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A Note on Fuzzy Soft Pre Continuity

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Abstract

Pre continuous functions, fuzzy pre continuous functions and soft pre continuous functions have been already investigated by topologists. In this paper the concept of a fuzzy soft pre continuous function is introduced and its relationship with the existing concepts in the literature of fuzzy soft topology is discussed.

Keywords: Fuzzy soft sets, Fuzzy soft topology, Fuzzy soft mapping, Fuzzy soft pre continuous.

1. INTRODUCTION

In the year 1982, Mashhour ,Abd-El-Mansef, and El-Deep [6] discussed the concept of pre continuous and weak pre continuous mapping. In 2014, Metin Akdag and Alkan ozkan [7] introduced the concept of soft pre open sets and soft pre separation axioms. In 1991 Singal and Niti Prakash [10] introduced the notion of Fuzzy pre open sets and fuzzy pre separation axioms. In this paper we introduce the notion of fuzzy soft pre continuity and some results along with examples have been discussed. Throughout this paper X and Y denote the initial sets. E and K denote the parameter spaces.

2. PRELIMINARIES

Definition 2.1

A pair (F,E) is called a soft set **[8]** over X where F is a mapping given by F:E→ 2^X and 2^X is the power set of X

Definition 2.2

A fuzzy set **[13]** of on X is a mapping f: $X \to I^X$ where I=[0,1].

Definition 2.3

A pair $\tilde{\lambda} = (\lambda, E)$ is called a fuzzy soft set **[11]** over (X,E) where $\lambda: E \to I^X$ is a mapping , I^X is the collection of all fuzzy subsets of X. FS(X,E) denotes the collection of all fuzzy soft sets over (X,E). We denote $\tilde{\lambda}$ by $\tilde{\lambda}=\{(e, \lambda(e)):e\in E\}$where $\lambda(e)$ is a fuzzy subset of X for each e in E.

Definition 2.4 [11]

For any two fuzzy soft sets $\tilde{\lambda}$ and $\tilde{\mu}$ over a common universe X and a common parameter space E, $\tilde{\lambda}$ is a fuzzy soft subset of $\tilde{\mu}$ if $\lambda(e) \leq \mu(e)$ for all $e \in E$.If $\tilde{\lambda}$ is a fuzzy soft subset of $\tilde{\mu}$ then we write $\tilde{\lambda} \subseteq \tilde{\mu}$ and $\tilde{\mu}$ contains $\tilde{\lambda}$.

Two fuzzy soft sets $\tilde{\lambda}$ and $\tilde{\mu}$ over (X,E) are soft equal if $\tilde{\lambda} \subseteq \tilde{\mu}$ and $\tilde{\mu} \subseteq \tilde{\lambda}$. That is $\tilde{\lambda} = \tilde{\mu}$ if and only if $\lambda(e) = \mu(e)$ for all $e \in E$. We use the following notations:

$\bar{0}(x) = 0$, for all x in X and $\bar{1}(x) = 1$, for all x in X.

Definition 2.5 [11]

A fuzzy soft set $\tilde{\varphi}_X$ over (X,E) is said to be a null fuzzy soft set if for all $e \in E$, $\varphi_X(e) = \bar{0}$ and $\tilde{\varphi}_X = (\varphi_X, E)$.

Definition 2.6 [11]

A fuzzy soft set $\tilde{1}_X$ over (X, E) is said to be absolute fuzzy soft set if for all $e \in E$, $1_X(e) = \bar{1}$ and

$\tilde{1}_X = (1_X, E)$.

Definition 2.7 [12]

The union of two fuzzy soft sets $\tilde{\lambda}$ and $\tilde{\mu}$ over (X,E) is defined as $\tilde{\lambda} \tilde{\cup} \tilde{\mu} = (\lambda \tilde{\cup} \mu, E)$ where $(\lambda \tilde{\cup} \mu)(e) = \lambda(e) \cup \mu(e)$ = the union of fuzzy sets $\lambda(e)$ and $\mu(e)$ for all $e \in E$.

Definition 2.8 [12]

The intersection of two fuzzy soft sets $\tilde{\lambda}$ and $\tilde{\mu}$ over (X,E) is defined as $\tilde{\lambda} \tilde{\cap} \tilde{\mu} = (\lambda \tilde{\cap} \mu, E)$ where $(\lambda \tilde{\cap} \mu)(e) = \lambda(e) \cap \mu(e)$ = the intersection of fuzzy sets $\lambda(e)$ and $\mu(e)$ for all $e \in E$.

The arbitrary union and arbitrary intersection of fuzzy soft sets over (X,E) are defined as

$\tilde{\cup}\{\tilde{\lambda}_\alpha : \alpha \in \Delta\} = (\tilde{\cup}\{\lambda_\alpha : \alpha \in \Delta\}, E)$ and $\tilde{\cap}\{\tilde{\lambda}_\alpha : \alpha \in \Delta\} = (\tilde{\cap}\{\lambda_\alpha : \alpha \in \Delta\}, E)$ where

$(\tilde{\cup}\{\lambda_\alpha : \alpha \in \Delta\})(e) = \cup\{\lambda_\alpha(e) : \alpha \in \Delta\}$ = the union of fuzzy sets λ_α(e),$\alpha \in \Delta$ and

$(\tilde{\cap}\{\lambda_\alpha : \alpha \in \Delta\})(e) = \cap\{\lambda(e) : \alpha \in \Delta\}$ = the intersection of fuzzy sets λ_α(e),$\alpha \in \Delta$, for all e∈E.

Definition 2.9 [12]

The complement of a fuzzy soft set (λ, E) over (X, E), denoted by $(\lambda, E)^C$ is defined as $(\lambda, E)^C = (\lambda^C, E)$ where $\lambda^C : E \to I^X$ is a mapping given by $\lambda^C(e) = 1-\lambda(e)$ for every e in E.

Definition 2.10 [12]

A fuzzy soft topology $\tilde{\tau}$ on (X,E) is a family of fuzzy soft sets over (X,E) satisfying the following axioms.

i. $\tilde{\varphi}_X, \tilde{1}_X$ belong to $\tilde{\tau}$,
ii. Arbitrary union of fuzzy soft sets in $\tilde{\tau}$,belongs to $\tilde{\tau}$,
iii. The intersection of any two fuzzy soft sets in $\tilde{\tau}$,belongs to $\tilde{\tau}$.

Members of $\tilde{\tau}$ are called fuzzy soft open sets in (X,$\tilde{\tau}$,E). A fuzzy soft set $\tilde{\lambda}$ over (X,E) is fuzzy soft closed in (X,$\tilde{\tau}$,E) if $(\tilde{\lambda})^C \in \tilde{\tau}$. The fuzzy soft interior of $\tilde{\lambda}$ in (X,$\tilde{\tau}$,E) is the union of all fuzzy soft open sets $\tilde{\mu} \subseteq \tilde{\lambda}$ denoted by $\widetilde{fs}$ int($\tilde{\lambda}$) = $\tilde{\cup}\{\tilde{\mu} : \tilde{\mu} \subseteq \tilde{\lambda}, \tilde{\mu} \in \tilde{\tau}\}$. The fuzzy soft closure of $\tilde{\lambda}$ in (X,$\tilde{\tau}$,E) is the intersection of all fuzzy soft closed sets $\tilde{\eta}$, $\tilde{\lambda} \subseteq \tilde{\eta}$ denoted by $\widetilde{fs}$ cl($\tilde{\lambda}$) = $\tilde{\cap}\{\tilde{\eta} : \tilde{\lambda} \subseteq \tilde{\eta}, (\tilde{\eta})^C \in \tilde{\tau}\}$.

Definition 2.11 [2]

Let (X,$\tilde{\tau}$,E). be a fuzzy soft topological space and let $\tilde{\lambda}$ be a fuzzy soft set over (X,E). Then $\tilde{\lambda}$ is fuzzy soft semi-open if $\tilde{\lambda} \subseteq \widetilde{fs}Cl(\widetilde{fs}Int(\tilde{\lambda}))$ and fuzzy soft semi closed if $\widetilde{fs}Int(\widetilde{fs}Cl(\tilde{\lambda})) \subseteq \tilde{\lambda}$.

Definition 2.12 [2]

Let (X,$\tilde{\tau}$,E). be a fuzzy soft topological space and let $\tilde{\lambda}$ be a fuzzy soft set over (X,E). Then $\tilde{\lambda}$ is fuzzy soft pre-open if $\tilde{\lambda} \subseteq \widetilde{fs}int(\widetilde{fs}cl(\tilde{\lambda}))$ and fuzzy soft pre- closed if $\widetilde{fs}cl(\widetilde{fs}Int(\tilde{\lambda})) \subseteq \tilde{\lambda}$.

Definition 2.13 [2]

Let (X,$\tilde{\tau}$,E) be a fuzzy soft topological space and let $\tilde{\lambda}$ be a fuzzy soft set over (X,E). Then $\tilde{\lambda}$ is fuzzy soft α-open if $\tilde{\lambda} \subseteq \widetilde{fs}Int(\widetilde{fs}Cl(\widetilde{fs}Int(\tilde{\lambda})))$ and fuzzy soft α-closed if $\tilde{\lambda} \supseteq \widetilde{fs}Cl(\widetilde{fs}Int(\widetilde{fs}Cl(\tilde{\lambda})))$.

The classes of all fuzzy soft α-open, fuzzy soft pre-open, fuzzy soft semi-open and fuzzy soft semi-pre-open sets over (X,E) are denoted as $\widetilde{FS}\alpha(X)$, $\widetilde{FS}SO(X)$, $\widetilde{FS}PO(X)$ and $\widetilde{FS}SP(X)$ respectively.

The fuzzy soft pre-interior, fuzzy soft pre-closure, fuzzy soft semi-interior, fuzzy soft semi-closure and fuzzy soft α-interior,fuzzy soft α-closure ,fuzzy soft semi-pre-interior, fuzzy soft semi-pre-closure of X are denoted by $\widetilde{fs}PCl(\tilde{\lambda})$, $\widetilde{fs}PInt(\tilde{\lambda})$, $\widetilde{fs}SInt(\tilde{\lambda})$, $\widetilde{fs}SCl(\tilde{\lambda})$, $\widetilde{fs}\alpha Cl(\tilde{\lambda})$, $\widetilde{fs}\alpha Int(\tilde{\lambda})$, $\widetilde{fs}SPInt(\tilde{\lambda})$, $\widetilde{fs}SPCl(\tilde{\lambda})$ respectively

Definition 2.14 [2]

Let (X,$\tilde{\tau}$,E). be a fuzzy soft topological space and let $\tilde{\lambda}$ be a fuzzy soft set over (X,E). Then its fuzzy soft pre-closure and fuzzy soft pre-interior are defined as:

$\widetilde{fs}PCl(\tilde{\lambda}) = \cap \{ \tilde{\mu} \mid \tilde{\mu} \supseteq \tilde{\lambda}, \tilde{\mu} \in \widetilde{FS}PC(X)\}$.

$\widetilde{fs}PInt(\tilde{\lambda}) = \cup \{ \tilde{\eta} \mid \tilde{\eta} \subseteq \tilde{\lambda}, \tilde{\eta} \in \widetilde{FS}PO(X)\}$.

The definitions for $\widetilde{fs}SCl$, $\widetilde{fs}SInt$, $\widetilde{fs}\alpha Cl$ and $\widetilde{fs}\alpha Int$ are similar.

The following extension principle is used to define the mapping between the classes of fuzzy soft sets.

Definition 2.15 [11]

Let X and Y be any two non-empty sets. Let g :X→Y be a mapping. Let λ be a fuzzy subset of X and $\tilde{\mu}$ be a fuzzy subset of Y. Then g(λ) is a fuzzy subset of Y and for y in Y

$$g(\lambda)(y)=\begin{cases} Sup\{\lambda(f(x)) : x \in g^{-1}(y)\}, & g^{-1}(y) \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

$g^{-1}(\mu)$ is a fuzzy subset of X, defined by $g^{-1}(\mu)(x)= \mu(f(x))$ for all $x \in X$.

Definition 2.16 [10]

Let FS(X,E) and FS(Y,K) be classes of fuzzy soft sets over (X,E) and (Y,K) respectively.

ρ: X→ Y and ψ : E → K be any two mappings. Then a fuzzy soft mapping g=(ρ, ψ) : FS(X,E)→ FS(Y,K) is defined as follows:

For a fuzzy soft set $\tilde{\lambda}$ in FS(X,E), g($\tilde{\lambda}$) is a fuzzy soft set in FS(Y,K) obtained as follows:

$$g(\tilde{\lambda})(k) = \begin{cases} \bigcup_{e\in\psi^{-1}(k)} \rho(\lambda(e)) \ , & \psi^{-1}(k) \neq \phi \\ \bar{0} \ , & \text{Otherwise} \end{cases}$$

For every y in Y,

$$\text{where } \rho(\lambda(e))(y) = \begin{cases} Sup\{\lambda(e)(x) : x \in \rho^{-1}(y)\} \ , & \rho^{-1}(y) \neq \phi \\ 0 \ , & \text{Otherwise} \end{cases}$$

$$\text{That is } g(\tilde{\lambda})(k)\ \mathbf{(y)} = \begin{cases} \underset{e\in\psi^{-1}(k)}{Sup}\left\{\underset{x\in\rho^{-1}(y)}{Sup}\ \lambda(e)(x)\right\}, & \rho^{-1}(y) \neq \phi, \psi^{-1}(k) \neq \phi \\ o \ , & \text{Otherwise} \end{cases}$$

g ($\tilde{\lambda}$) is the image of the fuzzy soft set $\tilde{\lambda}$ under the fuzzy mapping g=(ρ,ψ).

For a fuzzy soft set $\tilde{\mu}$ in FS(Y,K) , $g^{-1}(\tilde{\mu})$ is a fuzzy soft set in FS(X,E) obtained as follows:

$g^{-1}(\tilde{\mu})(e)(x) = \rho^{-1}(\tilde{\mu}(\psi(e)))(x)$ for every x in X

$g^{-1}(\tilde{\mu})$ is the inverse image of the fuzzy soft set $\tilde{\mu}$.

Lemma 2.17 [8]

Let (X,$\tilde{\tau}$,E). and (Y,$\tilde{\sigma}$,K) be fuzzy soft topological spaces. Let ρ: X→ Y and ψ : E → K be the two mappings and g = (ρ,ψ) : FS(X,E)→ FS(Y,K) be a fuzzy soft mapping. Let $\tilde{\lambda}, \tilde{\lambda}_1, (\tilde{\lambda})_i$ ∈ FS(X,E) and $\tilde{\mu}, \tilde{\mu}_1, (\tilde{\mu})_i$ ∈ FS(Y,K), where $i \in J$ is an index set.

i. If $\tilde{\lambda}_1 \subseteq \tilde{\lambda}_2$,then $g(\tilde{\lambda}_1) \subseteq g(\tilde{\lambda}_2)$.
ii. If $\tilde{\mu}_1 \subseteq \tilde{\mu}_2$,then $g^{-1}(\tilde{\mu}_1) \subseteq g^{-1}(\tilde{\mu}_2)$.
iii. $\tilde{\lambda} \subseteq g^{-1}(g(\tilde{\lambda}))$, the equality holds if g is injective.
iv. $g(g^{-1}(\tilde{\mu})) \subseteq \tilde{\mu}$, the equality holds if g is surjective.
v. $g^{-1}((\tilde{\mu})^C) = [g^{-1}(\tilde{\mu})]^C$.
vi. $[g(\tilde{\lambda})]^C \subseteq g((\tilde{\lambda})^C)$.
vii. $g^{-1}(\tilde{1}_K) = \tilde{1}_E, g^{-1}(\tilde{0}_K) = \tilde{0}_E$.

viii. $g(\tilde{1}_E) = \tilde{1}_K$ if g is surjective.
ix. $g(\tilde{0}_E) = \tilde{0}_K$.

Lemma 2.18 [8]

Let (X,$\tilde{\tau}$,E). and (Y,$\tilde{\sigma}$,K) be the two fuzzy soft topological spaces. Let ρ: X→ Y and ψ : E → K be the two mappings and g = (ρ,ψ) : FS(X,E)→ FS(Y,K) be a fuzzy soft mapping. Let $\tilde{\lambda}, \tilde{\lambda}_1, (\tilde{\lambda})_i \in$ FS(X,E) and $\tilde{\mu}, \tilde{\mu}_1, (\tilde{\mu})_i \in$ FS(Y,K), where J is an index set.

i. $g(\cup_{i\in J} \tilde{\lambda}_i) = \cup_{i\in J} g(\tilde{\lambda}_i)$.
ii. $g(\cap_{i\in J} \tilde{\lambda}_i) \subseteq \cap_{i\in J} g(\tilde{\lambda}_i)$, the equality holds if g is injective.
iii. $g^{-1}(\cup_{i\in J} \tilde{\mu}_i) = \cup_{i\in J} g^{-1}(\tilde{\mu}_i)$.
iv. $g^{-1}(\cap_{i\in J} \tilde{\mu}_i) = \cap_{i\in J} g^{-1}(\tilde{\mu}_i)$.

Definition 2.19 [12]

Fix $x \in X$, $0 < \alpha < 1$ .Then the fuzzy subset x^α of X is called fuzzy point if

$$x^\alpha(y) = \begin{cases} \alpha & \text{if } y = x \\ 0 & \text{if } y \neq x \end{cases}$$

Definition 2.20 [12]

Fix x∈X, 0< α < 1, e∈E.The fuzzy soft set x_e^α over (X,E) is called fuzzy soft point if

$$x_e^\alpha(e_1) = \begin{cases} x^\alpha & \text{for } e_1 = e \\ \bar{0} & \text{otherwise} \end{cases}$$

$$x_e^\alpha(e_1)(y) = \begin{cases} \alpha & \text{for } e_1 = e, y = x \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.21 [3]

Let (X,$\tilde{\tau}$,E). and (Y,$\tilde{\sigma}$,K) be the fuzzy soft topological spaces. Let ρ: X→ Y and ψ : E → K be the two mappings and g = (ρ,ψ) : FS(X,E)→ FS(Y,K) be a fuzzy soft mapping. Then g=(ρ,ψ) is said to be fuzzy soft continuous if the inverse image of every fuzzy soft open set in (Y,$\tilde{\sigma}$,K) is fuzzy soft open in (X,$\tilde{\tau}$,E). That is g^{-1}(μ̃)∈ $\tilde{\tau}$,for all μ̃∈ $\tilde{\sigma}$.

3. FUZZY SOFT PRE CONTINUITY

Definition 3.1

Let (X, $\tilde{\tau}$,E) and (Y,$\tilde{\sigma}, K$) be two fuzzy soft topological spaces.A fuzzy soft mapping g **:** (X, $\tilde{\tau}$,E) → (Y,$\tilde{\sigma}, K$) is said to be fuzzy soft pre continuous if for each fuzzy soft open set $\tilde{\mu}$ in (Y,$\tilde{\sigma}, K$), the inverse image $g^{-1}(\tilde{\mu})$ is fuzzy soft preopen set in (X, $\tilde{\tau}$,E).

Definition 3.2

Let (X, $\tilde{\tau}$,E) and (Y,$\tilde{\sigma}, K$) be two fuzzy soft topological spaces. A fuzzy soft mapping

g **:** (X, $\tilde{\tau}$,E) → (Y,$\tilde{\sigma}, K$) is said to be fuzzy soft pre irresolute if for each fuzzy soft pre open set $\tilde{\mu}$ in (Y,$\tilde{\sigma}, K$), the inverse image $g^{-1}(\tilde{\mu})$ is fuzzy soft pre open set in (X, $\tilde{\tau}$,E).

Definition 3.3

Let (X, $\tilde{\tau}$,E) and (Y,$\tilde{\sigma}, K$) be two fuzzy soft topological spaces. A fuzzy soft mapping

g **:** (X, $\tilde{\tau}$,E) → (Y,$\tilde{\sigma}, K$) is said to be fuzzy soft preopen if for each fuzzy soft open set $\tilde{\lambda}$ in (X, $\tilde{\tau}$,E), the image $g(\tilde{\lambda})$ is fuzzy soft preopen set in (Y,$\tilde{\sigma}, K$).

Definition 3.4

Let (X, $\tilde{\tau}$,E) and (Y,$\tilde{\sigma}, K$) be two fuzzy soft topological spaces. A fuzzy soft mapping

g **:** (X, $\tilde{\tau}$,E) → (Y,$\tilde{\sigma}, K$) is said to be fuzzy soft preclosed if for each fuzzy soft closed set $\tilde{\lambda}$ in (X, $\tilde{\tau}$,E), the image $g(\tilde{\lambda})$ is fuzzy soft preclosed set in (Y,$\tilde{\sigma}, K$).

Proposition 3.5

For a fuzzy soft mapping g = (ρ,ψ) **:** (X, $\tilde{\tau}$,E) → (Y,$\tilde{\sigma}, K$), the following are equivalent

(i) g is fuzzy soft pre-continuous.
(ii) The inverse image of every fuzzy soft closed set in (Y,$\tilde{\sigma}, K$) is fuzzy soft preclosed in (X, $\tilde{\tau}$,E).

Proof:

Suppose (i) holds. Let $\tilde{\mu}$ be a fuzzy soft closed in (Y,$\tilde{\sigma}, K$). Then $(\tilde{\mu})^C$ is fuzzy soft open in (Y,$\tilde{\sigma}, K$).Using definition 3.1, $g^{-1}((\tilde{\mu})^C)$ is fuzzy soft preopen. Since $g^{-1}((\tilde{\mu})^C) = [g^{-1}(\tilde{\mu})]^C$, $g^{-1}(\tilde{\mu})$ is fuzzy soft preclosed. This proves (i) ⟹ (ii).

Conversely we assume that (ii) holds. Let $\tilde{\mu}$ be fuzzy soft open in (Y,$\tilde{\sigma}, K$). Therefore $(\tilde{\mu})^C$ is fuzzy soft closed set in (Y,$\tilde{\sigma}, K$). Then by applying (ii), $[g^{-1}(\tilde{\mu})]^C$ is fuzzy soft preclosed in (X, $\tilde{\tau}$,E). That implies $g^{-1}(\tilde{\mu})$ is fuzzy soft preopen in (X, $\tilde{\tau}$,E). This proves (ii) ⟹ (i).

Proposition 3.6

For a fuzzy soft mapping **g** = (ρ,ψ) **:** (X, $\tilde{\tau}$,E) → (Y,$\tilde{\sigma}, K$). If g is fuzzy soft pre irresolute then it is fuzzy soft pre continuous.

Proof:

Suppose g is fuzzy soft pre irresolute. Let $\tilde{\mu}$ be a fuzzy soft open set in (Y,$\tilde{\sigma}, K$). Since every

fuzzy soft open set is fuzzy soft pre open and since g is fuzzy soft irresolute, by using Definition 3.2, $g^{-1}(\tilde{\mu})$ is fuzzy soft preopen. That implies g is fuzzy soft pre continuous.

Proposition 3.7

A fuzzy soft mapping **g** = (ρ,ψ) **:** (X, $\tilde{\tau}$,E) → (Y,$\tilde{\sigma}, K$) is fuzzy soft pre continuous iff $g^{-1}(\widetilde{fs}Int\tilde{\mu}) \tilde{\subseteq} \widetilde{fs}PInt(g^{-1}(\tilde{\mu}))$ for every fuzzy soft set $\tilde{\mu}$ in (Y,$\tilde{\sigma}, K$).

Proof:

Let g **:** (X, $\tilde{\tau}$,E) → (Y,$\tilde{\sigma}, K$) be fuzzy soft pre continuous. . Let $\tilde{\mu}$ be a fuzzy soft set in (Y,$\tilde{\sigma}, K$).

Then $\widetilde{fs}Int(\tilde{\mu})$ is fuzzy soft open in Y. Since g is fuzzy soft pre continuous, by using Definition 3.1, $g^{-1}(\widetilde{fs}Int(\tilde{\mu}))$is fuzzy soft pre open in (X, $\tilde{\tau}$,E). Then by using lemma 2.18, $g^{-1}\left(\widetilde{fs}Int(\tilde{\mu})\right) \subseteq g^{-1}(\tilde{\mu})$. This implies that $\widetilde{fs}PIntg^{-1}\left(\widetilde{fs}Int(\tilde{\mu})\right) \subseteq \widetilde{fs}Pint(g^{-1}(\tilde{\mu}))$.

Therefore $g^{-1}(\widetilde{fs}Int\tilde{\mu}) \tilde{\subseteq} \widetilde{fs}PInt(g^{-1}(\tilde{\mu}))$.

Conversely we assume that, $g^{-1}(\widetilde{fs}Int\tilde{\mu}) \tilde{\subseteq} \widetilde{fs}PInt(g^{-1}(\tilde{\mu}))$ for every fuzzy soft set $\tilde{\mu}$ in (Y,$\tilde{\sigma}, K$). In particular the above statement is true for fuzzy soft open sets in $\tilde{\mu}$. If

$\tilde{\mu}$ is fuzzy soft open sets in Y, $g^{-1}(\tilde{\mu}) \tilde{\subseteq} \widetilde{fs}PInt(g^{-1}(\tilde{\mu})) \tilde{\subseteq} g^{-1}(\tilde{\mu})$. That implies $g^{-1}(\tilde{\mu}) = \widetilde{fs}PInt(g^{-1}(\tilde{\mu}))$ is fuzzy soft pre open. Therefore g is fuzzy soft pre continuous.

Proposition 3.8

A fuzzy soft mapping **g** = (ρ,ψ) **:** (X, $\tilde{\tau}$,E) → (Y,$\tilde{\sigma}, K$) is fuzzy soft pre continuous iff $g(\widetilde{fs}Pcl\tilde{\lambda}) \subseteq \widetilde{fs}Cl(g(\tilde{\lambda}))$ for every fuzzy soft set $\tilde{\lambda}$ in (X, $\tilde{\tau}$,E).

Proof:

Let g **:** (X, $\tilde{\tau}$,E) → (Y,$\tilde{\sigma}, K$) be fuzzy soft pre continuous. Let $\tilde{\lambda}$ be fuzzy soft set in (X, $\tilde{\tau}$,E). Then $g(\tilde{\lambda})$ is fuzzy soft set in (Y,$\tilde{\sigma}, K$). Since g is fuzzy soft pre continuous, by using Definition 3.1, $g^{-1}(\widetilde{fs}Clg(\tilde{\lambda}))$ is fuzzy soft pre closed in (X, $\tilde{\tau}$,E). Since $g(\tilde{\lambda}) \subseteq (\widetilde{fs}Clg(\tilde{\lambda}))$,

$g^{-1}\left(g(\tilde{\lambda})\right) \subseteq g^{-1}\left(\widetilde{fs}Clg(\tilde{\lambda})\right)$, $\tilde{\lambda} \subseteq g^{-1}\left(g(\tilde{\lambda})\right) \subseteq g^{-1}\left(\widetilde{fs}Clg(\tilde{\lambda})\right)$. This implies that

$(\widetilde{fs}Pcl\tilde{\lambda}) \subseteq g^{-1}(\widetilde{fs}Clg(\tilde{\lambda}))$. Therefore $g(\widetilde{fs}Pcl\tilde{\lambda}) \subseteq g(g^{-1}(\widetilde{fs}Clg(\tilde{\lambda}))) \subseteq \widetilde{fs}Clg(\tilde{\lambda})$.

Conversely we assume that, $g(\widetilde{fs}Pcl\tilde{\lambda}) \subseteq \widetilde{fs}Cl(g(\tilde{\lambda}))$ for every fuzzy soft set $\tilde{\lambda}$ in (X, $\tilde{\tau}$,E).

Let $\tilde{\mu}$ be a fuzzy soft closed in (Y,$\tilde{\sigma}, K$). Let $\tilde{\lambda} = g^{-1}(\tilde{\mu})$. Since by our assumption,

$g(\widetilde{fs}Pcl\tilde{\lambda}) \subseteq \widetilde{fs}Cl(g(\tilde{\lambda}))$, $g(\widetilde{fs}Pclg^{-1}(\tilde{\mu})) \subseteq \widetilde{fs}Clg(g^{-1}(\tilde{\mu})) \subseteq \widetilde{fs}Cl\tilde{\mu}$.

$g\left(\widetilde{fs}Pclg^{-1}(\tilde{\mu})\right) \subseteq \widetilde{fs}Cl\tilde{\mu} = \tilde{\mu}$. $g^{-1}(g\left(\widetilde{fs}Pclg^{-1}(\tilde{\mu})\right) \subseteq g^{-1}(\tilde{\mu})$. $\widetilde{fs}Pclg^{-1}(\tilde{\mu}) \subseteq g^{-1}(\tilde{\mu})$.

This implies that $g^{-1}(\tilde{\mu}) = \widetilde{fs}Pclg^{-1}(\tilde{\mu})$. Therefore $g^{-1}(\tilde{\mu})$ is fuzzy soft pre closed. Hence

g is fuzzy soft pre continuous.

Proposition 3.9

For a fuzzy soft mapping **g** = (ρ,ψ) **:** (X, $\tilde{\tau}$,E) → (Y,$\tilde{\sigma}, K$). The following are equivalent.

i. g is fuzzy soft pre continuous.

ii. $g(\widetilde{fs}Pcl(\tilde{\lambda})) \tilde{\subseteq} \widetilde{fs}Pclg(\tilde{\lambda})$, for every fuzzy soft semi open set $\tilde{\lambda}$.

iii. $g(\widetilde{fs}Pcl(\tilde{\lambda})) \tilde{\subseteq} \widetilde{fs}\alpha clg(\tilde{\lambda})$, for every fuzzy soft semi pre open set $\tilde{\lambda}$.

Proof:

Assume (i) holds. By Proposition 3.8, $g(\widetilde{fs}Pcl\tilde{\lambda}) \subseteq \widetilde{fs}Cl(g(\tilde{\lambda}))$ for every fuzzy soft set $\tilde{\lambda}$ in (X, $\tilde{\tau}$,E) . Since $\widetilde{fs}Cl(g(\tilde{\lambda}))$ = $\widetilde{fs}Pclg(\tilde{\lambda})$, for every fuzzy soft semi open set $\tilde{\lambda}$.

This proves (i) ⟹ (ii).

Assume (ii) holds, $g(\widetilde{fs}Pcl(\tilde{\lambda})) \tilde{\subseteq} \widetilde{fs}Pclg(\tilde{\lambda})$, for every fuzzy soft semi open set $\tilde{\lambda}$. Let $\tilde{\mu}$ be fuzzy soft closed set in (Y,$\tilde{\sigma}, K$) and let $\tilde{\lambda}$ = $g^{-1}(\tilde{\mu})$.

$g(\widetilde{fs}Pcl(g^{-1}(\tilde{\mu}))) \tilde{\subseteq} \widetilde{fs}Pclg(g^{-1}(\tilde{\mu})) \tilde{\subseteq} \widetilde{fs}Pcl(\widetilde{\mu})$,

$g(\widetilde{fs}Pcl(g^{-1}(\tilde{\mu})))\tilde{\subseteq} \widetilde{fs}Pcl(\widetilde{\mu})$ = $\tilde{\mu}$,

$g^{-1}(g(\widetilde{fs}Pcl(g^{-1}(\tilde{\mu})))\tilde{\subseteq} g^{-1}(\tilde{\mu})$,

$g^{-1}(\tilde{\mu}) = \widetilde{fs}Pcl(g^{-1}(\tilde{\mu})$. That implies $g^{-1}(\tilde{\mu})$ is fuzzy soft pre closed. Therefore g is fuzzy soft pre continuous. This proves (ii) ⟹ (i).

Assume (i) holds. By Proposition 5.2.13, $g(\widetilde{fs}Scl\tilde{\lambda}) \subseteq \widetilde{fs}Cl(g(\tilde{\lambda}))$ for every fuzzy soft set $\tilde{\lambda}$ in (X, $\tilde{\tau}$,E). Since $\widetilde{fs}Cl(g(\tilde{\lambda}))$ = $\widetilde{fs}\alpha clg(\tilde{\lambda})$, for every fuzzy soft semi pre open set $\tilde{\lambda}$. This proves (i) ⟹ (iii).

Assume (iii) holds, $g(\widetilde{fs}Pcl(\tilde{\lambda})) \tilde{\subseteq} \widetilde{fs}\alpha clg(\tilde{\lambda})$, for every fuzzy soft semi pre open set $\tilde{\lambda}$. Let $\tilde{\mu}$ be fuzzy soft closed set in (Y,$\tilde{\sigma}, K$) and let $\tilde{\lambda}$ = $g^{-1}(\tilde{\mu})$.

$g(\widetilde{fs}Pcl(g^{-1}(\tilde{\mu}))) \tilde{\subseteq} \widetilde{fs}\alpha clg(g^{-1}(\tilde{\mu})) \tilde{\subseteq} \widetilde{fs}\alpha cl(\widetilde{\mu})$,

$g(\widetilde{fs}Pcl(g^{-1}(\tilde{\mu})))\tilde{\subseteq} \widetilde{fs}\alpha cl(\widetilde{\mu})$ = $\tilde{\mu}$,

$g^{-1}(g(\widetilde{fs}Pcl(g^{-1}(\tilde{\mu})))\tilde{\subseteq} g^{-1}(\tilde{\mu})$,

$g^{-1}(\tilde{\mu}) = \widetilde{fs}Pcl(g^{-1}(\tilde{\mu})$. That implies $g^{-1}(\tilde{\mu})$ is fuzzy soft pre closed. Therefore g is fuzzy soft pre continuous. This proves (iii) ⟹ (i).

Proposition 3.10

A fuzzy soft mapping g = (ρ,ψ) **:** (X, $\tilde{\tau}$,E) → (Y,$\tilde{\sigma}, K$) is fuzzy soft pre open iff

$g(\widetilde{fs}Int\tilde{\lambda}) \subseteq \widetilde{fs}PIntg(\tilde{\lambda})$ for every fuzzy soft set $\tilde{\lambda}$ in (X, $\tilde{\tau}$,E).

Proof:

Let g = (ρ,ψ) **:** (X, $\tilde{\tau}$,E) → (Y,$\tilde{\sigma}, K$) be fuzzy soft pre open. Let $\tilde{\lambda}$ be fuzzy soft open set in (X, $\tilde{\tau}$,E). The$\widetilde{fs}Int(\tilde{\lambda})$ n is fuzzy soft set in (X, $\tilde{\tau}$,E). Since g is fuzzy soft pre open, by Definition 3.4, $g(\widetilde{fs}Int(\tilde{\lambda}))$ is fuzzy soft pre open in (Y,$\tilde{\sigma}, K$). Then by using lemma 2.18,

$g(\widetilde{fs}Int(\tilde{\lambda})) \subseteq g(\tilde{\lambda})$, $\widetilde{fs}PIntg(\widetilde{fs}Int(\tilde{\lambda})) \subseteq \widetilde{fs}Pintg(\tilde{\lambda})$. Therefore $g(\widetilde{fs}Int\tilde{\lambda}) \subseteq \widetilde{fs}PIntg(\tilde{\lambda})$.

Conversely we assume that $g(\widetilde{fs}Int\tilde{\lambda}) \subseteq \widetilde{fs}PIntg(\tilde{\lambda})$ for every fuzzy soft set $\tilde{\lambda}$ in (X, $\tilde{\tau}$,E).

In particular the above statement is true for fuzzy soft open sets in $\tilde{\lambda}$. If $\tilde{\lambda}$ is fuzzy soft open in $\tilde{\lambda}$,

$g(\tilde{\lambda}) \subseteq \widetilde{fs}PIntg(\tilde{\lambda}) \subseteq g(\tilde{\lambda})$. That implies $g(\tilde{\lambda}) = \widetilde{fs}PIntg(\tilde{\lambda})$ is fuzzy soft pre open. Therefore g is fuzzy soft pre continuous.

Proposition 3.11

A fuzzy soft mapping g = (ρ,ψ) **:** (X, $\tilde{\tau}$,E) → (Y,$\tilde{\sigma}, K$) is fuzzy soft pre closed iff $\widetilde{fs}Pclg(\tilde{\lambda}) \subseteq g(\widetilde{fs}Cl\tilde{\lambda})$ for every fuzzy soft set $\tilde{\lambda}$ in (X, $\tilde{\tau}$,E).

Proof:

Let g = (ρ,ψ) **:** (X, $\tilde{\tau}$,E) → (Y,$\tilde{\sigma}, K$) be fuzzy soft pre closed. Let $\tilde{\lambda}$ be fuzzy soft set in (X, $\tilde{\tau}$,E). Then $\widetilde{fs}Cl(\tilde{\lambda})$ is fuzzy soft closed set in (X, $\tilde{\tau}$,E). Since g is fuzzy soft pre closed, by Definition 3. 5, $g(\widetilde{fs}Cl(\tilde{\lambda}))$ is fuzzy soft pre closed in (Y,$\tilde{\sigma}, K$). Since $g(\tilde{\lambda}) \subseteq g(\widetilde{fs}Cl(\tilde{\lambda}))$, $\widetilde{fs}Pclg(\tilde{\lambda}) \subseteq \widetilde{fs}PCl\, g\left(\widetilde{fs}Cl(\tilde{\lambda})\right) = g\left(\widetilde{fs}Cl(\tilde{\lambda})\right)$. Therefore $\widetilde{fs}Pclg(\tilde{\lambda}) \subseteq g\left(\widetilde{fs}Cl(\tilde{\lambda})\right)$.

Conversely we assume that, $\widetilde{fs}Pclg(\tilde{\lambda}) \subseteq g(\widetilde{fs}Cl\tilde{\lambda})$ for every fuzzy soft set $\tilde{\lambda}$ in (X, $\tilde{\tau}$,E).

Let $\tilde{\lambda}$ be fuzzy soft closed in (X, $\tilde{\tau}$,E). By our assumption, $\widetilde{fs}Pclg(\tilde{\lambda}) \subseteq g(\widetilde{fs}Cl\tilde{\lambda}) = g(\tilde{\lambda}) \subseteq \widetilde{fs}PClg(\tilde{\lambda})$. Therefore $g(\tilde{\lambda}) = \widetilde{fs}PClg(\tilde{\lambda})$. Therefore $g(\tilde{\lambda})$ is fuzzy soft pre closed.

Theorem 3.12

Let g = (ρ,ψ) **:** (X, $\tilde{\tau}$,E) → (Y,$\tilde{\sigma}, K$) be fuzzy soft mapping. Then the following are equivalent.

i. g is fuzzy soft pre continuous.
ii. The inverse image of every fuzzy soft closed set in (Y,$\tilde{\sigma}$,K) is fuzzy soft pre closed in (X, $\tilde{\tau}$,E).
iii. $g^{-1}(\widetilde{fs}Int\tilde{\mu}) \tilde{\subseteq} \widetilde{fs}PInt(g^{-1}(\tilde{\mu}))$ for every fuzzy soft set $\tilde{\mu}$ in (Y,$\tilde{\sigma}, K$).
iv. $g(\widetilde{fs}Pcl\tilde{\lambda}) \subseteq \widetilde{fs}Cl(g(\tilde{\lambda}))$ for every fuzzy soft set $\tilde{\lambda}$ in (X, $\tilde{\tau}$,E).
v. $g(\widetilde{fs}Pcl(\tilde{\lambda})) \tilde{\subseteq} \widetilde{fs}Pclg(\tilde{\lambda})$, for every fuzzy soft semi open set $\tilde{\lambda}$.
vi. $g(\widetilde{fs}Pcl(\tilde{\lambda})) \tilde{\subseteq} \widetilde{fs}\alpha clg(\tilde{\lambda})$, for every fuzzy soft semi pre open set $\tilde{\lambda}$.

Proof:

Follows from proposition 3.5, proposition 3.7, proposition 3.8, proposition 3.9.

Remark 3.13

The above discussions give the following implication diagram.

Fuzzy soft continuous mapping $\longrightarrow$ Fuzzy soft pre continuous mapping.

4. CONCLUSION

Fuzzy soft pre continuous mappings have been characterized using recent concepts in the literature of fuzzy soft topology.

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