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Computational complexity for minimizing wire length in two- and multi-layer channel routing

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Abstract

In this paper we have proved that two-layer (VH) no-dogleg channel routing problem of wire length minimization in absence of vertical constraints is NP-complete. Next, we have proved that the said problem is also NP-complete for a general channel instance containing both horizontal and vertical constraints in three-layer VHV and multi-layer $V_{i+1}H_i$, $2 \leq i < d_{max}$ no-dogleg channel routing models, where vertical and horizontal layers of interconnections alternate. We concluded that if the density of the channel (d_{max}) is same as the number of nets present in the channel, then the no-dogleg channel routing problem of wire length minimization in two-layer VH is polynomial time computable though channel has no vertical constraints.

Keywords: Channel routing problem; No-dogleg routing; NP-complete; VLSI; Wire length minimization.

INTRODUCTION

The wire length minimization problem is a longstanding problem in VLSI physical design automation [4], though a very little amount of work has been addressed so far in this theme of research [4-6]. Some channel routing problem (CRP) of wire length minimization is NP-hard [2,3,4,7,8]. In most of the cases, heuristics have been designed to obtain near optimal solutions for solving hard problems, but still there are some important problems in channel routing, whose nature is yet to know, whether these are polynomial time computable or beyond polynomial time computable (or intractable) [4]. This is the prime motivation to us to consider the addressed problems.

In this paper, we primarily address the computational complexity issues of the wire length minimization problem in two-layer channel routing in the absence of vertical constraints (*TNWLM*). We define this problem in no-dogleg reserved layer Manhattan routing model, when only horizontal constraints are there (in a channel) among the nets with two or more terminals. Later on, we have considered the wire length minimization problem in three-layer VHV, and multi-layer $V_{i+1}H_i$, $2 \leq i < d_{max}$, no-dogleg channel routing model for channels consisting of both horizontal and vertical constraints (*MNWLM*).

In this paper, we have proved that the stated problems are NP-complete. Sequencing to minimize weighted completion time (*SMWCT*) is an established NP-complete problem [1]. First of all, we reduce an instance of *SMWCT* to a corresponding instance of the first problem, *TNWLM* (of wire length minimization in routing a channel having no vertical constraints). As it is not possible to compute a solution of an instance of *SMWCT* in polynomial time, then eventually we prove that the problem *TNWLM* is at least as hard as *SMWCT*. Next, we prove that this result is equally applicable to *MNWLM*, which is the wire length minimization problem for a general channel instance (that contains both horizontal as well as vertical constraints).

Generally, a channel is a rectangular routing region that is formed when two blocks are placed sideways on a chip floor and a routing region is sandwiched in between. Terminals to be connected inside a channel are placed on the periphery of the blocks. A set of terminals that need to be electrically connected by wires is called a *net*. Channel is specified by net list. Here, initially we consider the reserved two-layer (VH) no-dogleg Manhattan channel routing model, where one layer is reserved for vertical wire segments and the other layer is reserved for horizontal wire segments. Later on, we have assumed multi-layer channel routing model with alternating vertical (V) and horizontal (H) layers of interconnect where both the top and bottom layers are vertical layers (and no extreme horizontal layer is allowed) [4].

Now, we formally define the two constraints present in general CRP [4]. These constraints are termed as horizontal constraints and vertical constraints. Two nets are said to be horizontally constrained, if their intervals overlap when they are assigned to the same track. Vertical constraints in a channel determine the order of placement of nets from top to bottom or vice versa. Horizontal and vertical constraints of the channel are represented by *horizontal constraint graph* (*HNCG*) and *vertical constraint graph* (*VCG*) [5]. The parametric difference of net n_i is denoted by pd_i and it represents the difference between the number of top terminals (TT_i) and the number of bottom terminals (BT_i) of the net.

Decision Versions of Different Associated Problems in Channel Routing for Minimizing Wire Length: Here we pose the decision versions of associated problems to the work under consideration.

Problem: Sequencing to minimize weighted completion time (*SMWCT*).

Instance: Set T of tasks, partial order $<\cdot$ on T, for each task $t \in T$ a length $l(t) \in Z^+$ and a weight $w(t) \in Z^+$, and a positive integer K.

Question: Is there a one processor schedule σ for T that obeys the precedence constraints and for which the sum of $(\sigma(t) + l(t)).w(t)$ is K or less for all $t \in T$?

Problem: Two-layer no-dogleg wire length minimization in channel routing in the absence of vertical constraint (*TNWLM*).

Instance: A channel specification of n multi-terminal nets that do not contain any vertical constraint whose channel density is d_{max} ($< n$), and a positive integer K'.

Question: Does there exist a two-layer no-dogleg routing solution such that the total wire length is K'or less?

Problem: Wire length minimization in multi-layer $V_{i+1}H_i$, $2 \leq i < d_{max}$, no-dogleg general channel routing (*MNWLM*).

Instance: A general channel specification of n multi-terminal nets for their assignment in the multi-layer $V_{i+1}H_i$, $2 \leq i < d_{max}$, routing model, where $d_{max} < n$ is the channel density, and a positive integer K'.

Question: Does there exist a multi-layer $V_{i+1}H_i$, $2 \leq i < d_{max}$, no-dogleg routing solution so that the total wire length is K' or less?

We may note that *SMWCT* is NP-complete in the strong sense and remains so even if all task lengths are 1 or all task weights are also 1 [1]. If the partial order $<\cdot$ is replaced by individual task deadline, the resulting problem remains NP-complete in the strong sense, but can be solved in polynomial time if all task weights are equal [1]. If there are individual task release times instead of deadlines, the resulting problem remains NP-complete in the strong sense, even if all task weights are same as 1 [1].

NP-completeness of Wire Length Minimization in Two-layer Channel Routing in the Absence of Vertical Constraints, Three-layer VHV and Multi-layer $V_{i+1}H_i$, $2 \leq i < d_{max}$ No-dogleg General Channel Routing: Here we shall first prove *TNWLM* is NP-complete by reducing an instance of *SMWCT* to the corresponding instance of this problem in polynomial time.

Theorem 1: *TNWLM* is NP-complete.

Proof: To proof this Theorem, we first show that *TNWLM* $\in$NP. Given a guessed two-layer no-dogleg routing solution *S* of some channel instance I' without having any vertical constraint, we can verify in polynomial time whether *S* is a valid routing solution of the given channel instance I', where total wire length required is less than an positive integer K'. Therefore, *TNWLM* $\in$ NP.

Here we establish a polynomial transformation over an instance *I* of the well-known NP-complete problem *SMWCT* to a corresponding instance of *TNWLM*. The transformation is as follows.

Let us assume that each task has same weight and is equal to 1, i.e., $w(t) = 1$ for all $t \in T$. Hence, according to problem *SMWCT*, $\sum_{t\in T}(\sigma(t)+l(t)).w(t)$ is less than or equal to *K*. Now as $w(t) = 1$, therefore, $\sum_{t\in T}(\sigma(t)+l(t)) \leq K$.

We presume that in a channel a set $N'$of nets contains *n* multi-terminal nets that do not contain any vertical constraints, where each net n_i, $1 \leq i \leq n$, has pd_i the parametric difference of n_i, which is the difference between top terminals (TT_i) and bottom terminals (BT_i), and $l(n_i)$ is the vertical wire length required to connect the top and bottom terminals of net n_i.

To show that *TNWLM* is NP-complete we consider the following transformation from *SMWCT* to *TNWLM*.

For all tasks $t \in T$, of *SMWCT* is transformed into a net n_i of *TNWLM* ($n_i \in N'$). Partial order <· on *T*, for associated pair of tasks in *T* is represented as horizontal non-constraint among the corresponding nets in *TNWLM*. For one processor schedule σ for *T* that obeys the precedence constraints and for which we have corresponding pd_i of net $n_i \in N'$). Here $\sigma(t_i) > \sigma(t_j)$ signifies that $pd_i > pd_j$. We assign net n_i either above or to the same track of net n_j if they do not have horizontal constraint. $l(t)$ is the length of task $t \in T$. We represent $l(t)$ as total vertical wire length of net $n_i \in N'$ (i.e., $l(n_i)$).

Now, we construct the net list (or channel specification) I' of *TNWLM* which is reduced from an instance *I* of *SMWCT*. We construct the channel specification in such a way that the channel does not have vertical constraints. As we discussed earlier that the partial orders among tasks $t \in T$ of instance *I* of *SMWCT* represent horizontal non-constraints among associated nets of instance I' of *TNWLM*, and $\sigma(t)$ of *t* represents the parametric difference pd_i of net n_i. We have written below the steps of the transformation that eventually produces a channel instance without any vertical constraint, where an *HNCG* ($G = (V, E)$) and pd_i of each net n_i are obtained by one-to-one correspondence from instance *I* of *SMWCT*. The subsequent steps of constructing I' are as follows.

1. Construct *HCG* ($G' = (V, E')$) [4] from *HNCG* ($G = (V, E)$) by complementing *G*.

2. Start from a simplicial vertex v_i in G' (that corresponds to net n_i) having highest pd_i.

3. Insert $pd_i + 1$ leftmost columns to the channel under construction and assign terminals of net n_i to the top net list in each of these columns where the bottom terminal in each associated column contains a non-terminal (i.e.,'0').

4. For a vertex v_j (corresponds to net n_j) with $\{v_j , v_i\} \in E'$ that has highest pd_j among the vertices whose corresponding nets are yet to assign but form a maximal clique including v_i (or have horizontal constraint with n_i), insert $pd_j + 1$ columns to the right of the rightmost column inserted up to this point in time (into channel under construction) and assign terminals of net n_j to the top net list in each of these columns where the bottom terminal in each associated column contains a non-terminal (i.e., '0').

5. As for all adjacent vertices v_i corresponding terminals get connected to the top of the channel after insertion of each of the associated nets to the channel under construction, a column is appended to the right of the rightmost column that containing a non-terminal to the top and a terminal of net n_i corresponding to vertex v_i to the bottom.

6. Delete vertex v_i from G'

7. Continue Step 2 through Step 6 until set *V* of G' gets empty.

Now, consider a (desired) routing solution of minimum wire length as follows for which we do the following.

Now, once the channel specification is completely obtained, we can employ *MCC1* [4] with necessary modifications as follows. Here instead of assigning a net to the topmost track that starts from the leftmost terminal of the net list, net n_i with highest pd_i among the nets having horizontal constraints, gets more privilege to be assigned to a track nearer the top than the remaining ones. Thus, in this way, we find track X_i, where net n_i is to be placed (for i =1, 2, ..., n) and the total number of tracks required to route the nets becomes *TN*. We may assume $\Im_H$ as the total horizontal wire needed to route all the nets and s_i being the set of nets assigned to track X_i. Hence, the total wire length required for routing is as follows:

$$\sum_{1\le j\le TN}\left\lfloor \sum_{\forall i\in S_j}\left[X_i TT_i+(TN-X_i+1)BT_i\right]\right\rfloor+\Im_H. \text{ Now, for } SMWCT,$$

$$\sum_{t\in T}(\sigma(t)+l(t)).w(t)\le K \text{ or } \sum_{t\in T}(\sigma(t)+l(t))\le K, \text{ where } w(t)=1, \text{ for all } t.$$

As stated earlier, for *TNWLM*, the aim is to minimize the total vertical wire length along with horizontal wire segment essential for a valid routing. Thus, the expression needed to be optimized as written below.

$$\sum_{1\le j\le TN}\left(\sum_{\forall i\in s_j}\left(X_i TT_i+(TN-X_i+1)BT_i\right)\right)+\Im_H$$

$$=\sum_{1\le j\le TN}\left(\sum_{\forall i\in s_j}\left(X_i TT_i+(TN-X_i+1)BT_i+\sigma(t_i)-\sigma(t_i)\right)\right)+\Im_H$$

$$=\sum_{1\le j\le TN}\left(\sum_{\forall i\in s_j}\left(X_i TT_i+(TN-X_i+1)BT_i+\sigma(t_i)-(TT_i-BT_i)\right)\right)+\Im_H$$

$$\le K-\sum_{1\le j\le TN}\left(\sum_{\forall i\in s_j}(pd_i)\right)+\Im_H=K', \text{ where } 1\le j\le TN$$

Here, $\sum_{1\le j\le TN}\left[\sum_{\forall i\in S_j}\left[pd_i\right]\right]$ must be less than or equal to $\Im_H$, hence, K' is a positive integer. On the opposite side, starting from $SMWCT$,

$\sum_{1\le i\le n}\left[\left(\left[\sigma(t_i)+l(t_i)\right].w(t)\right)\right]$, where $w(t)$ is one. Hence, replacing $\sum_{1\le i\le n} l(t_i)$ by vertical wire length, we get the following expression.

$$\sum_{1\le j\le TN}\left\lfloor\sum_{\forall i\in S_j}\left[X_i TT_i+(TN-X_i+1)BT_i\right]\right\rfloor+\sum_{1\le j\le TN}\left\lfloor\sum_{\forall i\in S_j}\left[\sigma(t_i)\right]\right\rfloor$$

$$=\sum_{1\le j\le TN}\left[\sum_{\forall i\in S_j}\left[X_i TT_i+(TN-X_i+1)BT_i\right]\right]+\sum_{1\le j\le TN}\left[\sum_{\forall i\in S_j} pd_i\right]$$

$$\le K'-\Im_H+\sum_{1\le j\le TN}\left[\sum_{\forall i\in S_j} pd_i\right]=K$$

We can easily prove that the time and space consumed by the reduction process from *I* of *SWMCT* to *I′* of *TNWLM* and vice-versa is bounded by some polynomial function of number of tasks existing is an instance of *SWMCT*.

If the density of the channel d_{max} is *n*, i.e., the number of nets present in the channel of two-layer VH no-dogleg wire length minimization in channel routing in absence of vertical constraint, we need *n* number of tracks to route the nets to the channel because all the nets present in the channel are horizontally constraint to each other. Hence, we can get minimum wire length routing solution in polynomial time only if we place the nets in different tracks according to their parametric differences.

In similar way we can prove that the wire length minimization problem in three-layer VHV and multi-layer no dogleg general channel routing *MNWLM* ($V_{i+1}H_i$, $2 \le i < d_{max}$) is NP-complete by reducing the instance of an NP-complete problem such as *SMWCT* to the instance of three-layer VHV and the instance of *MNWLM* ($V_{i+1}H_i$, $2 \le i < d_{max}$), in polynomial time, respectively.

CONCLUSIONS

We showed that the decision version of two-layer VH no-dogleg wire length minimization in channel routing (with $d_{max}<n$) in the absence of vertical constraints, three-layer VHV and multi-layer $V_{i+1}H_i$, $2 \le i < d_{max}$ no-dogleg wire length minimization in general channel routing problem are NP-complete by transforming the instance of the known NP-complete problem, i.e. sequencing to minimize weighted completion time problem to the instance of each problem in polynomial time. We also concluded that if the density of the channel d_{max} is *n*, i.e. the number of nets present in the channel then the two-layer VH no-dogleg channel routing problem for wire length minimization is polynomial time computable though channel has no vertical constraints.

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