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Characterizations of Extremally Disconnected and Submaximal Binary Fuzzy Soft Spaces

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Abstract

The aim of this work is to introduce extremally disconnected and submaximal binary fuzzy soft topological space in order to give a new characterization of them. First we discuss, more about the binary fuzzy soft interior and binary fuzzy soft closure operators in binary fuzzy soft topology and illustrate them with some examples. Extremally disconnected spaces are topological spaces with the property that the closure of every open set is open. This means that the open sets in the space are completely separated from each other. In binary fuzzy soft topological spaces, this notion can be studied. we introduce new classes of binary fuzzy soft sets, so-called: A -sets, B-sets, C -sets, S-sets, α∗-open sets, s-sets, t-sets, β-set, β∗∗-set and locally closed sets.

Keywords: extremally disconnected, submaximal, binary fuzzy soft set, binary fuzzy soft topology, open sets, closed sets

1 Introduction

Structures dealing with two universal sets are important in mathematics because they allow us
to study the relationships and interactions between two distinct mathematical objects or systems. These kinds of structures have applications in many fields, including mathematics, physics, computer sciences and engineering. Two common structures that deal with two universal sets are fuzzy sets, soft sets, fuzzy soft sets [1, 9, 16]. These structures provide a way to classify information, allowing for more nuanced and flexible reasoning and analysis. It is the case that the notion of binary topology introduced by Jothi and Thangavelu [6] plays an important role in providing notions of proximity between structures dealing with two universal sets. A binary topology M from a nonempty set X to a nonempty set Y is a subset of $P(X) \times$

P(Y) that satisfies some conditions of stability for operations related to unions and intersections. The binary topology was defined recently and has left the door open for the development of works related to this concept. To name a few instances, in [3–5] some notions of weaker binary open sets were introduced and some characterizations were obtained. In [7, 8, 10, 15], separation axioms associated with binary topological spaces, binary soft sets and weaker notions of binary open sets were studied respectively. [11, 12, 13, 14]The binary fuzzy soft topology was defined recently and has left the door open for the development of works related to this concept. Within this work, we establish additional properties of closure and interior for a binary fuzzy soft open sets and provide examples that these kinds of operations don't behave in the same way as the closure and interior operations of a topological space. After that, we introduce the notions of extremally disconnected and submaximal spaces in binary fuzzy soft topological spaces and provide some properties. Finally, we introduce new classes of binary fuzzy soft sets, so-called: A -sets, B-sets, C -sets, S-sets, $\alpha*$-open sets, s-sets, t-sets, β-set, $\beta**$-set and locally closed sets. We study the relationship between them and illustrate with some examples.

2 Preliminaries

Throughout this section, P(X) and P(Y) denote the power set of X and Y , respectively.For (A_e, B_e), $(C_e, D_e) \in P(X) \times P(Y)$, the following notations will be assumed:

(1) $(A_e, B_e) \subseteq (C_e, D_e)$ means that $A_e \subseteq C_e$ and $B_e \subseteq D_e$. It can be expressed as (A_e, B_e) is a subset of (C_e, D_e).

(2) $(X, Y) \setminus (A_e, B_e) = (X \setminus A_e, Y \setminus B_e)$.

All definitions and results given in this section were introduced and proved in [6].

Definition 2.1. Let X and Y be any two nonempty sets. A binary fuzzy soft topology from X to Y is a binary fuzzy soft structure $\mathfrak{I} \subset P(X) \times P(Y)$ that satisfies the following properties:

(1) $(\emptyset, \emptyset)$ and (X, Y) belong to $\mathfrak{I}$.

(2) If $(A1,B1) \in \mathfrak{I}$ and $(A2,B2) \in \mathfrak{I}$, then $(A1 \cap A2,B1 \cap B2) \in \mathfrak{I}$.

(3) If $\{(Ai,Bi)\}i\in I$ is a family of elements of $\mathfrak{I}$, then $(\cup i\in IAi,\cup i\in IBi) \in \mathfrak{I}$.

If $\mathfrak{I}$ is a binary fuzzy soft topology from X to Y, then the triple $(X, Y, E, \mathfrak{I})$ is called a binary fuzzy soft topological space, and the members of $\mathfrak{I}$, denoted by (A_e, B_e), are called the binary fuzzy soft open sets of the binary fuzzy soft topological space $(X, Y, E, \mathfrak{I})$. The pair (A_e, B_e) is called binary fuzzy soft closed in $(X, Y, E, \mathfrak{I})$ if $(X \setminus A, Y \setminus B) \in \mathfrak{I}$.

Note that a binary fuzzy soft topology $\mathfrak{I}$ from X to Y isn't a topology on $X \times Y$. Firstly, because $\mathfrak{I}$ is a subset of $P(X) \times P(Y)$, while a topology on $X \times Y$ is a subset of $P(X \times Y)$. Secondly, one must be very careful with the union of sets operation. Although the elements of $\mathfrak{I}$ can be understood as a Cartesian product, it is well known that the union and the Cartesian product do not distribute. Therefore the union of elements of $\mathfrak{I}$ is not necessarily an element of $\mathfrak{I}$.

Definition 2.2. Let $(X, Y, E, \mathfrak{I})$ be a binary fuzzy soft topological space, $(F_e, G_e) \subseteq (X, Y)$ and $(F_e, G_e)^{1\square} = \cup \{F_{e(\mu)}: (F_{e(\mu)}, G_{e(\mu)}) \in \mathfrak{I}$ and $(F_{e(\mu)}, G_{e(\mu)}) \subseteq (F_e, G_e)\}$,

$(F_e, G_e)^{2\square} = \cup \{G_{e(\mu)}: (F_{e(\mu)}, G_{e(\mu)}) \in \mathfrak{I}$ and $(F_{e(\mu)}, G_{e(\mu)}) \subseteq (F_e, G_e)\}$.

Then the pair $((F_e, G_e)^{1\square}, (F_e, G_e)^{2\square})$ is called the binary soft interior of (F_e, G_e) and denoted by Int $((F_e, G_e))$.

Definition 2.3. Let $(X, Y, E, \mathfrak{I})$ be a binary fuzzy soft topological space, $(F_e, G_e) \subseteq (X, Y)$ and $(F_e, G_e)^{1\blacksquare} = \cap \{F_{e(\mu)} : (X \setminus F_{e(\mu)}, Y \setminus G_{e(\mu)}) \in \mathfrak{I}$ and $(F_e, G_e) \subseteq (F_{e(\mu)}, G_{e(\mu)})\}$,

$(F_e, G_e)^{2\blacksquare} = \cap \{G_{e(\mu)} : (X \setminus F_{e(\mu)}, Y \setminus G_{e(\mu)}) \in \mathfrak{I}$ and $((F_e, G_e) \subseteq (F_{e(\mu)}, G_{e(\mu)})\}$. Then the pair $((F_e, G_e)^{1\blacksquare}, (F_e, G_e)^{2\blacksquare})$ is called the binary fuzzy soft closure of (F_e, G_e) and denoted by $Cl((F_e, G_e))$.

Some properties of binary fuzzy soft interior and binary fuzzy soft closure are listed below.

Theorem 2.4. Let $(X, Y, E, \mathfrak{I})$ be a binary fuzzy soft topological space and (A_e, B_e), (C_e, D_e) be the subsets of (X, Y). Then the following statements hold:
(1) $Int((A_e, B_e)) \in M$.
(2) $Int((A_e, B_e)) \subseteq (A_e, B_e)$.
(3) $(A_e, B_e) \in M$ if and only if $Int((A_e, B_e)) = (A_e, B_e)$.
(4) If $(A_e, B_e) \subseteq (C_e, D_e)$, then $Int((A_e, B_e)) \subseteq Int((C_e, D_e))$.
(5) $Int(Int((A_e, B_e))) = Int((A_e, B_e))$.
(6) $Cl((A_e, B_e))$ is a binary fuzzy soft closed set.
(7) $(A_e, B_e) \subseteq Cl((A_e, B_e))$.
(8) If $(A_e, B_e) \subseteq (C_e, D_e)$, then $Cl((A_e, B_e)) \subseteq Cl((C_e, D_e))$.
(9) (A_e, B_e) is a binary fuzzy soft closed set if and only if $Cl((A_e, B_e)) = (A_e, B_e)$.
(10) $Cl(Cl((A_e, B_e))) = Cl((A_e, B_e))$.

Like in general topology, many authors have defined the sets in terms of interior and closure, as shown in the following

Definition 2.5. Let $(X, Y, E, \mathfrak{I})$ be a binary fuzzy soft topological space and $(A_e, B_e) \subseteq (X, Y)$. Then, (A_e, B_e) is said to be:
(1) A binary fuzzy soft regular open set [8] if $(A_e, B_e) = Int(Cl((A_e, B_e)))$.
(2) A binary fuzzy soft regular closed set [8] if $(A_e, B_e) = Cl(Int((A_e, B_e)))$.
(3) A binary fuzzy soft semi pre open set [5] if $(A_e, B_e) \subseteq Cl(Int(Cl((A_e, B_e))))$.
(4) A binary fuzzy soft semi pre closed set [5] if $Int(Cl(Int((A_e, B_e)))) \subseteq (A_e, B_e)$.
(5) A binary fuzzy soft semi open set [3] if $(A_e, B_e) \subseteq Cl(Int((A_e, B_e)))$.
(6) A binary fuzzy soft semi closed set [3] if $Int(Cl((A_e, B_e))) \subseteq (A_e, B_e)$.
(7) A binary fuzzy soft pre open set [4] if $(A_e, B_e) \subseteq Int(Cl((A_e, B_e)))$.
(8) A binary fuzzy soft pre closed set [4] if $Cl(Int((A_e, B_e))) \subseteq (A_e, B_e)$.
(9) A binary fuzzy soft α-open set if $(A_e, B_e) \subseteq Int(Cl(Int((A_e, B_e))))$.

3. Extremally Disconnected and Submaximal Binary Fuzzy Soft topological Spaces

Extremally disconnected spaces are topological spaces with the property that the closure of every open set is open. This means that the open sets in the space are completely separated from each other. In binary fuzzy soft topological spaces, this notion can be studied.

Definition 3.1. Let $(X, Y, E, \mathfrak{I})$ be a binary fuzzy soft topological space. We said that $(X, Y, E, \mathfrak{I})$ is an extremally disconnected binary fuzzy soft space if the binary fuzzy soft closure of every binary fuzzy soft open set is a binary fuzzy soft open set.

The following result provides a characterization of extremely disconnected binary fuzzy soft spaces in terms of binary fuzzy soft semi-open and binary fuzzy soft pre-open sets.

Theorem 3.2. Let $(X, Y, E, \Im)$ be a binary fuzzy soft topological space. Then $(X, Y, E, \Im)$ is an extremally disconnected binary fuzzy soft space if and only if every binary fuzzy soft semi-open set is a binary fuzzy soft pre-open set.

Proof. Let (A_e, B_e) be a binary fuzzy soft semi open set, since $(X, Y, E, \Im)$ is an extremally disconnected binary fuzzy soft space, we have

$$Cl(Int((A_e, B_e))) = Int(Cl(Int((A_e, B_e))))$$

and therefore

$$(A_e, B_e) \subseteq Cl(Int((A_e, B_e))) = Int(Cl(Int((A_e, B_e)))) \subseteq Int(Cl((A_e, B_e))),$$

hence (A_e, B_e) is a binary fuzzy soft pre-open set.

Now, let (A_e, B_e) be a binary fuzzy soft open set, then $(A_e, B_e) \subseteq Cl(Int((A_e, B_e)))$, that means (A_e, B_e) is a binary fuzzy soft semi-open set and by the hypothesis, (A_e, B_e) is a binary fuzzy soft pre-open set. It follows that

$$Cl(Int((A_e, B_e))) \subseteq Cl(Int(Cl(Int((A_e, B_e))))).$$

So, $Cl(Int((A_e, B_e)))$ is a binary fuzzy soft semi-open set and by the hypothesis, is a binary fuzzy soft pre-open set, therefore $Cl(A_e, B_e) = Cl(Int((A_e, B_e)))$
$\subseteq Int(Cl(Cl(Int((A_e, B_e)))))$
$= Int(Cl((A_e, B_e)))$.

Hence $(X, Y, E, \Im)$ is an extremally disconnected binary fuzzy soft space.

A submaximal topological space is a topological space satisfying that every dense subset is open. Now, we are going to define a binary fuzzy soft dense set and, consequently, a submaximal binary fuzzy soft space. A subset (A_e, B_e) of a binary fuzzy soft topological space is said to be a binary fuzzy soft dense set, if $(X, Y) = Cl((A_e, B_e))$.

Definition 3.3. Let $(X, Y, E, \Im)$ be a binary fuzzy soft topological space. We say that $(X, Y, E, \Im)$ is a submaximal binary fuzzy soft space if every binary fuzzy soft dense set is a binary fuzzy soft open set.

Now, we are going to characterize the submaximal binary fuzzy soft spaces.

Theorem 3.4. Let $(X, Y, E, \Im)$ be a binary fuzzy soft topological space. $(X, Y, E, \Im)$ is a submaximal binary fuzzy soft space if and only if for $(A_e, B_e) \subseteq (X, Y)$ such that $Int(A_e, B_e) = (\emptyset, \emptyset)$, it is true that (A_e, B_e) is a binary fuzzy soft closed set.

Proof. Suppose that $(X, Y, E, \Im)$ is a submaximal binary fuzzy soft space and let (A_e, B_e) be a binary fuzzy soft subset of (X, Y) such that $Int(A_e, B_e) = (\emptyset, \emptyset)$, then

$Cl(X \setminus A_e, Y \setminus B_e) = (X, Y)$ and by the hypothesis, $(X \setminus A_e, Y \setminus B_e) \in \Im$, hence (A_e, B_e) is a binary fuzzy soft closed set.

Now, let (A_e, B_e) be a binary fuzzy soft subset of (X, Y) such that $Cl(A_e, B_e) = (X, Y)$, then $Int(X \setminus A_e, Y \setminus B_e) = (\emptyset, \emptyset)$ and by the hypothesis, it is true that

$(X \setminus A_e, Y \setminus B_e) = (X \setminus A_e, Y \setminus B_e)$ so, $Int(A_e, B_e) = (A_e, B_e)$ and therefore (X, Y) is a binary fuzzy soft submaximal space.

4. New Classes of Binary Fuzzy Soft Sets and their Properties

In this section, using binary fuzzy soft closure and binary fuzzy soft interior, we are going to introduce a new class of binary fuzzy soft sets and to study the relationships between them.

Definition 4.1. Let $(X, Y, E, \mathfrak{I})$ be a binary fuzzy soft topological space and $(A_e, B_e) \subseteq (X, Y)$. Then (A_e, B_e) is said to be:

(1) A binary fuzzy soft $\alpha*$-set, if $Int(A_e, B_e) = Int(Cl(Int((A_e, B_e))))$.

(2) A binary fuzzy soft t-set, if $Int((A_e,B_e)) = Int(Cl((A_e, B_e)))$.

(3) A binary fuzzy soft s-set, if $Int((A_e, B_e)) = Cl(Int((A_e, B_e)))$.

(4) A binary fuzzy soft $\beta*$-set, if $Int((A_e, B_e)) = Cl(Int(Cl((A_e,B_e))))$.

Now, the existing relationships between the defined binary fuzzy soft sets are presented in

Theorem 4.2. Let $(X, Y, E, \mathfrak{I})$ be a binary fuzzy soft topological space and $(A_e, B_e) \subseteq (X, Y)$. Then the following statements hold:

(1) If (A_e, B_e) is a binary fuzzy soft t-set, then (A_e, B_e) is a binary fuzzy soft $\alpha*$-set.

(2) If (A_e, B_e) is a binary fuzzy soft s-set, then (A_e, B_e) is a binary fuzzy soft $\alpha*$-set.

(3) If (A_e, B_e) is a binary fuzzy soft $\beta*$-set, then (A_e, B_e) is both a binary fuzzy soft t-set and s-set.

(4) If (A_e, B_e) is a binary fuzzy soft regular open set, then (A_e, B_e) is a binary fuzzy soft t-set.

(5) If (A_e, B_e) is a binary fuzzy soft closed set, then (A_e, B_e) is a binary fuzzy soft t-set.

Proof.

(1) Note that $Int((A_e, B_e)) \subseteq Int(Cl(Int((A_e, B_e))))$.
Suppose that (A_e, B_e) is a binary fuzzy soft t-set, then
$Int((A_e, B_e)) = Int(Cl((A_e, B_e))) \supseteq Int(Cl(Int((A_e, B_e))))$,
hence $Int((A_e, B_e)) = Int(Cl(Int((A_e, B_e))))$ and therefore (A_e, B_e) is a binary fuzzy soft $\alpha*$-set.

(2) Note that $Int((A_e, B_e)) \subseteq Int(Cl(Int((A_e, B_e))))$.
Suppose that (A_e, B_e) is a binary fuzzy soft s-set, then
$Int((A_e, B_e)) = Cl(Int((A_e, B_e))) \supset Int(Cl(Int((A_e, B_e))))$,
hence $Int(Cl(Int((A_e, B_e)))) = Int((A_e, B_e))$ and therefore (A_e, B_e) is a binary fuzzy soft $\alpha*$-set.

(3) Let (A_e, B_e) be a binary fuzzy soft $\beta*$-set, then
$Int((A_e, B_e)) = Cl(Int(Cl((A_e, B_e)))) \supset Int(Cl((A_e, B_e)))$,
hence $Cl(Int((A_e, B_e))) = Int((A_e, B_e))$, and therefore (A_e, B_e) is a binary fuzzy soft $\beta*$-set. In a similar way, since $Int((A_e, B_e)) = Cl(Int(Cl((A_e, B_e)))) \supset Int(Cl((A_e, B_e)))$,
hence $Int(Cl((A_e, B_e))) = Int((A_e, B_e))$, and therefore (A_e, B_e) is a binary fuzzy soft t-set.

(4) Suppose that (A_e, B_e) is a binary fuzzy soft regular open set, then
$(A_e, B_e) = Int(Cl((A_e, B_e)))$ and therefore $Int((A_e, B_e)) = Int(Cl((A_e, B_e)))$,
that is (A_e, B_e) is a binary fuzzy soft t-set.

(5) Suppose that (A_e, B_e) is a binary fuzzy soft closed set, then
$Int((A_e, B_e)) = Int(Cl(A_e, B_e)))$, hence (A_e, B_e) is a binary fuzzy soft t-set.

Note. The notions of a binary fuzzy soft t-set and binary fuzzy soft s-set are independent, as well the reciprocal propositions of Theorem 5.1, are not true.

The next result gives us the conditions for the reciprocal propositions of Theorem 5.1, came to be true.

Theorem 4.3. Let $(X, Y, E, \mathfrak{I})$ be a binary fuzzy soft topological space and $(A_e, B_e) \subseteq (X, Y)$. Then the following statements hold:

(1) If (A_e, B_e) is a binary fuzzy soft $\alpha*$-set and binary fuzzy soft semi-open, then (A_e, B_e) is a binary fuzzy soft t-set.

(2) If (A_e, B_e) is a binary fuzzy soft $\alpha*$-set and $(X, Y, E, \mathfrak{I})$ is a extremally disconnected binary fuzzy soft space, then (A_e, B_e) is a binary fuzzy soft s-set.

(3) If (A_e, B_e) is a binary fuzzy soft t-set and a binary fuzzy soft open set, then (A_e, B_e) is a binary fuzzy soft regular open set.

Proof.

(1) Note that $Int((A_e, B_e)) \subseteq Int(Cl((A_e, B_e)))$. Suppose that (A_e, B_e) is a binary fuzzy soft $\alpha*$-set and binary fuzzy soft semi-open, then

$Int(Cl((A_e, B_e))) \subseteq Int(Cl(Int((A_e, B_e)))) = Int((A_e, B_e))$.

It follows that $Int(Cl((A_e, B_e))) = Int((A_e, B_e))$ and therefore (A_e, B_e) is a binary fuzzy soft t-set.

(2) Suppose that (A_e, B_e) is a binary fuzzy soft $\alpha*$-set, then $Int(Cl(Int((A_e, B_e)))) = Int((A_e, B_e))$. Since $(X, Y, E, \mathfrak{I})$ is an extremally disconnected space

$Int(Cl(Int((A_e, B_e)))) = Cl(Int((A_e, B_e)))$,

therefore $Cl(Int((A_e, B_e))) = Int((A_e, B_e))$, that is, (A_e, B_e) is a binary fuzzy soft s-set.

(3) Suppose that (A_e, B_e) is a binary fuzzy soft t-set and binary fuzzy soft open set, then $Int(Cl((A_e, B_e))) = Int((A_e, B_e)) = (A_e, B_e)$,

hence (A_e, B_e) is a binary fuzzy soft regular open set.

Theorem 4.4. Let $(X, Y, E, \mathfrak{I})$ be a binary fuzzy soft topological space and $(A_e, B_e) \subseteq (X, Y)$. Then the following statements hold:

(1) (A_e, B_e) is a binary fuzzy soft α-open set and a binary fuzzy soft $\alpha*$-set if and only if it is a binary fuzzy soft regular open set.

(2) (A_e, B_e) is a binary fuzzy soft α-open set if and only if it is a binary fuzzy soft semi-open set and a binary fuzzy soft pre-open set.

Proof.

(1) Since (A_e, B_e) is a binary fuzzy soft α-open set and a binary fuzzy soft $\alpha*$-set, then (A_e, B_e) is a binary fuzzy soft open set.

It follows that $(A_e, B_e) \subseteq Int(Cl((A_e, B_e)))$, and then $Int(Cl((A_e, B_e))) = Int(Cl(Int((A_e, B_e)))) = Int((A_e, B_e)) = (A_e, B_e)$. Reciprocally, since (A_e, B_e) is a binary fuzzy soft α-open set, it follows that $Int((A_e, B_e)) \subseteq Int(Cl(Int((A_e, B_e))))$. Now, since (A_e, B_e) is a binary fuzzy soft regular open set, it follows that $Int((A_e, B_e)) = Int(Cl((A_e, B_e))) \supseteq Int(Cl(Int((A_e, B_e))))$ and then (A_e, B_e) is a binary fuzzy soft $\alpha*$-set.

(2) It is clear that every binary fuzzy soft α-open set is a binary fuzzy soft semi open set and a binary fuzzy soft pre-open set.

Conversely, if (A_e, B_e) is a binary fuzzy soft semi-open set and a binary fuzzy soft pre-open set, then $(A_e, B_e) \subseteq Cl(Int((A_e, B_e)))$
and $(A_e, B_e) \subseteq Int(Cl((A_e, B_e)))$. It follows that $(A_e, B_e) \subseteq Int(Cl((Cl(Int((A_e, B_e)))))$ and then $(A_e, B_e) \subseteq Int(Cl(Int((A_e, B_e))))$.

Definition 4.5. Let $(X, Y, E, \Im)$ be a binary fuzzy soft topological space and (A_e, B_e) be a subset of (X, Y). Then (A_e, B_e) is said to be:

(1) A binary fuzzy soft A -set if $(A_e, B_e) = (U \cap C, V \cap D)$, where $(U, V) \in \Im$ and (C,D) is a binary fuzzy soft regular closed set.

(2) A binary fuzzy soft B-set if $(A_e, B_e) = (U \cap C, V \cap D)$, where $(U, V) \in \Im$ and (C,D) is a binary fuzzy soft t- set.

(3) A binary fuzzy soft C -set if $(A_e, B_e) = (U \cap C, V \cap D)$, where $(U, V) \in \Im$ and (C,D) is a binary fuzzy soft $\alpha*$- set.

(4) A binary fuzzy soft locally closed set if $(A_e, B_e) = (U \cap C, V \cap D)$, where $(U, V) \in \Im$ and (C,D) is a binary fuzzy soft closed set.

(5) A binary fuzzy soft β-set if $(A_e, B_e) = (U \cap C, V \cap D)$, where $(U, V) \in \Im$ and (C,D) is a binary fuzzy soft $\beta*$-set.

(6) A binary fuzzy soft S-set if $(A_e, B_e) = (U \cap C, V \cap D)$, where $(U, V) \in \Im$ and (C,D) is a binary fuzzy soft s-set.

Theorem 4.6. Let $(X, Y, E, \Im)$ be a binary fuzzy soft topological space. The following statements hold:

(1) Every binary fuzzy soft A -set is a binary fuzzy soft B-set.
(2) Every binary fuzzy soft B-set is a binary fuzzy soft C -set.
(3) Every binary fuzzy soft S-set is a binary fuzzy soft C -set.
(4) Every binary fuzzy soft locally closed set is a binary fuzzy soft B-set.
(5) Every binary fuzzy soft A -set is a binary fuzzy soft locally closed set.
(6) Every binary fuzzy soft β-set is both a binary fuzzy soft B-set and a binary fuzzy soft
C -set.
(7) Every binary fuzzy soft t-set is a binary fuzzy soft B-set.
(8) Every binary fuzzy soft $\alpha*$-set is a binary fuzzy soft C -set.
(9) Every binary fuzzy soft closed set is a binary fuzzy soft B-set.
(10) Every binary fuzzy soft $\beta*$-set is a binary fuzzy soft β-set.

Proof.

(1) Let (A_e, B_e) be a binary fuzzy soft A -set, then $(A_e, B_e) = (U \cap C, V \cap D)$ for some $(U, V) \in \Im$ and some (C,D) binary fuzzy soft regular closed set. As every binary fuzzy soft regular closed set is a binary fuzzy soft t-set, then $(A_e, B_e) = (U \cap C, V \cap D)$, for some $(U, V) \in \Im$ and some (C,D) binary fuzzy soft t-set, and therefore (A_e, B_e) is a binary fuzzy soft B-set.

(2) Let (A_e, B_e) be a binary fuzzy soft B-set, then $(A_e, B_e) = (U \cap C, V \cap D)$ for some $(U, V) \in \Im$ and some (C,D) binary fuzzy soft t-set. As every binary fuzzy soft t-set is a binary fuzzy soft $\alpha*$-set, then $(A_e, B_e) = (U \cap C, V \cap D)$, for some $(U, V) \in \Im$ and some (C,D) binary fuzzy soft $\alpha*$-set, and therefore (A_e, B_e) is a binary fuzzy soft C -set.

(3) Let (A_e, B_e) be a binary fuzzy soft S-set, then $(A_e, B_e) = (U \cap C, V \cap D)$ for some $(U, V) \in \Im$ and some (C,D) binary fuzzy soft s-set. As every binary fuzzy soft s-set is

a binary fuzzy soft α∗-set, then $(A_e, B_e) = (U \cap C, V \cap D)$, for some $(U, V) \in \Im$ and some (C,D) binary fuzzy soft α∗-set, and therefore (A_e, B_e) is a binary fuzzy soft C -set.

(4) Let (A_e, B_e) be a binary fuzzy soft locally closed set, then $(A_e, B_e) = (U \cap C, V \cap D)$, for some $(U, V) \in \Im$ and some (C,D) binary fuzzy soft closed set. As every binary fuzzy soft closed set is a binary fuzzy soft t-set, then $(A_e, B_e) = (U \cap C, V \cap D)$, for some $(U, V) \in \Im$ and some (C,D) binary fuzzy soft t-set, and therefore (A_e, B_e) is a binary fuzzy soft B-set.

(5) Let (A_e, B_e) be a binary fuzzy soft A -set, then $(A_e, B_e) = (U \cap C, V \cap D)$ for some $(U, V) \in \Im$ and some (C,D) binary fuzzy soft regular closed set. As every binary fuzzy soft regular closed set is a binary fuzzy soft closed set, then $(A_e, B_e) = (U \cap C, V \cap D)$, for some $(U, V) \in \Im$ and some (C,D) binary fuzzy soft closed set, and therefore (A_e, B_e) is a binary fuzzy soft B-set.

(6) Let (A_e, B_e) be a binary fuzzy soft β-set, then $(A_e, B_e) = (U \cap C, V \cap D)$ for some $(U, V) \in \Im$ and some (C,D) binary fuzzy soft β∗-set. As every binary fuzzy soft β∗-set is both a binary fuzzy soft t-set and a binary fuzzy soft s-set, and every binary fuzzy soft s-set is α∗-set, then $(A_e, B_e) = (U \cap C, V \cap D)$, for some $(U, V) \in \Im$ and some (C,D) binary fuzzy soft t-set; $(A_e, B_e) = (U \cap C, V \cap D)$, for some $(U, V) \in \Im$ and some (C,D) binary fuzzy soft α∗-set. Therefore (A_e, B_e) is a binary fuzzy B-set and a binary fuzzy soft C -set.

(7) Let (A_e, B_e) be a binary fuzzy soft t-set, then $(A_e, B_e) = (X \cap A_e, Y \cap B_e)$, note that $(X, Y) \in \Im$ and therefore $(A_e, B_e) = (X \cap A_e, Y \cap B_e)$, for $(X, Y) \in \Im$ and (A_e, B_e) binary fuzzy soft t-set; hence (A_e, B_e) is a binary fuzzy soft B-set.

(8) Let (A_e, B_e) be a binary fuzzy soft α∗-set, then $(A_e, B_e) = (X \cap A_e, Y \cap B_e)$, note that $(X, Y) \in \Im$ and therefore $(A_e, B_e) = (X \cap A_e, Y \cap B_e)$, for $(X, Y) \in \Im$ and (A_e, B_e) binary fuzzy soft α∗-set; hence (A_e, B_e) is a binary fuzzy soft C -set.

(9) Let (A_e, B_e) be a binary fuzzy soft closed set, then (A_e, B_e) is a binary fuzzy soft t-set and $(A_e, B_e) = (X \cap A_e, Y \cap B_e)$, note that $(X, Y) \in \Im$, and therefore $(A_e, B_e) = (X \cap A_e, Y \cap B_e)$, for $(X, Y) \in \Im$ and (A_e, B_e) binary fuzzy soft t-set; hence (A_e, B_e) is a binary fuzzy soft B-set.

(10) Let (A_e, B_e) be a binary fuzzy soft β∗-set, then $(A_e, B_e)=(X \cap A_e, Y \cap B_e)$, note that $(X, Y) \in \Im$, and therefore $(A_e, B_e)=(X \cap A_e, Y \cap B_e)$, for $(X, Y) \in \Im$ and (A_e, B_e) binary fuzzy soft β∗-set; hence (A_e, B_e) is a binary fuzzy soft β-set.

Theorem 4.7. Let $(X, Y, E, \Im)$ be a binary fuzzy soft topological space. If every subset (A_e, B_e) of (X, Y) is a binary fuzzy soft locally closed set, then $(X, Y, E, \Im)$ is a binary fuzzy soft submaximal space.

Proof. Let (A_e, B_e) be a binary fuzzy soft dense set. By the hypothesis, there exists (U, V) binary fuzzy soft open set and (C,D) binary fuzzy soft closed set such that $(A_e, B_e) = (U \cap C, V \cap D)$.

Note that $(X, Y) = Cl(A_e, B_e) = Cl(U \cap C, V \cap D) \subseteq Cl((C,D)) = (C,D)$.

So, $(A_e, B_e) = (U \cap X, V \cap Y) = (U, V) \in \Im$, and therefore $(X, Y, E, \Im)$ is a binary fuzzy soft submaximal space.

In the topological spaces, submaximal spaces are characterized in terms of locally closed sets, that is, X is a submaximal space if and only if every subset of X is a locally closed set.

5. Conclusions

In this work, making use of binary fuzzy soft topological spaces and associated notions of interior and closure, new classes of binary fuzzy soft sets are introduced, the relationship between them is studied, and the conditions such as extremely disconnected binary fuzzy soft spaces or submaximal binary fuzzy soft spaces are established in order to obtain characterizations of certain notions of binary fuzzy soft sets.

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