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An Interpolation Process on Laguerre Polynomial

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Abstract

In the present paper ,we have considered the problem in which $\{\xi_i\}_{i=1}^n$ and $\{\xi_i^*\}_{i=1}^n$ the two sets of interscaled nodal points on the interval [0,∞) . Here we deal with the problem in which function values are prescribed at the zeros of L_n^k(x) and the first derivative values are prescribed on the zeros L_n^{k-1}(x) . We investigate the existence, uniqueness explicit representation of interpolatory polynomial. Estimation of the fundamental polynomials leading to a convergence theorem have also been obtained.

Keywords: Lacunary interpolation, Pál - Type interpolation, Laguerre Polynomial
MSC 2000: 41 A 05 65 D 32.

1. INTRODUCTION

Pál [10], Mathur P. and Datta S. [8] and many other authors [1][2][6][7] [12] [14] have discussed about various kind of interpolation problems. . In 1975 Pál [10] proved that when the function values are prescribed on one set of n points and derivative values on other set of n-1 points , then there exist no unique polynomial of degree ≤ 2n-2 , but prescribing function value at one more point not belonging to former set of n points there exists a unique polynomial of degree ≤ 2n-1. Lénárd M. [5] investigated the Pál – type interpolation problem on the nodes of Laguerre abscissas. In this paper we study Pál – type interpolational polynomial with $\omega_{n+k}(x) = x^k L_n^{(k)}(x)$.we have examined the existence, uniqueness ,explicit representation and estimation of fundamental polynomials of such special kind of mixed type of interpolation on interval $[0,\infty)$. For this we have considered the problem in which $\{\xi_i\}_{i=1}^{n}$ and $\{\xi_i^*\}_{i=1}^{n}$ the two sets of interscaled nodal points.

(1.1) $$0 \leq \xi_0 < \xi_1^* < \xi_1 < \cdots < \xi_{n-1} < \xi_n^* < \xi_n < \infty$$

on the interval $[0,\infty)$. We seek to determine a polynomial $R_n(x)$ of minimal possible degree 3n+k satisfying the interpolatory conditions :

(1.2) $$R_n{}'(\xi_i) = g_i\ , \quad R_n(\xi_i^*) = g_i^* \quad , \ R_n{}'(\xi_i^*) = g_i^{**} \quad , \quad for\ i = 1(1)n$$

(1.3) $$R_n^{(j)}(\xi_0) = g_0^{(j)} \qquad , \quad j = 0,1,\dots,k$$

where g_i , g_i^*, $g_i^{**}$and $g_0^{(j)}$ are arbitrary real numbers. Here Laguerre polynomials $L_n^{(k)}(x)$ and $L_n^{(k-1)}(x)$ have zeroes $\{\xi_i\}_{i=1}^{n}$ and $\{\xi_i^*\}_{i=1}^{n}$ respectively and $x_0 = 0$. We prove existence, uniqueness , explicit representation and estimation of fundamental polynomials.

2. PRELIMINARIES

In this section we shall give some well-known results which are as follws :
As we know that the Laguerre polynomial is a constant multiple of a confluent hypergeometric function so the differential equation is given by

(2.1) $$xD^2L_n^k(x) + (1 + k - x)DL_n^k(x) + nL_n^k(x) = 0$$

(2.2) $$L_n^{(k-1)'}(x) = -L_{n-1}^{(k)}(x)$$

Also using the identities

(2.3) $$L_n^{(k)}(x) = L_n^{(k+1)}(x) - L_{n-1}^{(k+1)}(x)$$

(2.4) $$xL_n^{(k)'}(x) = nL_n^{(k)}(x) - (n + k)L_{n-1}^{(k)}(x)$$

We can easily find a relation

(2.5) $$\frac{d}{dx}[x^k L_n^k(x)] = (n + k)x^{k-1}L_n^{(k-1)}(x)$$

By the following conditions of orthogonality and normalization we define Laguerre polynomial $L_n^{(k)}(x)$, $for\ k > -1$

(2.6) $\int_0^\infty e^{-x}x^k L_n^{(k)}(x)L_m^{(k)}(x)dx = \Gamma k+1 \binom{n+k}{n}\delta_{nm}$ $\quad n,m = 0,1,2,\ldots..$

(2.7) $L_n^{(k)}(x) = \sum_{\mu=0}^{n}\binom{n+k}{n}\frac{(-x)^\mu}{\mu!}$

The fundamental polynomials of Lagrange interpolation are given by

(2.8) $l_j(x) = \frac{L_n^{(k)}(x)}{L_n^{(k)'}(x_j)(x-x_j)} = \delta_{i,j}$

(2.9) $l_j^*(x) = \frac{L_n^{(k-1)}(x)}{L_n^{(k-1)'}(x_j)(x-x_j)} = \delta_{i,j}$

(2.10) $l_j^{*'}(y_j) = \begin{cases} \frac{L_n^{(k-1)'}(y_i)}{L_n^{(k-1)'}(y_j)(y_i-y_j)} & i \neq j \\ -\frac{(k-y_j)}{2y_j} & i = j \end{cases} \quad i,j = 1(1)n$

(2.12) $l_j'(y_j) = \frac{1}{(y_j-x_j)}\left[\frac{L_n^{(k)'}(y_j)}{L_n^{(k)'}(x_j)} - \frac{L_n^{(k)}(y_j)}{L_n^{(k)'}(x_j)(y_j-x_j)}\right] \quad , \quad j = 1(1)n$

For the roots of $L_n^{(k)}(x)$ we have

(2.13) $x_k^2 \sim \frac{k^2}{n}$

(2.14) $\eta(x)\left|S_n^{(l)}(x)\right| = O(1)$ where $\eta(x)$ is the weight function

(2.15) $\left|L_n^{(k)'}(x_j)\right| \sim j^{-k-\frac{3}{2}}n^{k+1}, (0 < x_j \leq \Omega, n = 1,2,3,\ldots\ldots.)$

(2.16) $\left|L_n^k(x_j)\right| = \begin{cases} x^{-\frac{k}{2}-\frac{1}{4}}O\left(n^{\frac{k}{2}-\frac{1}{4}}\right), cn^{-1} \leq x \leq \Omega \\ O(n^k), \quad 0 \leq x \leq cn^{-1} \end{cases}$

(2.17) $\left|l_j(x)\right| = O\left(\frac{nj^{k+\frac{3}{2}}}{(k+n)}\right) \quad , \quad$ for $0 \leq x \leq cn^{-1}$

$= O\left(\frac{j^{k+\frac{1}{2}}x^{\frac{-k}{2}-\frac{1}{4}}n^{-\frac{k}{2}+\frac{1}{2}}}{(k+n)}\right) \quad , \quad cn^{-1} \leq x \leq \Omega$

(2.18) $\left|l_j^*(x)\right| = O\left(\frac{nj^{k+\frac{1}{2}}}{(k+n-1)}\right)$, for $0 \leq x \leq cn^{-1}$

$= O\left(\frac{j^{k+\frac{1}{2}}x^{\frac{-k}{2}+\frac{1}{4}}n^{-\frac{k}{2}+\frac{1}{2}}}{(k+n-1)}\right)$, for $cn^{-1} \leq x \leq \Omega$

3. NEW RESULT

Theorem 1 : For n >1 fixed integer let $\{g_i\}_{i=1}^n$, $\{g_i^*\}_{i=1}^n$, $\{g_i^{**}\}_{i=1}^n$ and , $\{g_0^{(j)}\}_{j=0}^k$ are arbitrary real numbers then there exists a unique polynomial $R_n(x)$ of minimal possible degree ≤ 3n+k on the nodal points (1.1) satisfying the condition (1.2) and (1.3). The polynomial $R_n(x)$ can be written in the form

(3.1) $\boldsymbol{R_n(x) = \sum_{j=1}^n U_j(x)g_j + \sum_{j=1}^n V_j(x)g_j^* + \sum_{j=1}^n W_j(x)g_j^{**} + \sum_{j=0}^k C_j(x)g_0^{(j)}}$

where $U_j(x)$, $V_j(x\ )$, $W_j(x)$ and $C_j(x)$ are fundamental polynomials of degree ≤ 3n+k given by

(3.2) $U_j(x) = \frac{x^{(k+1)}l_j(x)[L_n^{(k-1)}(x)]^2}{x_j^{(k+1)}[L_n^{(k-1)}(x_j)]^2}$

(3.3) $V_j(x) = \frac{x^{k+1}l_j^*(x)L_n^k(x)}{y_j^{(k+1)}[L_n^{(k)}(y_j)]^2}[L_n^{(k)}(x) + \frac{y_j-3k+2}{2y_j}L_n^{(k-1)}(x)]$

(3.4) $W_j(x) = \frac{x^{k+1}l_j^*(x)L_n^{(k)}(x)L_n^{(k-1)}(x)}{y_j^{k+1}L_n^k(y_j)L_n^{(k-1)'}(y_j)}$

(3.5) $C_j(x) = p_j(x)x^j[L_n^k(x)]^2L_n^{(k-1)}(x)+x^kL_n^{(k)}(x)L_n^{(k-1)}(x)[c_j$

$-\frac{L_n^k(x)p_j(x)+q_j(x)L_n^{(k-1)}(x)}{x^{k-j}}]$

vc , $j = 0,1,\dots,k-1$

(3.6) $C_k(x) = \frac{1}{\binom{n+k}{k}k!L_n^{(k-1)}(0)}x^kL_n^{(k)}(x)[L_n^{(k-1)}(x)]^{2}$

where $p_j(x)$ and $q_j(x)$ are polynomials of degree at most k-j-1 . c_j is defined in (4.14)

Theorem 2 Let the interpolatory function $f: R \to R$ be continuously differentiable such that,

$C(m) = \{f(x): f\ is\ continuous\ in[0,\infty), f(x) = O(x^m) as\ x \to \infty; m \geq 0\ is\ an\ integer\}$

For every $f \in C(m)$ and $\alpha \geq 0$, Then

$$(3.7) \quad R_n(x) = \sum_{j=1}^{n} U_j(x) g_j + \sum_{j=1}^{n} V_j(x) g_j^* + \sum_{j=1}^{n} W_j(x) g_j^{**} + \sum_{j=0}^{k} C_j(x) g_0^{(j)}$$

satisfies the relations

$$(3.8) \quad |R_n(x) - \mathrm{f(x)}| = O\left(\frac{j^j n^{\frac{3k}{2}-1-\frac{j}{2}}}{(k+n)}\right) \omega\left(f, \frac{\log n}{\sqrt{n}}\right) \quad , \quad \text{for } 0 \leq x \leq cn^{-1}$$

$$(3.9) \quad |R_n(x) - \mathrm{f(x)}| = O\left(\frac{j^{\frac{k}{2}+1} x^{-\frac{k}{2}-\frac{1}{4}} n^{-\frac{k}{2}}}{(k+n)(k+n-1)}\right) \omega\left(f, \frac{\log n}{\sqrt{n}}\right) \quad , \quad \text{for } cn^{-1} \leq x \leq \Omega$$

where ω is the modulus of continuity.

4. PROOF OF THEOREM 1

Let $U_j(x)$, $V_j(x)$, $W_j(x)$ and $C_j(x)$ are polynomials of degree ≤ 3n+k satisfying conditions (4.1) , (4.2) , (4.3) and (4.4) respectively.

(4.1)

$$\begin{cases} U_j(x_i) = \begin{matrix} 0 \\ 1 \end{matrix} \quad \begin{matrix} when\ i \neq j \\ when\ i = j \end{matrix} \qquad U_j(x_i) = 0 \quad , \ U_j'(x_i) = 0 \\ U_j^{(l)}(0) = 0 \qquad , \quad i = 1(1)n \quad and \quad l = 0,1,\ldots,k \end{cases}$$

(4.2)

$$\begin{cases} V_j(y_i) = 0 \qquad V_j(y_i) = \begin{matrix} 0 \\ 1 \end{matrix} \quad \begin{matrix} when\ i \neq j \\ when\ i = j \end{matrix} \quad , \ V_j'(y_i) = 0 \\ V_j^{(l)}(0) = 0 \qquad , \quad i = 1(1)n \quad and \quad l = 0,1,\ldots,k \end{cases}$$

(4.3)

$$\begin{cases} W_j(y_i) = 0 \qquad W_j(y_i) = 0 \quad , \ W_j'(y_i) = \begin{matrix} 0 \\ 1 \end{matrix} \quad \begin{matrix} when\ i \neq j \\ when\ i = j \end{matrix} \\ W_j^{(l)}(0) = 0 \qquad , \quad i = 1(1)n \quad and \quad l = 0,1,\ldots,k \end{cases}$$

(4.4) $\begin{cases} C_k(x_i) = 0 & , \quad C_k(y_i) = 0 \quad , \quad {C_k}'(y_i) = 0 \\ C_k^{(l)}(0) = 0 & , \quad i = 1(1)n \;\; and \quad l = 0,1,\ldots,k \end{cases}$

To determine $U_j(x)$ let

(4.5) $U_j(x) = C_1 x^{k+1} l_j(x)[L_n^{(k-1)}(x)]^2$

where C_1 is a constant . $l_j(x)$ is defined in (2.8) . $U_j(x)$ is a polynomial of degree ≤ 3n+k

By using (2.8) we determine

(4.6) $C_1 = \frac{1}{x_j^{(k+1)} L_n^{(k-1)}(x_j)}$

Hence we find the first fundamental polynomial $U_j(x)$of degree ≤ 3n+k

To find second fundamental polynomial let

(4.7) $V_j(x) = C_2 x^{k+1} l_j^*(x)[L_n^{(k)}(x)]^2 + C_3 x^{k+1} l_j^*(x) L_n^{(k)}(x) L_n^{(k-1)}(x)$

where C_2 and C_3 are arbitrary constants. By using (2.9) and (4.2) we determine

(4.8) $C_2 = \frac{1}{y_j^{(k+1)}[L_n^k(y_j)]^2}$ and

(4.9) $C_3 = \frac{y_j - 3k + 2}{2 y_j y_j^{(k+1)}[L_n^{(k)}(y_j)]^2}$

Hence we find the first fundamental polynomial $V_j(x)$of degree ≤ 3n+k

Again let

(4.10) $W_j\,(x) = C_4 x^{k+1} l_j(x) L_n^{(k)}(x) L_n^{(k-1)}(x)$

where C_4 is a constant , $l_j(x)$ is defined in (2.8) .$W_j(x)$ is polynomial of degree ≤ 3n+k satisfying the conditions (4.3) by which we obtain

(4.11) $C_4 = \frac{1}{y_j^{(k+1)}) L_n^k(y_j) L_n^{(k-1)'}(y_j)}$

Hence we find the third fundamental polynomial $W_j(x)$ of degree ≤ 3n+k

To find $C_j(x)$, we assume $C_j(x)$ for fixed $j \in \{0,1,\ldots\ldots..,k-1\}$ in the form

(4.12) $C_j(x) = p_j(x)x^j[L_n^k(x)]^2 L_n^{(k-1)}(x) + x^k L_n^{(k)}(x) L_n^{(k-1)}(x) g_n(x)$

Where $p_j(x)$ and $g_n(x)$ are polynomials of degree k-j-1 and n respectively . Now it is clear that $C_j^{(l)}(0) = 0 \; for \; (l = 0, \dots\dots, j-1)$ and since $L_n^{(k)}(x_i) = 0$ and $L_n^{(k-1)}(y_i) = 0$ we get

$C_j(x_i) = 0 \; and \; C_j(y_i) = 0 \; for \; i = 1(1)n$. The coefficient of the polynomial $p_j(x)$ are calculated by the system

(4.13) $C_j^{(l)}(0) = \frac{d^l}{dx^l}\left[p_j(x)x^j[L_n^k(x)]^2 L_n^{(k-1)}(x)\right]_{x=0} = \delta_{i,j}$ $(l = j, \dots\dots, k-1)$

now from the equation $C_j^{(k)}(0) = 0$ we get

(4.14) $c_j = g_n(0) = \frac{-1}{\binom{n+k}{k} k! L_n^{(k-1)}(0)} \frac{d^k}{dx^k}[p_j(x)x^j[L_n^k(x)]^2 L_n^{(k-1)}(x)]_{x=0}$

Now using the condition $C_j(y_i)' = 0$ of (4.7) , we get

(4.15) $g_n(y_i) = -(y_i)^{j-k} L_n^k(y_i) p_j(y_i)$ Which implies $g_n(x)$ as follows

(4.16) $g_n(x) = -\frac{L_n^k(x)p_j(x) + q_j(x)L_n^{(k-1)}(x)}{x^{k-j}}$

Where $q_j(x)$ is a polynomial of degree k-j and function $g_n(x)$ will be a polynomial iff for $r = 0,1,\dots,k-j-1$

(4.17) $\frac{d^r}{dx^r}[L_n^k(x)p_j(x) + q_j(x)L_n^{(k-1)}(x)]_{x=0} = 0$

The coefficients of $q_j(x)$ are uniquely calculated by this system (4.13)
Using (4.12) and (4.14) we obtain $C_j(x)$ of degree ≤ 3n+k satisfying the conditions (4.4)

5. ESTIMATION OF THE FUNDAMENTAL POLYNOMIALS

Lemma 5.1. For $j = 1(1)n$ and $[0,\infty)$, we have

(5.1) $\sum_{j=1}^{n}|U_j(x)| \le O\left(\frac{nj^{k+\frac{3}{2}}}{(k+n)}\right)$, for $0 \le x \le cn^{-1}$

(5.2) $\sum_{j=1}^{n}|U_j(x)| \le O\left(\frac{j^{\frac{k}{2}+1}x^{\frac{-k}{2}-\frac{1}{4}}n^{\frac{k}{2}}}{(k+n)}\right)$, for $cn^{-1} \le x \le \Omega$

Wher $U_j(x)$ is given in equqtion (3.2)

Proof : from (3.2) we have

(5.3) $$\sum_{i=1}^{n}\left|U_j(x)\right| \le \frac{|x^{(k+1)}||l_j(x)|\left|L_n^{(k-1)}\right|^2}{\left|x_j^{(k+1)}\right|\left|L_n^{(k-1)}(x_j)\right|^2}$$

where $U_j(x)$ is given in equqtion (3.2)

using equations (4.1) (2.13) and (2.17) we yield the result.

Lemma 5.2. For $j = 1(1)n$ and $[0, \infty)$, we have

(5.4) $$\sum_{j=1}^{n}\left|V_j(x)\right| \le O\left(\frac{nj^{k+\frac{1}{2}}}{(k+n-1)}\right) \quad , \quad \text{for } 0 \le x \le cn^{-1}$$

(5.5) $$\sum_{j=1}^{n}\left|V_j(x)\right| \le O\left(\frac{j^{2k+1}x^{-k}n^{\frac{-5k}{4}+\frac{5}{8}}}{(k+n-1)}\right) \quad , \quad \text{for } cn^{-1} \le x \le \Omega$$

Where $V_j(x)$ is given in equqtion (3.3)

Proof : from (3.3) we have

(5.6) $$\sum_{j=1}^{n}\left|V_j(x)\right| \le \frac{|x^{k+1}|\left|l_j^*(x)\right||L_n^k(x)|}{\left|y_j^{(k+1)}\right|\left|L_n^{(k)}(y_j)\right|^2}\left[\left|L_n^{(k)}(x)\right| + \left|\frac{y_j-3k+2}{2y_j}L_n^{(k-1)}(x)\right|\right]$$

using equations (2.16) and (2.18) we yield the result.

Lemma 5.3. For $j = 1(1)n$ and $[0, \infty)$, we have

(5.7) $$\sum_{j=1}^{n}\left|W_j(x)\right| \le O\left(\frac{j^{\frac{3k}{2}+\frac{5}{2}}}{(k+n-1)}\right) \quad , \quad \text{for } 0 \le x \le cn^{-1}$$

(5.8) $$\sum_{j=1}^{n}\left|W_j(x)\right| \le O\left(\frac{j^{2k+\frac{7}{4}}x^{\frac{-3k}{2}-\frac{1}{4}}n^{\frac{-k}{4}-\frac{7}{8}}}{(k+n-1)}\right) \quad , \quad \text{for } cn^{-1} \le x \le \Omega$$

where $W_j(x)$ is given in equqtion (3.4)

Proof : from (3.3) we have

(5.9) $$\sum_{j=1}^{n}\left|W_j(x)\right| = \frac{|x^{k+1}|\left|l_j^*(x)\right|\left|L_n^{(k)}(x)\right|\left|L_n^{(k-1)}(x)\right|}{\left|y_j^{k+1}\right||L_n^k(y_j)|\left|L_n^{(k-1)'}(y_j)\right|}$$

By equations (2.15), (2.16) and (2.17) we yield the result.

Lemma 5.4. For $j = 0,1,2,\dots,k$ and $[0,\infty)$, we have

(5.8) $\sum_{j=0}^{k}\left|C_j(x)\right| \leq O\left(j^j\, n^{\,3k-2-\frac{j}{2}}\right)$, for $0 \leq x \leq cn^{-1}$

(5.9) $\sum_{j=0}^{k}\left|C_j(x)\right| \leq O\left(j^j\, x^{\frac{-3k}{2}} n^{\frac{1}{2}(3k-j)-\frac{7}{4}}\right)$, for $cn^{-1} \leq x \leq \Omega$

Proof : by using equation (3.5) dealing with similar method we get the result.

6. PROOF OF THEOREM 2

We prove theorem 2 with the help of certain theorem mentioned as below

Theorem (5.5): Let $C(m) = \{f(x): f \text{ is continuous in}[0,\infty), f(x) = O(x^m) \text{as } x \to \infty; m \geq 0 \text{ is an integer}\}$ Then by Szego[12] is

$$\lim_{n\to\infty}\left\|f(x) - H_n^{(\alpha)}(f,x)\right\|_I = 0$$

For every $f \in C(s)$ and $\subset (0,\infty) for\ \alpha \geq 0$, or $I \subset (0,\infty) for -1 < \alpha < 0$. furthermore there exists a function in C(m) such that $\left\{H_n^{(\alpha)}(f,x)\right\}$ diverges for $\alpha \geq 0$ at x=0 .As for thr rate of convergence the following result is due to Vertesi

(5.10) $$\left\|f(x) - H_n^{(\alpha)}(f,x)\right\|_I = \begin{cases} O\left(\omega(f, n^{-1-\alpha})\right); & -1 < \alpha < 0 \\ O\left(\omega\left(f, \frac{\log n}{\sqrt{n}}\right)\right); & \alpha \geq -\frac{1}{2} \end{cases}$$

Proof of main theorem 2 : Since $R_n(x)$ given by equation (3.1) is exact for all polynomial $S_n(x)$ of degree ≤ 3n+k , we have

(5.11) $$S_n(x) = \sum_{j=1}^{n} S_n(x_j)U_j(x) + \sum_{j=1}^{n} S_n(y_j)V_j(x) + \sum_{j=1}^{n} S_n{}'(y_j)W_j(x) + \sum_{j=0}^{k} S_n(y_i)C_j(x)$$

From equation (5.11) and (3.1) we get

$$|R_n(x) - (\mathrm{x})| = |S_n(x) - f(x)| + \sum_{j=1}^{n} \left|f(x_j) - S_n(x_j)\right| \left|U_j(x)\right|$$

$$+ \sum_{j=1}^{n} \left|f(y_j) - S_n(y_j)\right| \left|V_j(x)\right|$$

$$+ \sum_{j=1}^{n} \left|f'(y_j) - S_n{}'(y_j)\right| \left|W_j(x)\right| + \sum_{l=1}^{k} \left|f^l(y_j) - S_n{}^l(y_j)\right| \left|C_j(x)\right|$$

Owing to equations (2.14) and lemmas (5.1), (5.2), (5.3), (5.4) we get the result.

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