

**Proposition (i):** If the total rate effect of impreciseness caused by sine having function is  $\left\{ \frac{2 \sum (2n-1)\pi}{\text{Total area of the experiment}} ; n \in Z^+ \right\} \leq 1$ , then the sound or the signal covered by this object will be audible or completely visible for that place.

**Proposition (ii):** If the total rate effect of impreciseness caused by is  $\left\{ \frac{2 \sum (2n-1)\pi}{\text{Total area of the experiment}} ; n \in Z^+ \right\} > 1$ , then the sound or the signal covered by this object will not be audible or completely visible for that place. So we have to minimize our effect of impreciseness by multiplying it with suitable factor. It is nothing but the maintaining of the level of the volume of sound or light by using the regulator of the devices in the level which are less than or equal to the audible or visible range.

Variance of effect of impreciseness for this type of function can be obtained as  $\sigma^2 = \frac{4 \sum \{(2n-1)\pi\}^2}{n}$

Standard deviation of effect of imprecise of this type of function is  $\sqrt{\frac{4 \sum \{(2n-1)\pi\}^2}{n}}$

Area bounded by the imprecise function  $y = x \cos(x)$  along the right part of the x-axis is obtained by interval  $\left[0, \frac{\pi}{2}\right]$  and the imprecise numbers,  $\left[\frac{\pi}{2}, \pi, \frac{3\pi}{2}\right], \left[\frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}\right], \dots, \left[\frac{(2n-1)\pi}{2}, n\pi, \frac{(2n+1)\pi}{2}\right]$ , for  $n \in N$ . Here angle of cosine is the integral multiple of 1(one), we take integral multiple 1(one) with  $\frac{\pi}{2}$  as the limit of the integration, which is shown below as,

$$\int_0^{\frac{\pi}{2}} x \cos(x) dx = \frac{\pi}{2} - 1, \quad \text{sq. unit}, \quad \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} x \cos(x) dx = -2\pi \quad \text{sq. unit},$$

$$\int_{\frac{3\pi}{2}}^{\frac{5\pi}{2}} x \cos(x) dx = 4\pi \text{ sq. unit}, \dots \text{so on.}$$

In general, Area bounded by the imprecise function  $y = x \sin(x)$  in the imprecise number or the interval  $\left[\frac{(2n-1)\pi}{2}, n\pi, \frac{(2n+1)\pi}{2}\right]$  is

$$\int_{\frac{(2n-1)\pi}{2}}^{\frac{(2n+1)\pi}{2}} x \sin(x) dx = (-x \cos x) \frac{(2n+1)\pi}{2} + \int_{\frac{(2n-1)\pi}{2}}^{\frac{(2n+1)\pi}{2}} \cos x dx = (-1)^{n+2} 2n\pi + 0$$

$$= (-1)^{n+2} 2n\pi$$

Where negative sign and positive sign showing the area bounded by the above and below the axis

So, the total area bounded by  $y = x \sin(x)$  along the positive axis is

$$1 - \frac{\pi}{2} + 2\pi + 4\pi + 6\pi + \dots \dots \dots \text{to } \infty$$

$$= 1 - \frac{\pi}{2} + \pi(1 + 3 + 5 + 7 + \dots \dots \dots \text{to } \infty)$$

Thus the rate effect of impreciseness over the axis is given by

$$\frac{1 - \frac{\pi}{2} + (2n - 1)\pi}{\text{Total area of the experiment}} ; n \in I^+$$

And the total rate effect of impreciseness over the axis is given by

$$\frac{2[1 - \frac{\pi}{2} + \sum(2n - 1)\pi]}{\text{Total area of the experiment}} ; n \in Z^+$$

Thus it can be observed that imprecise function formed by multiplication of sine function is forming a regular imprecise function as every oscillation there is always form a section of normal imprecise number having indicator function,

$$\mu_N(x) = \begin{cases} \mu_1(x), & \alpha \leq x \leq \beta \\ \mu_2(x), & \beta \leq x \leq \gamma \end{cases}$$

$$\text{Such that } \mu_1(\alpha) = \mu_2(\beta) = 0$$

$$\text{and } \mu_1(\beta) = \mu_2(\beta) = 1$$

**Proposition (iii):** If the total rate effect of impreciseness caused by cosine having function is  $\left\{ \frac{2[1 - \frac{\pi}{2} + \sum(2n - 1)\pi]}{\text{Total area of the experiment}} ; n \in Z^+ \right\} \leq 1$ , then the sound or the signal covered by this object will be audible or completely visible for that place.

**Proposition (iv):** If the total rate effect of impreciseness caused by is  $\left\{ \frac{2[1 - \frac{\pi}{2} + \sum(2n - 1)\pi]}{\text{Total area of the experiment}} ; n \in Z^+ \right\} > 1$ , then the sound or the signal covered by this object will not be audible or completely visible for that place. So we have to minimize our effect of impreciseness by multiplying it with a suitable factor. Simply here we required to maintain the level of the volume of sound or light by using the regulator of the devices in the level which are less than or equal to the audible or visible range.

Variance of effect of impreciseness for this type of function can be obtained as  $\sigma^2 = \frac{4\left[\left(1 - \frac{\pi}{2}\right)^2 + \sum\{(2n - 1)\pi\}^2\right]}{n}$

Standard deviation of effect of imprecise of this type of function is

$$\sqrt{\frac{4\left[\left(1-\frac{\pi}{2}\right)^2 + \sum\{(2n-1)\pi\}^2\right]}{n}}$$

## VII. DIVERSION POINT

It is a point from where controllable function or the normal imprecise function comes back to the original function i.e. un-controllable function. Thus the diversion point is the main reason for why recovered function from imprecise function may create problems in our system. For example the eye problem of a person may recreate more problems when he gives up habit of the using optical glass. Whenever we remove all the conversion points from the graph it can be obtained original function of the graph. In reverse sense, conversion point is also known as diversion point of the original function.

## VIII. CONCLUSION

Here, the possible applications of imprecise function have been discussed in details. Activity of any object is obtained as a graph of the function. When this function is not imprecise form we cannot study their behavior easily and are not easy to apply for our practical purposes. So to control those functions, sine and cosine functions are used as a multiplication factor for multiplication. Successfully we have obtained multiplication factors are the functions to transform any function into oscillation nature known as normal imprecise function. This new function starts to form their nature both normal imprecise function and the original function from the conversion point and diversion point, which are also discussed. Searching of another multiplication factors of the functions that can convert uncontrollable function into controllable to form a normal imprecise function is the future prospect of our research, which will be benefitted for the future researchers.

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