Advances in Computational Sciences and Technology
ISSN 0973-6107 Volume 10, Number 9 (2017) pp. 2801-2810

http://www.ripublication.com

# Applications of Distance - 2 Dominating Sets of Graph in Networks

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**Abstract**

The aim of the paper is to impart the importance of graph theoretical concepts and the applications of distance - 2 domination in graphs to various real life situations in the areas of science and engineering. A set D is a distance - 2 dominating set if for every vertex $u \varepsilon V - D$, d(u, D) $\leq 2$and is denoted by $\gamma_{\leq 2}(G)$. In this paper,we get many bounds and exact values for some standard graphs. Also this paper explores mainly on the applications of distance - 2 dominating sets in networks. Nordhaus – Gaddum type results are obtained for this parameter. Distance - 2 dominating sets are identified in School Bus Routing, Radio Stations, Communication Networks and Mobile adhoc Networks.

**Keywords** – Dominating set, connected dominating set, Mobile adhoc network, distance -2 dominating set.

## I. INTRODUCTION

"Graph Theory" is an important branch of Mathematics. It has grown rapidly in recent times with a lot of research activities. In the last few decades, at the international level, one third of the Mathematics research papers are from Graph Theory and combinatorics.

Applications of graph theory in computer and communication, social networks,Molecular physics and chemistry, Biological sciences, Engineering and in other numerous areas [3].

In graph theory, one of the extensively researched branches is domination in graphs. This is largely due to a variety of new parameters that can be developed from the basic definition of domination. The NP- completeness in basic domination problems and its close relationship to other NP- completeness problems have contributed to the enormous growth of research activity in Domination theory.

All graphs considered here are simple, finite and undirected. The greatest (least) integer less (greater) than or equal to x is $\lfloor x \rfloor (\lceil x \rceil)$. In this paper, the terms and notations used may be found in Haynes[3] or Harary[2].

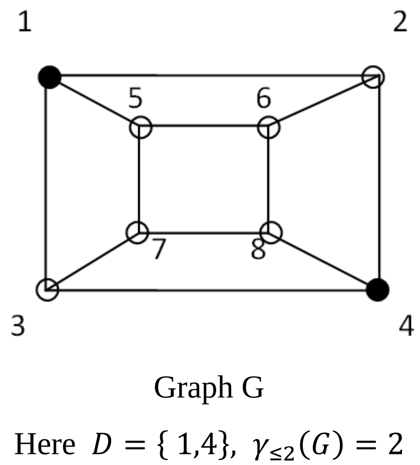
A set D$\subseteq$ Vof vertices in a graph G = (V,E) is called a dominating set if every vertex v$\varepsilon$V is either an element of D or it is adjacent to an element of D. The domination number of G is the minimum cardinality of a dominating set and it is denoted by $\gamma$ (G). A recent survey of $\gamma$ (G) can be found in [5].

In [11], Sampathkumar and Walikar defines a connected dominating set D to be a dominating set D whose induced subgraph $\langle D \rangle$ is connected. The connected domination number $\gamma_c(G)$ of a connected graph G is the minimum cardinality of a connected dominating set of G.

**Definition 1.1**

A set D of vertices in a graph G = (V,E) is a distance - 2 dominating set if every vertex in V-D is within distance 2 of atleast one vertex in D. The distance -2 domination number $\gamma_{\leq 2}(G)$ of G equals the minimum cardinality of a distance 2-dominating set in G.

**Example 1.2**




Graph G

Here $D = \{1,4\}$, $\gamma_{\leq 2}(G) = 2$

**Figure 1.**

**MAIN RESULTS**

**Observation: 1.3**

1. For any path $P_n$ , for $n \geq 2$

$$\gamma_{\leq 2}(P_n) = \left\lceil \frac{n}{5} \right\rceil$$

2. For any Cycle $C_n$ , $for\ n \geq 3$ ,

$$\gamma_{\leq 2}(C_n) = \left\lceil \frac{n+1}{5} \right\rceil$$

3. For any Star $K_{1,n}$,for n $\geq 2$

$$\gamma_{\leq 2}(K_{1,n}) = 1$$

4. For any Complete graph $K_n$ , for n $\geq 2$

$$\gamma_{\leq 2}(K_n) = 1$$

5. For any Wheel graph $W_n$ , $for\ n \geq 4$,

$$\gamma_{\leq 2}(W_n) = 1$$

6. For any Helm graph $H_n$ , for n $for\ n \geq 3$,

$$\gamma_{\leq 2}(H_n) = 1$$

7. For any Book graph $B_n$ , for n $\geq 3$

$$\gamma_{\leq 2}(B_n) = 1$$

8. For any Friendship graph $F_n$ , for n $\geq 2$

$$\gamma_{\leq 2}(F_n) = 1$$

9. For any Complete bipartite graph $K_{n,m}$ , $for\ n, m \geq 1$

$$\gamma_{\leq 2}(K_{n,m}) = 1$$

**Proposition 1.4**

For any graph G , $\gamma_{\leq 2}(G) \leq \gamma(G)$.

The converse of the above proposition need not be true.

**Proposition 1.5**

For a Grid graph $G_{2,k}$ , for k $for\ k \geq 1$,

$$\gamma_{\leq 2}(G_{2,k}) = \left\lfloor \frac{2+k}{3} \right\rfloor$$

**Proposition 1.6**

Let D be a distance - 2 dominating set of a graph G. Then D is a minimal distance - 2 dominating set if and only if each vertex u$\varepsilon$D satisfies at least one of the following conditions:

(a). there exists a vertex v$\varepsilon$ V(G) – D for which $N_{\leq 2}(v) \cap D = \{u\}$.

(b). d(u, w) > 2 for every vertex wε D-{u}.

**Theorem 1.7**

If G is a connected graph with at least 3 vertices and diam(G) ≥ 2, then G has a minimum distance - 2 dominating set D such that every vertex u$\varepsilon$D satisfies condition (a) and has a private 2 neighbor u′$\varepsilon$ V-D for which d(u, u′)=2.

The proof of the above proposition and theorem can be found in [3].

***Nordhaus – Gaddum Type Results***

**Theorem 1.8**

For any graph G and $\overline{G}$ with $n \geq 3$,

(a) $2 \leq \gamma_{\leq 2}(G) + \gamma_{\leq 2}(\bar{G}) \leq n + 1$

(b) $1 \leq \gamma_{\leq 2}(G).\gamma_{\leq 2}(\bar{G}) \leq n$

**Theorem 1.9**

If both the graphs G and $\bar{G}$ are connected with n≥ 3 then

(a) $2 \leq \gamma_{\leq 2}(G) + \gamma_{\leq 2}(\bar{G}) \leq \frac{n}{3} + 1$

(b) $1 \leq \gamma_{\leq 2}(G).\gamma_{\leq 2}(\bar{G}) \leq \frac{n}{3}$

**Applications of distance - 2 dominating sets**

When we extend the concept of dominating sets to distance - 2 dominating sets, there are more useful models to many real- world problems. Indeed, much of the motivation for the study of domination arises from problems involving locating optimally a hospital, police station, fire station, or any other emergency service facility.

**School Bus Routing**

Nowadays, almost all schools operate school buses for transporting children for to and fro services. Among many points, three important points to be noted are (i) The running time of a bus between school and its terminus (ii) Maximum number of students on a bus at any time and (iii) the maximum distance a student has to walk to board a school bus.Consider a street map of a city shown in fig.2 where each edge represents one city block. Let us assume that the school is located at the vertex starting point and the management committee of the school decides that no student shall walk more than two blocks to board a school bus. Construct a route for a school bus that leaves the school, gets within two blocks of every child that uses the school bus and returns to the school. Clearly this bus route forms a distance - 2 dominating set.

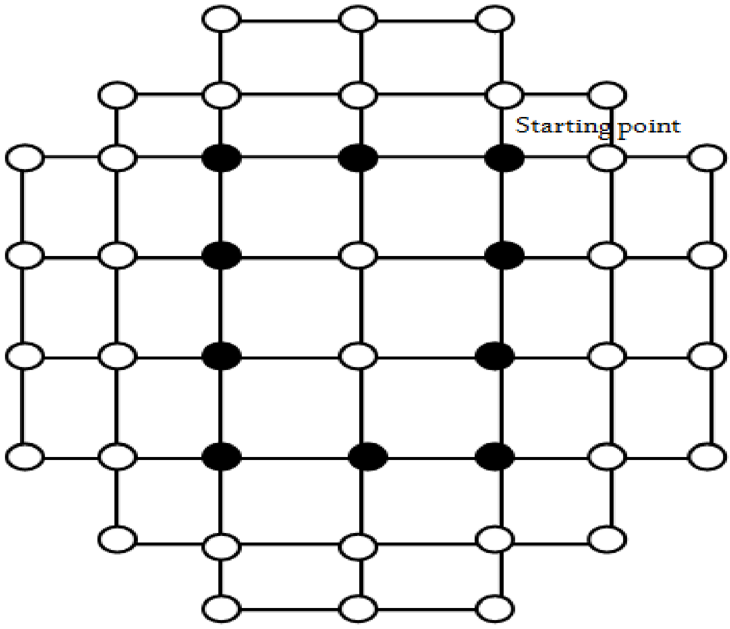



**Figure 2.**

**Radio stations**

Suppose that we have a collection of small villages in a remote part of the world. We would like to locate radio stations in some of these villages so that messages can be broadcasted to all the villages in the region. But since the installations of radio stations are costly, we want to locate as few as possible which can cover all other villages. Let each village be represented by a vertex. An edge between two villages is labeled with the distance, say in kilometers. The distance between the two villages is shown in fig.3. Let us assume that a radio station has a broadcast range of hundred kilometers. In this case we seek a distance - 2 dominating set among all the vertices within the distance of 100 kilometers. Clearly this fig.4 gives the distance - 2 dominating set.

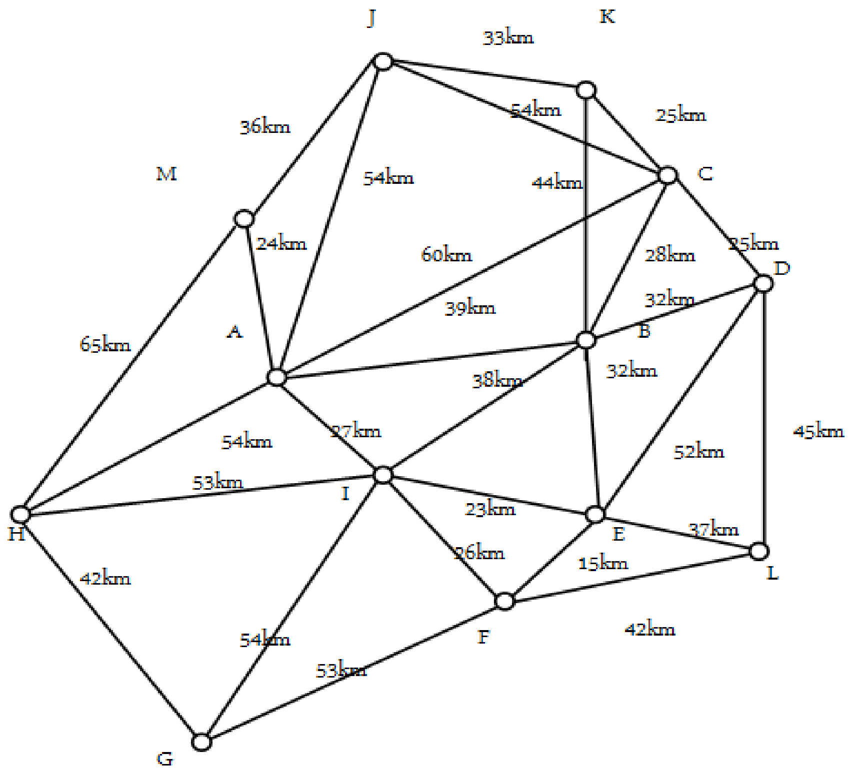



**Figure 3.**

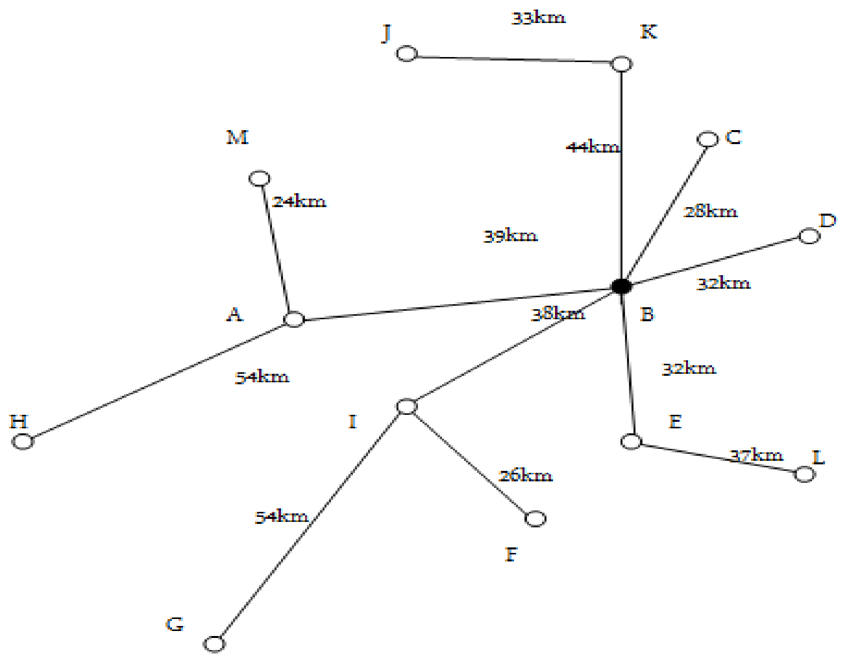



**Figure 4.**

**Computer Communication Networks**

The distance - 2 dominating set plays an important role in computer and communication networks to route the information between the nodes. We consider a computer network modeled by a Hyper cube. The vertices of the Hyper cube represents computer and edges represent direct communication link between two computers. So, in this model we have 16 computers or processors and each processor can pass information to the processor to which it is directly connected. Our problem is to collect information from all processors and we would like to do it relatively often and relatively fast. So, we identify a small set of processors called collecting processors and ask each processor to send its information to one of the small sets of collecting processors. We assume that at most a two – unit delay between the time a processor sends its information and the time it arrives at a nearest collector is allowed. In this case, we have to find a distance – 2 dominating set of all processors. The set of vertices {marked in dark} forms a distance 2 dominating set in the hypercube network in fig.5

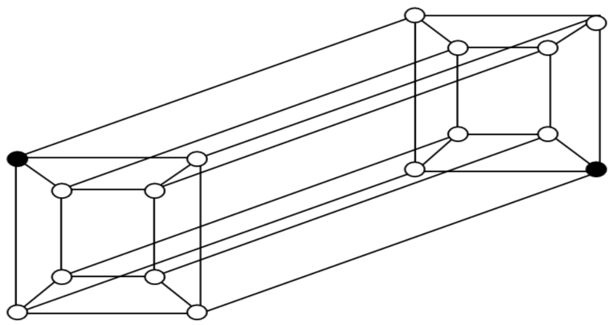


**Figure 5.**

**Mobile Ad-Hoc Network**

A mobile ad hoc network (MANET) is defined as a collection of mobile hosts that dynamically form a wireless network without any backbone infrastructure and centralized administration. The mobile hosts in a MANETs will be in situations where there is no fixed backbone infrastructure like battlefield scenarios, natural calamities such as earthquakes and hurricanes. MANET is considered to be adaptable and convenient because it consists of hosts which are heterogeneous nature.

MANET operates on the concept of flooding where each host, after receiving a message, broadcasts it to the entire network. This could result in waste of valuable resources such as network bandwidth and battery power of the devices. One of the greatest challenges in forming this type of network is to involve the minimum number of hosts in the routing process because not every node in the network may be required to forwardthe messages. A solution to this challenge is to identify minimum connected dominating set[4] among the hosts in a given area. Here we use a distance – 2 dominating sets, since it has minimum number of hosts than a connected dominating set. It is usefull in reducing the communication and the storage overhead by keeping its size to be minimized. The following fig.6 is an example of a distance - 2 dominating set in Mobile Ad- hoc networks.

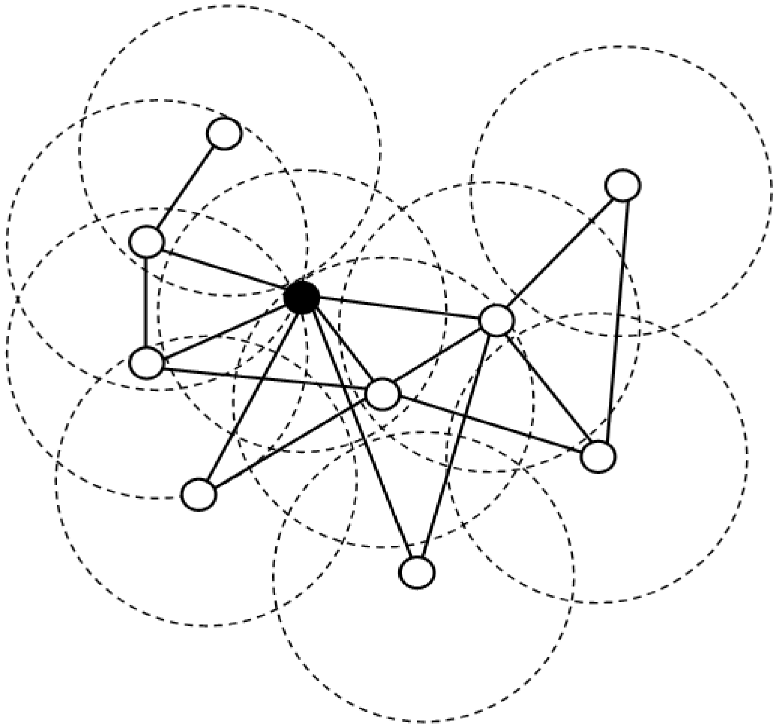


**Figure 6.**

## CONCLUSION

In this paper, we have discussed the use of distance – 2 dominating sets in computer and communication networks. In future we propose to extend this work with an algorithm to identify a distance - 2 dominating set in any communication network with different transmission radius. We need to verify the effectiveness of our updated strategies when the topology of the underlying network changes.

## ACKNOWLEDGMENT

The authors are highly thankful to everyone who contributed to the success of this work.

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