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On Minimal Prime filters of Almost Distributive Lattices

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Abstract

A set of equivalent conditions is derived for the class of all $\mathfrak{M}$-filters to become a sublattice of the lattice of filters. An equivalency is obtained between prime $\mathfrak{M}$-filters and minimal prime filters. Finally, the $\mathfrak{M}$-filters are characterized in terms of minimal prime filters.

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1. Introduction

After Booles axiomatization of two valued propositional calculus as a Boolean algebra, a number of generalizations both ring theoretically and lattice theoretically have come into being. The concept of an Almost Distributive Lattice (ADL) was introduced by Swamy and Rao [6] as a common abstraction of many existing ring theoretic generalizations of a Boolean algebra on one hand and the class of distributive lattices on the other. In that paper, the concept of an ideal in an ADL was introduced analogous to that in a distributive lattice and it was observed that the set PI(L) of all principal ideals of L forms a distributive lattice. In [5], the authors studied the properties of 0-ideals of an ADL. In [6], rafi introduced the concept of $\mathfrak{M}$-filters in an Almost Distributive Lattice (ADL) and studied their properties. In this paper, we first obtain that the class of all $\mathfrak{M}$-filters forms a complete distributive lattice on their own. Later, we obtain that every minimal prime filter of an ADL is an $\mathfrak{M}$-filters and also observed that the converse is not true. However, it is then derived a necessary and sufficient condition for an $\mathfrak{M}$-filter of an ADL to become a prime filter. An equivalency between the class of all prime $\mathfrak{M}$-filters and the class of all minimal prime filters is obtained. Though the lattice of $\mathfrak{M}$-filters is not a sublattice of the lattice of filters, we derive a set of equivalent conditions for the lattice of $\mathfrak{M}$-filters to become a sublattice of the lattice of filters. Finally, the class of all $\mathfrak{M}$-filters are characterized in terms of minimal prime filters.

2. Preliminaries

First, we recall certain definitions and properties of ADLs, Pseudo-complemented ADLs and Stone ADLs that are required in the paper. We begin with ADL definition as follows.

Definition 2.1. [7] An Almost Distributive Lattice with zero or simply ADL is an algebra $(L, \vee, \wedge, 0)$ of type (2, 2, 0) satisfying:

1. $(x \vee y) \wedge z = (x \wedge z) \vee (y \wedge z)$
2. $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
3. $(x \vee y) \wedge y = y$
4. $(x \vee y) \wedge x = x$
5. $x \vee (x \wedge y) = x$
6. $0 \wedge x = 0$
7. $x \vee 0 = x$, for all $x, y, z \in L$.

Example 2.2. Every non-empty set X can be regarded as an ADL as follows. Let $x_0 \in X$. Define the binary operations $\vee$, $\wedge$ on X by

$$x \vee y = \begin{cases} x & \text{if } x \neq x_0 \\ y & \text{if } x = x_0 \end{cases} \qquad x \wedge y = \begin{cases} y & \text{if } x \neq x_0 \\ x_0 & \text{if } x = x_0. \end{cases}$$

Then $(X, \vee, \wedge, x_0)$ is an ADL (where x_0 is the zero) and is called a discrete ADL.

If $(L, \vee, \wedge, 0)$ is an ADL, for any $a, b \in L$, define $a \leq b$ if and only if $a = a \wedge b$ (or equivalently, $a \vee b = b$), then $\leq$ is a partial ordering on L.

Theorem 2.3. [7] If $(L, \vee, \wedge, 0)$ is an ADL, for any $a, b, c \in L$, we have the following:

(1) $a \vee b = a \Leftrightarrow a \wedge b = b$

(2) $a \vee b = b \Leftrightarrow a \wedge b = a$

(3) $\wedge$ is associative in L

(4) $a \wedge b \wedge c = b \wedge a \wedge c$

(5) $(a \vee b) \wedge c = (b \vee a) \wedge c$

(6) $a \wedge b = 0 \Leftrightarrow b \wedge a = 0$

(7) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

(8) $a \wedge (a \vee b) = a,\ (a \wedge b) \vee b = b$ and $a \vee (b \wedge a) = a$

(9) $a \leq a \vee b$ and $a \wedge b \leq b$

(10) $a \wedge a = a$ and $a \vee a = a$

(11) $0 \vee a = a$ and $a \wedge 0 = 0$

(12) If $a \leq c,\ b \leq c$ then $a \wedge b = b \wedge a$ and $a \vee b = b \vee a$

(13) $a \vee b = (a \vee b) \vee a$.

It can be observed that an ADL L satisfies almost all the properties of a distributive lattice except the right distributivity of $\vee$ over $\wedge$, commutativity of $\vee$, commutativity of $\wedge$. Any one of these properties make an ADL L a distributive lattice. That is

Theorem 2.4. [7] Let $(L, \vee, \wedge, 0)$ be an ADL with 0. Then the following are equivalent:

1) $(L, \vee, \wedge, 0)$ is a distributive lattice

2) $a \vee b = b \vee a$, for all $a, b \in L$

3) $a \wedge b = b \wedge a$, for all $a, b \in L$

4) $(a \wedge b) \vee c = (a \vee c) \wedge (b \vee c)$, for all $a, b, c \in L$.

As usual, an element $m \in L$ is called maximal if it is a maximal element in the partially ordered set $(L, \leq)$. That is, for any $a \in L$, $m \leq a \Rightarrow m = a$.

Theorem 2.5. [7] Let L be an ADL and $m \in L$. Then the following are equivalent:

1) m is maximal with respect to $\leq$

2) $m \vee a = m$, for all $a \in L$

3) $m \wedge a = a$, for all $a \in L$

4) $a \vee m$ is maximal, for all $a \in L$.

As in distributive lattices [1, 2], a non-empty subset I of an ADL L is called an ideal of L if $a \vee b \in I$ and $a \wedge x \in I$ for any $a, b \in I$ and $x \in L$. Also, a non-empty subset F of L is said to be a filter of L if $a \wedge b \in F$ and $x \vee a \in F$ for $a, b \in F$ and $x \in L$.

The set $I(L)$ of all ideals of L is a bounded distributive lattice with least element $\{0\}$ and greatest element L under set inclusion in which, for any $I, J \in I(L)$, $I \cap J$ is the infimum of I and J while the supremum is given by $I \vee J := \{a \vee b \mid a \in I, b \in J\}$. A proper ideal P of L is called a prime ideal if, for any $x, y \in L$, $x \wedge y \in P \Rightarrow x \in P$ or $y \in P$. A proper ideal M of L is said to be maximal if it is not properly contained in any proper ideal of L. It can be observed that every maximal ideal of L is a prime ideal. Every proper ideal of L is contained in a maximal ideal. For any subset S of L the smallest ideal containing S is given by $(S] := \{(\bigvee_{i=1}^{n} s_i) \wedge x \mid s_i \in S, x \in L$ and $n \in N\}$. If $S = \{s\}$, we write $(s]$ instead of $(S]$. Similarly, for any $S \subseteq L$, $[S) := \{x \vee (\bigwedge_{i=1}^{n} s_i) \mid s_i \in S, x \in L$ and $n \in N\}$. If $S = \{s\}$, we write $[s)$ instead of $[S)$.

Theorem 2.6. [7] For any x, y in L the following are equivalent:

1) $(x] \subseteq (y]$

2) $y \wedge x = x$

3) $y \vee x = y$

4) $[y) \subseteq [x)$.

For any $x, y \in L$, it can be verified that $(x] \vee (y] = (x \vee y]$ and $(x] \wedge (y] = (x \wedge y]$. Hence the set $PI(L)$ of all principal ideals of L is a sublattice of the distributive lattice $I(L)$ of ideals of L.

Theorem 2.7. [3] Let I be an ideal and F a filter of L such that $I \cap F = \emptyset$. Then there exists a prime ideal P such that $I \subseteq P$ and $P \cap F = \emptyset$.

3. On Minimal Prime filters of ADLs

In this paper, derived a necessary and sufficient condition for an $\mathfrak{M}$-filter of an ADL to become a prime filter. An equivalency between the class of all prime $\mathfrak{M}$-filters and the class of all minimal prime filters is obtained. Though the lattice of $\mathfrak{M}$-filters is not a sublattice of the lattice of filters, we derive a set of equivalent conditions for the lattice of $\mathfrak{M}$-filters to become a sublattice of the lattice of filters. Finally, the class of all $\mathfrak{M}$-filters are characterized in terms of minimal prime filters.

Lemma 3.1. Let L be an ADL with maximal elements. For any subset A of L which is closed under $\vee$, the set $\mathfrak{M}(A)$ is a filter of L.

Proof. Let m be any maximal element of an ADL L. Then clearly $m \in \mathfrak{M}(A)$. Let $x, y \in \mathfrak{M}(A)$. Then $x \vee a$ and $y \vee b$ are maximal elements of L, for some $a, b \in A$. Since $a, b \in A$, we have $a \vee b \in A$. Now for any $s \in L$, $((x \wedge y) \vee (a \vee b)) \wedge s = ((x \vee a \vee b) \wedge (y \vee a \vee b)) \wedge s = (y \vee a \vee b) \wedge s = s$. Therefore $(x \wedge y) \vee (a \vee b)$ is a maximal element of L and hence $x \wedge y \in \mathfrak{M}(A)$. Let $x \in \mathfrak{M}(A)$ and $r \in L$. Then $x \vee a$ is maximal element, for some $a \in A$. That implies $(r \vee x) \vee a$ is also a maximal element. Hence $r \vee x \in \mathfrak{M}(A)$. Therefore $\mathfrak{M}(A)$ is a filter of L. ■

Note for any ideal A of L, we have (i). $\mathfrak{M}(A)$ is a filter of L (ii). $I \cap \mathfrak{M}(I) \neq \emptyset \Rightarrow I = \mathfrak{M}(I) = L$. Also for any family $\{I_\alpha\}_{\alpha \in \Delta}$ of ideals of L, $\bigcap\limits_{\alpha \in \Delta} \mathfrak{M}(I_\alpha) = \mathfrak{M}(\bigcap\limits_{\alpha \in \Delta} I_\alpha)$.

Theorem 3.2. Let L be an ADL with maximal elements. If A is a subset of L which is closed under $\vee$ and I is an ideal of L such that $I \cap \mathfrak{M}(A) = \emptyset$, then there exists a prime filter P of L such that P containing $\mathfrak{M}(A)$ and disjoint from I.

Proof. Consider $\mathcal{F} = \{F \mid F \text{ is a filter of } L, \mathfrak{M}(A) \subseteq F \text{ and } F \cap I = \emptyset\}$. Clearly a filter $\mathfrak{M}(A) \in \mathcal{F}$ and satisfies the hypothesis of Zorn's lemma. Then $\mathcal{F}$ has maximal element P say. That implies $\mathfrak{M}(A) \subseteq P$ and $F \cap P = \emptyset$. Now we prove that P is a prime filter of L. Let $x, y \in L$ with $x \vee y \in P$. Suppose $x \notin P$ and $y \notin P$. Then $P \subsetneq P \vee [x)$ and $P \subsetneq P \vee [y)$ and hence $(P \vee [x)) \cap I \neq \emptyset$ and $(P \vee [y)) \cap I \neq \emptyset$. Now choose $a \in (P \vee [x)) \cap I$ and $b \in (P \vee [y)) \cap I$. That implies $a \vee b \in (P \vee [x \vee y)) \cap I$. If $x \vee y \in I$, then $P \cap I \neq \emptyset$, which is a contradiction. Therefore either $x \in P$ or $y \in P$. Hence P is a prime filter of L. ■

Definition 3.3. A filter F od an ADL L is called an $\mathfrak{M}$-filter of L if $F = \mathfrak{M}(I)$, for some ideal I of L.

Lemma 3.4. For any $x \in L$, the filter $(x)^+$ is an $\mathfrak{M}$-filter of L.

Proof. Let $a \in (x)^+$. Then $a \vee x$ is a maximal element of L. That implies $a \in \mathfrak{M}((x])$, since $x \in (x]$. Therefore $(x)^+ \subseteq \mathfrak{M}((x])$. Let $a \in \mathfrak{M}((x])$. Then $a \vee y$ is a maximal element of L, for some $y \in (x]$. Since $y \in (x]$, we have $x \wedge y = y$. Now $(a \vee x) \wedge s = (a \vee (x \vee y)) \wedge s = s$. That implies $a \vee x$ is a maximal element of L. Therefore $a \in (x)^+$

and hence $(x)^+ \subseteq \mathfrak{M}((x])$. Thus $(x)^+$ is an $\mathfrak{M}$-filter of L. ■

Theorem 3.5. Every minimal prime filter of an ADL L is an $\mathfrak{M}$-filter of L.

Proof. Let P be a minimal prime filter of L. Let $x \in P$. Then there exists an element $y \notin P$ such that $x \vee y$ is a maximal element of L. That implies $x \in \mathfrak{M}(L \setminus P)$ and hence $P \subseteq \mathfrak{M}(L \setminus P)$. Let $x \in \mathfrak{M}(L \setminus P)$. Then $x \vee y$ is maximal element of L, for some $y \in L \setminus P$. That implies $x \in P$. Therefore $\mathfrak{M}(L \setminus P) \subseteq P$ and hence $\mathfrak{M}(L \setminus P) = P$. Thus P is an $\mathfrak{M}-$filter of L. ■

We now turn our intension towards the converse of above theorem. In general, every $\mathfrak{M}-$filter of an ADL need not be a minimal prime filter. In fact it need not even be a prime filter. In distributive Lattice, it can be observed in the following example.

Example 3.6. Let $X = \{1, 2, 3, 4\}$ be a set and L the sublattice of the power set of X, which is generated by the sets $\{1\}$, $\{2\}$ and $\{3\}$. That is, $L = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$. Consider $F = \{\{1, 2\}, \{1, 2, 3\}\}$ and $I = \{\emptyset, \{2\}, \{3\}, \{2, 3\}\}$. Clearly F is a filter and I an ideal of L. Now $\mathfrak{M}(I) = \{1, 2\}, \{1, 2, 3\}\} = F$. Therefore F is an $\mathfrak{M}$-filter of L. It can be observed that F is not a prime filter of L, because $\{2, 3\} \notin F$ and $\{1, 3\} \notin F$ but $\{2, 3\} \vee \{1, 3\} = \{1, 2, 3\} \in F$. Though every $\mathfrak{M}$-filter need not be a prime filter, we derive a necessary and sufficient condition for an $\mathfrak{M}$-filter to become a prime filter in L.

Theorem 3.7. Let F be a proper $\mathfrak{M}$-filter of L. Then F is prime if and only if F contains a prime filter.

Proof. Let F be a proper $\mathfrak{M}$-filter of L. That is $F = \mathfrak{M}(I)$, for some ideal I. Assume that F is a prime filter of L. Then clearly F contains a prime filter of L. Conversely assume that a proper $\mathfrak{M}$-filter F contains a prime filter P. That is $P \subseteq F \subseteq \mathfrak{M}(I)$. Let $x, y \in L$ with $x \vee y \in P$. Suppose $x \notin P$ and $y \notin P$. Then $(x)^+ \subseteq P$ and $(y)^+ \subseteq P$. That implies $(x)^+ \cap (y)^+ \subseteq P$ and hence $(x \vee y)^+ \subseteq P$. Since $x \vee y \in P \subseteq F \subseteq \mathfrak{M}(I)$, we have $(x \vee y) \vee i$ is maximal element, for some $i \in I$. That implies $i \in (x \vee y)^+ \subseteq P \subseteq \mathfrak{M}(I)$. Therefore $i \in \mathfrak{M}(I) \cap I$ and hence $I = \mathfrak{M}(I) = L$, which is a contradiction. Thus P is a prime filter of L. ■

Theorem 3.8. Every prime $\mathfrak{M}$-filter of L is a minimal prime filter of L.

Proof. Let P be a prime $\mathfrak{M}$-filter of L. Then $P = \mathfrak{M}(I)$, for some ideal I of L. Let $x \in P = \mathfrak{M}(I)$. Then $x \vee y$ is maximal element, for some $y \in I$. Suppose that $y \in P$. Then $y \in I \cap \mathfrak{M}I$. That implies $I = \mathfrak{M}(I) = P = L$, which is a contradiction. Hence $y \notin P$. Therefore P is a minimal prime filter of L. ■

Lemma 3.9. Every proper $\mathfrak{M}$-filter is contained in a minimal prime filter.

Proof. Let F be a proper $\mathfrak{M}$-filter of L. Then $F = \mathfrak{M}(I)$, for some ideal I of L. Clearly $F \cap I = \mathfrak{M}(I) \cap I = \emptyset$. Consider $\mathcal{F} = \{J \mid J \text{ is an ideal of L}, I \subseteq J \text{ and } F \cap J = \emptyset\}$.

Clearly, $\mathcal{F}$ satisfies the hypothesis of Zorn's lemma. Then $\mathcal{F}$ has a maximal element M say. We prove that M is a maximal ideal of L. Suppose that M is not a maximal ideal of L. Then there exists a proper ideal N of L such that $M \subsetneq N$. Then $F \cap N \neq \emptyset$. Choose $x \in F \cap N$. Then $x \in F$ and $x \in N$. Since $x \in F$ and F is an $\mathfrak{M}$-filter, we have $x \vee y$ is a maximal element, for some $y \in I \subseteq M \subsetneq N$. That implies $x, y \in N$ and hence $x \vee y \in N$, which is a contradiction. That implies M is a maximal ideal such that $F \cap M = \emptyset$. Therefore $L \setminus M$ is a minimal prime filter such that $F \subseteq L \setminus M$. ■

Theorem 3.10. Let L be an ADL with maximal elements. Then the following are equivalent in L :

(i) $\mathcal{F}_0(L)$ is a sublattice of $\mathcal{F}(L)$

(ii) for any $a, b \in L$, $a \vee b$ is maximal element$\Rightarrow$ $(a)^+ \vee (b)^+ = L$

(iii) for any $a, b \in L$, $(a)^+ \vee (b)^+ = (a \vee b)^+$

(iv) for any two ideals I, J of L, $I \vee J = L \Rightarrow \mathfrak{M}(I) \vee \mathfrak{M}(J) = L$

(v) for any two ideals I, J of L, $\mathfrak{M}(I \vee J) = \mathfrak{M}(I) \vee \mathfrak{M}(J)$.

Proof. $(i) \Rightarrow (ii)$: Assume that (i). Let $a, b \in L$ with $a \vee b$ is maximal element. Suppose $(a)^+ \vee (b)^+ \neq L$. Since $(a)^+$ and $(b)^+$ are $\mathfrak{M}-$filters of L, $(a)^+ \vee (b)^+$ is a proper $\mathfrak{M}-$filter of L. Then by theorem—,$(a)^+ \vee (b)^+$ contained in a minimal prime filter say P. That is $(a)^+ \vee (b)^+ \subseteq P$. Then $(a)^+ \subseteq P$ and $(b)^+ \subseteq P$. That implies $a \notin P$ and $b \notin P$, since P is a minimal prime filter. Therefore $a \vee b \notin P$, which is a contradiction because $a \vee b$ is a maximal element. Hence $(a)^+ \vee (b)^+ = L$.

$(ii) \Rightarrow (iii)$: Assume that (ii). For any $a, b \in L$, we have $(a)^+ \vee (b)^+ \subseteq (a \vee b)^+$. Let $x \in (a \vee b)^+$. Then $x \vee (a \vee b)$ is maximal and hence by our assumption, $(x \vee a)^+ \vee (x \vee b)^+ = L$. For any $y \in L$, $y \in (x \vee a)^+ \vee (x \vee b)^+$. Then $y = r \wedge s$, for some $r \in (x \vee a)^+$ and $s \in (x \vee b)^+$. Since $r \in (x \vee a)^+$ and $s \in (x \vee b)^+$, we get that $r \vee x \in (a)^+$ and $s \vee x \in (b)^+$. Now $y \vee y = y \vee (r \vee s) = (x \vee r) \wedge (x \vee s) \in (a)^+ \vee (b)^+$. Therefore $(a)^+ \vee (b)^+ = (a \vee b)^+$.

$(iii) \Rightarrow (iv)$: Assume that (iii). Let I, J be two ideals of L with $I \vee J = L$. Let m be any maximal element of L. Then $m \in I \vee J$. That implies $m = i \vee j$, for some $i \in I$ and $j \in J$. So that $(i)^+ \vee (j)^+ = (i \vee j)^+ = (m)^+ = L$. Therefore $L = (i)^+ \vee (j)^+ \subseteq \mathfrak{M}(I) \vee \mathfrak{M}(J)$. Hence $\mathfrak{M}(I) \vee \mathfrak{M}(J) = L$.

$(iv) \Rightarrow (v)$: Assume that (iv). Let I, J be two ideals of L. we have always that $\mathfrak{M}(I) \vee \mathfrak{M}(J) \subseteq \mathfrak{M}(I \vee J)$. Let $x \in \mathfrak{M}(I \vee J)$. Then $x \vee a$ is a maximal element, for some $a \in I \vee J$. Since $a \in I \vee J$, $a = i \vee j$, for some $i \in I$ and $j \in J$. Since $x \vee a$ is maximal, $(x \vee i) \vee (x \vee j)$ is also a maximal element. That implies $[(x \vee i) \vee (x \vee j)) = L$. Implies that $[x \vee i) \vee [x \vee j) = L$. Implies $\mathfrak{M}([x \vee i)) \vee \mathfrak{M}([x \vee j)) = L$. Hence

$(x \vee i)^+ \vee (x \vee j)^+ = L$. Therefore $x \in (x \vee i)^+ \vee (x \vee j)^+$. That implies $x = a \wedge b$, for some $a \in (x \vee i)^+$ and $b \in (x \vee j)^+$. Now $x = x \vee x = x \vee (a \wedge b) = (x \vee a) \vee (x \vee b) \in (i)^+ \vee (j)^+ \subseteq \mathfrak{M}(I) \vee \mathfrak{M}(J)$. Therefore $\mathfrak{M}(I \vee J) \subseteq (x \vee i)^+ \vee (x \vee j)^+$. Hence $\mathfrak{M}(I \vee J) = (x \vee i)^+ \vee (x \vee j)^+$.

$(v) \Rightarrow (i)$: Clear. ■

Lemma 3.11. Let I be an ideal of L. If P is a minimal prime filter containing $\mathfrak{M}(I)$ then $I \cap P = \emptyset$.

Proof. Let P be a minimal prime filter of L with $\mathfrak{M}(I) \subseteq P$. Suppose that $I \cap P \neq \emptyset$. Choose $x \in I \cap P$. Then $x \in I$ and $x \in P$. Since P is a minimal prime filter of L and $x \in P$, there exists an element $y \notin P$ such that $x \vee y$ is maximal element of L. That implies $x \vee y \in \mathfrak{M}(I)$. That implies $(x \vee y) \vee i$ is a maximal element, for some $i \in I$ and hence $y \vee (x \vee i)$ is a maximal element and $x \vee i \in I$. So that $y \in \mathfrak{M}(I) \subseteq P$, which is a contradiction. Therefore $I \cap P = \emptyset$. ■

Lemma 3.12. Every minimal prime filter of L containing an $\mathfrak{M}$-filter is a minimal prime filter of L.

Proof. Let F be an $\mathfrak{M}$-filter of L. Then we have $F = \mathfrak{M}(I)$, for some ideal I of L. Let P be a minimal prime filter containing $F = \mathfrak{M}(I)$. Then by above Lemma, $I \cap P = \emptyset$. Let $x \in P$. Then there exists $y \notin P$ such that $x \vee y \in \mathfrak{M}(I)$. Hence $(x \vee y) \vee i$ is a maximal element, for some $i \in I$. Thus $x \vee (y \vee i)$ is a maximal element and $y \vee i \notin P$. Hence P is a minimal prime filter of L. ■

Finally, $\mathfrak{M}$-filters are characterized in terms of minimal prime filters.

Theorem 3.13. Every $\mathfrak{M}$-filter of an ADL L is the intersection of all minimal prime filters containing it.

Proof. Let F be an $\mathfrak{M}$-filter of L. Then $F = \mathfrak{M}(I)$, for some ideal I of L. Let $\mathcal{F}_0 = \bigcap\{P \mid P \text{ is a minimal prime filter containing } F\}$. Clearly $F \subseteq \mathcal{F}_0$. Conversely, let $a \notin F = \mathfrak{M}(I)$. Then $a \vee t$is not maximal element, for all $x \in I$. Then there exists a minimal prime filter P such that $a \vee t \notin P$. Hence $a \notin P$ and $t \notin P$. Since P is prime, $(t)^+ \subseteq P$, for all $t \in I$. Therefore $I = \mathfrak{M}(I) \subseteq P$. Thus P is a minimal prime filter containing F and $a \notin P$. Therefore we get $a \notin \mathcal{F}_0$, which yields that $\mathcal{F}_0 \subseteq F$. Therefore $F = \mathcal{F}_0$. ■

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