Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 10 (2017), pp. 7301–7314

http://www.ripublication.com/gjpam.htm

On intuitionistic $\widehat{\mathcal{B}}$ structure normal spaces

T. Ramya and M.K. Uma

Department of Mathematics,
Sri Sarada College for Women, Salem-636016.
Tamil Nadu, India.
E-mail: tramyamath@gmail.com

Abstract

In this paper, the concepts of the concepts of intuitionistic $\widehat{\mathcal{B}}$ structure Normal spaces, intuitionistic $\widehat{\mathcal{B}}$ structure shrinkable, intuitionistic $\widehat{\mathcal{B}}$ structure open covering, intuitionistic $\widehat{\mathcal{B}}$ structure refinement, intuitionistic $\widehat{\mathcal{B}}$ structure subspaces and intuitionistic $\widehat{\mathcal{B}}$ structure totally normal spaces spaces are introduced. Some interesting properties and characterizations are established.

AMS subject classification: 54A40, 54E55.
Keywords: The concepts of intuitionistic $\widehat{\mathcal{B}}$ structure Normal spaces, intuitionistic $\widehat{\mathcal{B}}$ structure shrinkable, intuitionistic $\widehat{\mathcal{B}}$ structure open covering, intuitionistic $\widehat{\mathcal{B}}$ structure refinement, intuitionistic $\widehat{\mathcal{B}}$ structure subspaces and intuitionistic $\widehat{\mathcal{B}}$ structure totally normal spaces are introduced and some of their properties are also studied.

1. Introduction

The concept of an intuitionistic set was introduced by D.Coker in [2]. The intuitionistic set is the discrete form of intuitionistic fuzzy set. The concept of normal spaces was discussed by A.R. Pears [3]. It has many applications in life sciences, neural networking and medical image processing.

In this chapter the concepts of intuitionistic $\widehat{\mathcal{B}}$ structure Normal spaces, intuitionistic $\widehat{\mathcal{B}}$ structure shrinkable, intuitionistic $\widehat{\mathcal{B}}$ structure open covering, intuitionistic $\widehat{\mathcal{B}}$ structure refinement, intuitionistic $\widehat{\mathcal{B}}$ structure subspaces and intuitionistic $\widehat{\mathcal{B}}$ structure totally normal spaces are introduced and studied. Some interesting properties are also established.

2. Preliminaries

Definition 2.1. Let X be a nonempty fixed set. An *intuitionistic set* (IS for short) A is an object having the form $A = \langle X, A^1, A^2\rangle$, for all $x \in X$ where A^1 and A^2 are subsets of X satisfying $A^1 \cap A^2 = \phi$. The set A^1 is called the set of members of A while A^2 is called the set of nonmembers of A. Every crisp set A on a non empty set X is obviously an intuitionistic set having the form $A = \langle x, A, A^c\rangle$.

Definition 2.2. Let X be a nonempty set and the Intuitionistic sets A and B in the form $A = \langle x, A^1, A^2\rangle$, $B = \langle x, B^1, B^2\rangle$. Then

(i) $A \subseteq B$ iff $A^1 \subseteq B^1$ and $B^2 \subseteq A^2$;

(ii) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;

(iii) $A \subseteq B$ iff $A^1 \cup A^2 \supseteq B^1 \cup B^2$;

(iv) $\overline{A} = \langle x, A^2, A^1\rangle$;

(v) $\cup A_i = \langle x, \cup A_{i1}, \cap A_{i2}\rangle$;

(vi) $\cap A_i = \langle x, \cap A_{i1}, \cup A_{i2}\rangle$;

(vii) $A - B = A \cap \overline{B}$

(viii) $\phi_\sim = \langle x, \phi, X\rangle$ and $X_\sim = \langle x, X, \phi\rangle$.

Definition 2.3. Let X and Y be two nonempty sets and $f : X \to Y$ a function

(i) If $B = \langle x, B^1, B^2\rangle$ is an intuitionistic set in Y, then the *preimage* of B under f, denoted by $f^{-1}(B)$, is the intuitionistic set in X defined by

$$f^{-1}(B) = \langle x, f^{-1}(B_1), f^{-1}(B_2)\rangle.$$

(ii) If $A = \langle x, A^1, A^2\rangle$ is an intuitionistic set in X, then the *image* of A under f, denoted by $f(A)$, is the intuitionistic set in Y defined by

$$f(A) = \langle y, f(A_1), f_-(A_2)\rangle,$$

where $f_-(A^2) = (f(A^{2c}))^c$.

Definition 2.4. An *intuitionistic topology* (IT) on a nonempty set X is a family T of ISs in X satisfying the following axioms:

(i) $\phi_\sim$ and $X_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;

(iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.

In this case the ordered pair (X, T) is called an intuitionistic topological space (ITS for short) on X and any intuitionistic set in T is known as an intuitionistic open set in X. The complement $\overline{A}$ of an intuitionistic open set A is called an intuitionistic closed set(ICS for short) in X.

Definition 2.5. Let x be a point in a topological spaces (X, T). A set U in X is said to be a neighbourhood of x if there exists an open set G in X such that $x \in G \subseteq U$.

Definition 2.6. A topological space X is said to be normal if for each pair of disjoined closed sets A and B of X there exist disjoint open sets U and V such that $A \subset U$ and $B \subset V$.

Definition 2.7. Let (X, τ) be an intuitionistic topological space. An intuitionistic set $A = \langle x, A^1, A^2\rangle$ in X is said to be intuitionistic t-open if $Iint(A) = Iint(Icl(A))$. The complement of an intuitionistic t-open set is an intuitionistic t-closed set.

Definition 2.8. Let (X, τ) be an intuitionistic topological space. An intuitionistic set $A = \langle x, A^1, A^2\rangle$ in X is said to be intuitionistic B-open if $A = U \cap V$ where $U = \langle x, U^1, U^2\rangle \in \tau$ and $V = \langle x, V^1, V^2\rangle$ is an intuitionistic t-open set. The complement of an intuitionistic B-open set is an intuitionistic B-closed set.

3. Properties of intuitionistic $\widehat{\mathcal{B}}$ structure normal spaces

In this section, the concepts of intuitionistic $\widehat{\mathcal{B}}$ structure shrinkable, intuitionistic $\widehat{\mathcal{B}}$ structure open covering, intuitionistic $\widehat{\mathcal{B}}$ structure refinement and intuitionistic $\widehat{\mathcal{B}}$ structure subspaces are introduced and some of their properties are also studied.

Notation 3.1. Let (X, T) be an intuitionistic topological space. An intuitionistic set $A = \langle x, A^1, A^2\rangle$ in X.

(i) $Icl(A)$ denotes intuitionistic closure of A.

(ii) $Iint(A)$ denotes intuitionistic interior of A.

Definition 3.2. Let (X, T) be an intuitionistic topological space. An intuitionistic set A in X is said to be an intuitionistic $\widehat{B}$-closed set if $Icl(A) \subseteq U$, whenever $A \subseteq U$ and U is an intuitionistic B-open set. The complement of an intuitionistic $\widehat{B}$-closed set is an intuitionistic $\widehat{B}$-open set.

Definition 3.3. Let X be a non-empty set. An intuitionistic $\widehat{\mathcal{B}}$ structure on X is a family $\mathcal{B}$ of intuitionistic $\widehat{B}$-closed sets in X satisfying the following conditions:

(i) $X_{\sim}, \phi_{\sim} \in \mathcal{B}$.

(ii) $B_1 \cap B_2 \in \mathcal{B}$ for any $B_1, B_2 \in \mathcal{B}$ where $B_1 = \langle x, {B_1}^1, {B_1}^2 \rangle$, $B_2 = \langle x, {B_2}^1, {B_2}^2 \rangle$.

(iii) $\cup B_i \in \mathcal{B}$ for arbitrary family $\{G_i, i \in J\} \subseteq \mathcal{B}$.

Then the ordered pair $(X, \mathcal{B})$ is called an intuitionistic $\widehat{\mathcal{B}}$ structure space. Every member in $(X, \mathcal{B})$ is called an intuitionistic $\widehat{\mathcal{B}}$ structure open set. The complement of an intuitionistic $\widehat{\mathcal{B}}$ structure open set $A = \langle x, A^1, A^2 \rangle$ is an intuitionistic $\widehat{\mathcal{B}}$ structure closed set.

Definition 3.4. An intuitionistic $\widehat{\mathcal{B}}$ structure space $(X, \mathcal{B})$ is said to be intuitionistic $\widehat{\mathcal{B}}$ structure normal if for every intuitionistic $\widehat{\mathcal{B}}$ structure closed set $A = \langle x, A^1, A^2 \rangle$ and intuitionistic $\widehat{\mathcal{B}}$ structure open set $U = \langle x, U^1, U^2 \rangle$ such that $A \subseteq U$, there exists an intuitionistic $\widehat{\mathcal{B}}$ structure open set $G = \langle x, G^1, G^2 \rangle$ such that $A \subseteq G \subseteq I\widehat{\mathcal{B}}cl(G) \subseteq U$.

Definition 3.5. Let $(X, \mathcal{B})$ be an intuitionistic $\widehat{\mathcal{B}}$ structure space and Y be a subset of $(X, \mathcal{B})$. Then $T_Y = \{A \cap Y_\sim : A = \langle x, A^1, A^2 \rangle \in T\}$ is an intuitionistic topology on Y and is called the induced or relative intuitionistic topology. The pair (Y, T_Y) is called an intuitionistic subspace of $(X, \mathcal{B})$.

Definition 3.6. If $(X, \mathcal{B}_1)$ and $(Y, \mathcal{B}_2)$ be any two intuitionistic $\widehat{\mathcal{B}}$ structure spaces then $f : (X, \mathcal{B}_1) \to (Y, \mathcal{B}_2)$ is said to be an intuitionistic $\widehat{\mathcal{B}}$ structure continuous function if $f^{-1}(U)$ is an intuitionistic $\widehat{\mathcal{B}}$ structure open set of X for each intuitionistic $\widehat{\mathcal{B}}$ structure open set $U = \langle x, U^1, U^2 \rangle$ of Y.

Definition 3.7. Let $\mathcal{R}$ be a real line. An interval $I = \langle R, (a, b), (-\infty, a] \rangle$ where $a, b \in \mathcal{R}$ is said to be an open intuitionistic interval in $\mathcal{R}$. Throughout this chapter the usual left intuitionistic topology is defined on $\mathcal{R}$.

Notation 3.8. Let (X, T) be an intuitionistic topological space and the usual left intuitionistic topology is defined on $\mathcal{R}$. Let $f : X \to \mathcal{R}$ be a real valued function. For $r > 0$,

(i) $f(x) > r$ denotes $f(x)$ is a member in the intuitionistic interval $\langle R, (r, \infty), (-\infty, r] \rangle$.

(ii) $f(x) < r$ denotes $f(x)$ is a member in the intuitionistic interval $\langle R, (0, r), (-\infty, 0] \rangle$.

(iii) $f(x) \geq r$ denotes $f(x)$ is a member in the intuitionistic interval $\langle R, [r, \infty), [-\infty, r) \rangle$.

(iv) $f(x) \leq r$ denotes $f(x)$ is a member in the intuitionistic interval $\langle R, [0, r), [-\infty, 0) \rangle$.

Notation 3.9. Let $[a, b]$ be an intuitionistic closed interval in the real line induced with the intuitionistic subspace topology.

(i) $\{f(x) \in [a,b]/f(x) \leq r\}$ denotes $\langle [a,b], [a,r], (-\infty, a]\rangle$.

(ii) $\{f(x) \in [a,b]/f(x) < r\}$ denotes $\langle [a,b], (a,r), (-\infty, a]\rangle$.

(iii) $\{f(x) \in [a,b]/f(x) \geq r\}$ denotes $\langle [a,b], [r,b], (-\infty, r)\rangle$.

(iv) $\{f(x) \in [a,b]/f(x) > r\}$ denotes $\langle [a,b], (r,b], (-\infty, r]\rangle$.

Notation 3.10. Let $[a,b]$ be a subset of $\mathcal{R}$, intuitionistic open interval is defined as follows.

(i) $\langle R, (c,d), (a,c]\rangle$ where $c, d \in [a,b]$.

(ii) $\langle R, [c,d], (a,c)\rangle$ where $c, d \in [a,b)$.

Notation 3.11. Inverse image of an open interval I of R under the real valued function is defined as,
$f^{-1}(I) = \langle x, f^{-1}(I^1), f^{-1}(I^2)\rangle$ where $f^{-1}(I^1) = \{x \in X/f(x) \in I\}$ and
$f^{-1}(I^2) = X - f^{-1}(I^1)$.

Definition 3.12. Let $(X, \mathcal{B})$ be an intuitionistic $\widehat{\mathcal{B}}$ structure space and function $f : X \to R$ is said to be an intuitionistic continuous real valued function on X if $f^{-1}(I)$ is intuitionistic $\widehat{\mathcal{B}}$ structure open in X for every open intuitionistic interval I of R.

Definition 3.13. Let $(X, \mathcal{B})$ be an intuitionistic $\widehat{\mathcal{B}}$ structure space, and let $f, g : X \to R$ be intuitionistic $\widehat{\mathcal{B}}$ structure continuous functions, where R has the intuitionistic usual topology. If $x \in X$, then the real-valued functions $|f|$, $f + g$, fg, given by

(i) $|f|(x) = |f(x)|$.

(ii) $(f+g)(x) = f(x) + g(x)$.

(iii) $(fg)(x) = f(x)g(x)$

are intuitionistic $\widehat{\mathcal{B}}$ structure continuous.

Definition 3.14. Let (X, T) be an intuitionistic topological space. An intuitionistic set $A = \langle x, A^1, A^2\rangle$ of X is said to be an intuitionistic $\widehat{\mathcal{B}}$ structure neighborhood of x if there exists an intuitionistic $\widehat{\mathcal{B}}$ structure open set $U = \langle x, U^1, U^2\rangle$ such that $x \in U \subseteq A$.

Proposition 3.15. Let X be an intuitionistic $\widehat{\mathcal{B}}$ structure normal space. Let $A \subset \overline{B}$ where $A = \langle x, A^1, A^2\rangle$ and $B = \langle x, A^1, A^2\rangle$ be any two intuitionistic $\widehat{\mathcal{B}}$ structure closed sets of X. Let $[a,b]$ be a closed interval in the real line. Then there exists an intuitionistic $\widehat{\mathcal{B}}$ structure continuous function $f : X \to [a,b]$ such that, $f(x) = a$ for every $x \in A$ and $f(x) = b$ for every $x \in B$.

Proof. Instead of $[a,b]$ the proposition is proved for $[0,1]$. The general case follows. Let D be the set of rational numbers in $[0,1]$. Arrange D in some order. Let it be

$\{1, 0, 1/2, 1/3, 2/3, 1/4, 3/4, 1/5, 2/5......\}$. Define, for each $p \in D$ an intuitionistic $\widehat{\mathcal{B}}$ structure open set $U_p = \langle x, U_p^1, U_p^2 \rangle$ of the intuitionistic $\widehat{\mathcal{B}}$ structure normal space X. It is defined in such a manner that if $p, q \in D$ with $p < q$ then $I\widehat{\mathcal{B}}cl(U_p) \subseteq U_q$.

Construct a sequence of intuitionistic $\widehat{\mathcal{B}}$ structure open sets in X as follows. First define $U_1 = \overline{B}$ where $U_1 = \langle x, U_1^1, U_1^2 \rangle$. Here A is an intuitionistic $\widehat{\mathcal{B}}$ structure closed set contained in the intuitionistic $\widehat{\mathcal{B}}$ open set U_1. Using intuitionistic $\widehat{\mathcal{B}}$ structure normality of X choose an intuitionistic $\widehat{\mathcal{B}}$ structure open set $U_0\langle x, U_0^1, U_0^2 \rangle$ such that $A \subseteq U_0$ and $I\widehat{\mathcal{B}}cl(U_0) \subseteq U_1$. In general let D_n denote the set consisting of the first n rational numbers in the sequence.

By assumption U_p is defined for all rational numbers $p \in D$, with the condition that if

$$p < q \Rightarrow I\widehat{\mathcal{B}}cl(U_p) \subseteq U_q. \tag{1}$$

Define U_r where r is the next rational number in the sequence. Consider the set $D_{n+1} = D_n \cup \{r\}$. It is a finite subset of the interval [0, 1] and as such it has a simple ordering derived from the usual order relation $<$ on the real line. In a finite simply ordered set every element has an immediate predecessor and an immediate successor. The number 0 is the smallest element and 1 is the largest element of the simply ordered set D_{n+1} and r is neither 0 nor 1. Let r has the immediate predecessor p in D_{n+1} and an immediate successor q in D_{n+1}. The sets U_p and U_q are already defined and by induction hypothesis $I\widehat{\mathcal{B}}cl(U_p) \subseteq U_q$. Using intuitionistic $\widehat{\mathcal{B}}$ structure normality of X, find an intuitionistic $\widehat{\mathcal{B}}$ open set set U_r of X such that $I\widehat{\mathcal{B}}cl(U_p) \subseteq U_r$ and $I\widehat{\mathcal{B}}cl(U_r) \subseteq U_q$.

It can be concluded that (3.1) holds for every pair of elements D_{n+1}. If both the elements lie in D_n, then (3.1) holds by induction hypothesis. Let r and s be such a pair from D_n. Then either $s \leq p$, in which case $I\widehat{\mathcal{B}}cl(U_s) \subseteq I\widehat{\mathcal{B}}cl(U_p) \subseteq U_r$, or $s \geq q$, in which case $I\widehat{\mathcal{B}}cl(U_r) \subseteq U_q \subseteq U_s$ respectively. Thus for every pair of elements of D_{n+1}, relation (3.1) holds. By mathematical induction U_p is defined for every $p \in D$. Extend this definition to all rational numbers $p \in R$ by defining $U_p = \phi_\sim$ if $p < 0$ and

$$U_p = X_\sim \tag{2}$$

if $p > 1$. The relation (3.1) is still true for any pair of rational numbers with $p < q$ implies $I\widehat{\mathcal{B}}cl(U_p) \subseteq U_q$. Let a point x of X. Let us Define the set of those rational numbers p such that the corresponding intuitionistic $\widehat{\mathcal{B}}$ structure open U_p contain x: $Q(x) = \{p : x \in U_p\}$.

From (3.2) This set contains no number less than 0, since no x is in U_p for $p < 0$ and every number greater than 1, since every x is in U_p for $p > 0$. Therefore, $Q(x)$ is bounded below and its g.l.b. is a point in [0, 1] say $f(x)$. Therefore, $f(x) = g.l.b.\ Q(x) = inf\, Q(x)$.

Case (i): $x \in A$ then $x \in Up$ for every $p \geq 0$ implies $Q(x)$ equals the set of non negative rationals implies $f(x) = inf\, Q(x) = 0$.

Case (ii): $x \in B$ then $x \notin U_p$, $p \leq 1$ implies $x \in U_p$ for $p > 1$ implies $Q(x)$ consists of all rational numbers greater than 1 implies $f(x) = 1$.

Consider the intuitionistic open interval $\langle [0,1], (c,d), (-\infty, c] \rangle$ in $[0,1]$, if $x \in f^{-1}(I)$, $f(x) \in I$, $f(x) \leq d$ implies $d \in Q(x)$. Therefore $x \in U_d$. Hence $f^{-1}(I) \subset U_d$. If $x \in U_d$, $f(x) \leq d$, $f(x) \in I$. Therefore, $x \in f^{-1}(U_d) \subset f^{-1}(I)$. Therefore $f^{-1}(I) = U_d$. Therefore U_d is an intuitionistic $\widehat{\mathcal{B}}$ structure open set. Hence f is continuous. ■

Notation 3.16. Let $[a,b]$ be a subset of $\mathcal{R}$, the intuitionistic open interval is defined as follows.

(i) $\widetilde{0}$ denotes $\langle R, (-\infty, 0], (0, \infty) \rangle$.

Definition 3.17. Let $(X, \mathcal{B})$ be an intuitionistic $\widehat{\mathcal{B}}$ structure space and an intuitionistic set $A = \langle x, A^1, A^2 \rangle$ in $(X, \mathcal{B})$ is called an intuitionistic $\widehat{\mathcal{B}}$ structure zero set of X if there exists a continuous real valued function f on X such that $A^1 = \{x \in X / f(x) \in \widetilde{0}\}$ and $A^2 = X - A^1$.

The complement of an intuitionistic $\widehat{\mathcal{B}}$ structure zero set is called an intuitionistic $\widehat{\mathcal{B}}$ structure cozero set.

Remark 3.18. If f is an intuitionistic continuous real valued function on intuitionistic $\widehat{\mathcal{B}}$ structure space X, let us put $Z^1(f) = \{x \in X / f(x) \in \widetilde{0}\}$, $Z^2(f) = X - Z^1(f)$ and call $Z(f)$ the intuitionistic $\widehat{\mathcal{B}}$ structure zero set of f. If A is an intuitionistic $\widehat{\mathcal{B}}$ structure zero set of a space X, then there exists an intuitionistic $\widehat{\mathcal{B}}$ structure continuous function g on X such that $g(x) \geq 0$ for all x in X and $A = Z(f)$, we can take $g = \mid f \mid$. Similarly there exists a continuous function $h : X \to I$ such that $A = Z(h)$, for we can take $h(x) = min\{g(x), 1\}$, if $x \in X$.

Definition 3.19. Let $(X, \mathcal{B})$ be an intuitionistic $\widehat{\mathcal{B}}$ structure space. Let A be any intuitionistic set. Then $A = \langle x, A^1, A^2 \rangle$ is said to be an intuitionistic $\widehat{\mathcal{B}}$ structure G_δ set if $A = \cap_{i=1}^{\infty} U_i$, where each U_i is an intuitionistic $\widehat{\mathcal{B}}$ structure open set. The complement of an intuitionistic $\widehat{\mathcal{B}}$ structure G_δ set is an intuitionistic $\widehat{\mathcal{B}}$ structure F_σ set.

Notation 3.20. Let $\{x \in X / f(x) < r\}$ denotes $\langle \mathcal{R}, (0, r), (-\infty, 0] \rangle$.

Proposition 3.21. An intuitionistic $\widehat{\mathcal{B}}$ structure zero set of an intuitionistic $\widehat{\mathcal{B}}$ structure topological space is an intuitionistic $\widehat{\mathcal{B}}$ structure closed G_δ set. An intuitionistic $\widehat{\mathcal{B}}$ structure closed G_δ set in an intuitionistic $\widehat{\mathcal{B}}$ structure normal space is an intuitionistic $\widehat{\mathcal{B}}$ structure zero set.

Proof. Let $A = \langle x, A^1, A^2 \rangle$ be an intuitionistic $\widehat{\mathcal{B}}$ structure zero set in an intuitionistic $\widehat{\mathcal{B}}$ structure topological space. Then A is an intuitionistic $\widehat{\mathcal{B}}$ structure closed set and if $A^1 = Z^1(g)$, where g is an intuitionistic continuous real valued function such that $g(x) \geq 0$ if $x \in X$, then $A = \cap_{n \in N} G_n$, where $G_n = \langle x, G_n^1, G_n^2 \rangle$, $G_n^1 = \{x \in X / g(x) < 1/n\}$, so that A is an intuitionistic $\widehat{\mathcal{B}}$ structure G_δ set.

Let A be an intuitionistic $\widehat{\mathcal{B}}$ structure closed set of an intuitionistic $\widehat{\mathcal{B}}$ structure normal space X such that $A = \cap_{n \in N} G_n$, where each G_n is an intuitionistic $\widehat{\mathcal{B}}$ structure open set. By Proposition 3.1., for each n there exists an intuitionistic $\widehat{\mathcal{B}}$ structure continuous function $f : X \to [0, 1/2^n]$ such that $f_n(x) = 0$ if $x \in A$ and $f_n(x) = (1/2^n)$ if $x \in \overline{G_n}$. Then the function $f : X \to I$ defined by $f(x) = \sum_{n=1}^{\infty} f_n(x)$ if $x \in X$, is an intuitionistic $\widehat{\mathcal{B}}$ structure continuous, and it is clear that $A = Z(f)$. ■

Remark 3.22. If $(X, \mathcal{B})$ is an intuitionistic $\widehat{\mathcal{B}}$ structure normal space and A is an intuitionistic $\widehat{\mathcal{B}}$ structure closed subspace of X, then each intuitionistic continuous real valued function on A has an intuitionistic $\widehat{\mathcal{B}}$ structure extension to X.

Definition 3.23. An intuitionistic $\widehat{\mathcal{B}}$ structure covering of an intuitionistic $\widehat{\mathcal{B}}$ structure space is a family $\mathcal{A} = \{A_\lambda\}_{\lambda\in\Lambda}$ where A_λ's are intuitionistic sets such that $\cup_{\lambda\in\Lambda}(A_\lambda) = X_\sim$. If each set $A_\lambda = \langle x, A_\lambda^1, A_\lambda^2\rangle$ is an intuitionistic $\widehat{\mathcal{B}}$ structure open set, then $\mathcal{A}$ is an intuitionistic $\widehat{\mathcal{B}}$ structure open covering and if each set A_λ is an intuitionistic $\widehat{\mathcal{B}}$ structure closed, then $\mathcal{A}$ is an intuitionistic $\widehat{\mathcal{B}}$ structure closed covering.

Definition 3.24. An intuitionistic $\widehat{\mathcal{B}}$ structure covering $\{B_\gamma\}_{\gamma\in\Gamma}$ is said to be intuitionistic $\widehat{\mathcal{B}}$ structure refinement of an intuitionistic $\widehat{\mathcal{B}}$ structure covering $\{A_\lambda\}_{\lambda\in\Lambda}$ if for each γ in Γ there exists some λ in Λ such that $B_\nu \subseteq A_\lambda$.

Definition 3.25. A family $\{A_\lambda\}_{\lambda\in\Lambda}$ of intuitionistic $\widehat{\mathcal{B}}$ structure sets of an intuitionistic $\widehat{\mathcal{B}}$ structure space X is said to be an intuitionistic $\widehat{\mathcal{B}}$ structure point finite if for each point x of X the intuitionistic $\widehat{\mathcal{B}}$ structure set $\{\lambda \in \Lambda/x \in A_\lambda\}$ is finite.

Definition 3.26. A family $\{A_\lambda\}_{\lambda\in\Lambda}$ is said to be an intuitionistic $\widehat{\mathcal{B}}$ structure locally finite if for each point x of X there exists an intuitionistic $\widehat{\mathcal{B}}$ structure neighbourhood N_x of x such that $\{\lambda \in \Lambda/N_x \cap A_\lambda \neq \phi_\sim\}$ is finite.

Proposition 3.27. If $\{A_\lambda\}_{\lambda\in\Lambda}$ where $A_\lambda = \langle x, A_\lambda^1, A_\lambda^2\rangle$ is an intuitionistic $\widehat{\mathcal{B}}$ structure locally finite family of intuitionistic sets of a space X, then $\{I\widehat{\mathcal{B}}cl A_\lambda\}_{\lambda\in\Lambda}$ is an intuitionistic $\widehat{\mathcal{B}}$ structure locally finite family of intuitionistic sets of X.

Proof. Let x be a point of X and let V_x be an intuitionistic $\widehat{\mathcal{B}}$ structure open neighbourhood of x such that $\{\lambda \in \Lambda/ V_x \cap A_\lambda \neq \phi_\sim\}$ is finite. If $V_x \cap A_\lambda = \phi_\sim$, then $V_x \cap I\widehat{\mathcal{B}}cl A_\lambda = \phi_\sim$ so that $\{\lambda \in \Lambda/ V_x \cap I\widehat{\mathcal{B}}cl A_\lambda \neq \phi_\sim\}$ is finite. Thus $\{I\widehat{\mathcal{B}}cl A_\lambda\}_{\lambda\in\Lambda}$ is an intuitionistic $\widehat{\mathcal{B}}$ structure locally finite. ■

Definition 3.28. Let X be a set and let $\{A_\lambda\}_{\lambda\in\Lambda}$ be an intuitionistic $\widehat{\mathcal{B}}$ structure cover of X, where for each $\lambda \in \Lambda$ there is an intuitionistic topology defined on A_λ such that the following condition is satisfied, if $\lambda, \mu \in \lambda$, then the intuitionistic set $A_\lambda \cap A_\mu$ is an intuitionistic $\widehat{\mathcal{B}}$ structure closed in A_λ and in A_μ, and the intuitionistic topology induced on $A_\lambda \cap A_\mu$ by the intuitionistic topologies of A_λ and A_μ coincide. Let $\mathcal{F}$ be the family of intuitionistic sets F of X such that $F \cap A_\lambda$ is an intuitionistic $\widehat{\mathcal{B}}$ structure closed in A_λ for every λ. It is easily verified that $\mathcal{F}$ is the family of intuitionistic $\widehat{\mathcal{B}}$ structure closed sets for an intuitionistic topology on X. In this intuitionistic topology each intuitionistic set A_λ is intuitionistic $\widehat{\mathcal{B}}$ structure closed. An intuitionistic topology thus defined on X is called the intuitionistic weak topology with respect to the intuitionistic $\widehat{\mathcal{B}}$ structure covering $\{A_\lambda\}_{\lambda\in\Lambda}$. It is the largest intuitionistic topology on X including the given intuitionistic topology A_λ for every $\lambda \in \Lambda$.

Definition 3.29. Let $(X, \mathcal{B}_1)$ and $(Y, \mathcal{B}_2)$ be any two intuitionistic $\widehat{\mathcal{B}}$ structure spaces. A function $f : (X, T) \to (Y, S)$ is said to be an intuitionistic $\widehat{\mathcal{B}}$ structure continuous function if the inverse image of every intuitionistic $\widehat{\mathcal{B}}$ structure open set in (Y, S) is an intuitionistic $\widehat{\mathcal{B}}$ structure open set in (X, T).

Proposition 3.30. If f is a function with domain an intuitionistic topological space X which has the intuitionistic $\widehat{\mathcal{B}}$ structure weak topology with respect to an intuitionistic $\widehat{\mathcal{B}}$ structure covering $\{A_\lambda\}_{\lambda\in\Lambda}$, and f/A_λ is intuitionistic $\widehat{\mathcal{B}}$ structure continuous for every λ, then f is intuitionistic $\widehat{\mathcal{B}}$ structure continuous.

Proof. Let $E = \langle x, E^1, E^2\rangle$ be an intuitionistic $\widehat{\mathcal{B}}$ structure closed set of the range of f. Then $(f^{-1}(E)) \cap A_\lambda = (f/A_\lambda)^{-1}(E)$. which is intuitionistic $\widehat{\mathcal{B}}$ structure closed in A_λ since f/A_λ is intuitionistic $\widehat{\mathcal{B}}$ structure continuous. Since X has the intuitionistic $\widehat{\mathcal{B}}$ structure weak topology with respect to $\{A_\lambda\}_{\lambda\in\Lambda}$, it follows that $f^{-1}(E)$ is intuitionistic $\widehat{\mathcal{B}}$ structure closed in X. Hence f is intuitionistic $\widehat{\mathcal{B}}$ structure continuous. ■

Definition 3.31. An intuitionistic $\widehat{\mathcal{B}}$ structure open covering $\{U_\lambda\}_{\lambda\in\Lambda}$ of an intuitionistic $\widehat{\mathcal{B}}$ structure space X is said to be intuitionistic $\widehat{\mathcal{B}}$ structure shrinkable if there exists an intuitionistic $\widehat{\mathcal{B}}$ structure open covering $\{V_\lambda\}_{\lambda\in\Lambda}$ of X such that $I\widehat{\mathcal{B}}cl\{V_\lambda\} \subseteq U_\lambda$ for each λ in Λ.

Definition 3.32. An intuitionistic $\widehat{\mathcal{B}}$ structure covering $\{B_\gamma\}_{\gamma\in\Gamma}$ of an intuitionistic $\widehat{\mathcal{B}}$ structure space X is said to be intuitionistic $\widehat{\mathcal{B}}$ structure refinement of an intuitionistic $\widehat{\mathcal{B}}$ structure covering $\{A_\lambda\}_{\lambda\in\Lambda}$ if for each γ in Γ there exists some λ in Λ such that $B_\gamma \subseteq A_\lambda$.

Proposition 3.33. Let $(X, \mathcal{B})$ be an intuitionistic $\widehat{\mathcal{B}}$ structure space. Let $A = \langle x, A^1, A^2\rangle$ be an intuitionistic set in X. If X is an intuitionistic $\widehat{\mathcal{B}}$ structure normal space then each intuitionistic $\widehat{\mathcal{B}}$ structure point-finite open covering of X is an intuitionistic $\widehat{\mathcal{B}}$ structure shrinkable.

Proof. Let $\{U_\lambda\}_{\lambda\in\Lambda}$ be an intuitionistic $\widehat{\mathcal{B}}$ structure point finite open covering of an intuitionistic $\widehat{\mathcal{B}}$ structure normal space X and let λ be well-ordered. Construct an intuitionistic $\widehat{\mathcal{B}}$ structure shrinking of $\{U_\lambda\}_{\lambda\in\Lambda}$ where $U_\lambda = \langle x, U_\lambda^1, U_\lambda^2\rangle$ and $V_\lambda = \langle x, V_\lambda^1, V_\lambda^2\rangle$ by transfinite induction. Let μ be an element of Λ and suppose that for each $\lambda < \mu$ we have an intuitionistic $\widehat{\mathcal{B}}$ structure open set V_λ such that $I\widehat{\mathcal{B}}cl\{V_\lambda\} \subseteq U_\lambda$ and for each $\nu < \mu$, $\cup_{\lambda\leq\nu} V_\lambda \cup \cup_{\lambda>\nu} U_\lambda = X_\sim$. Let x be a point of X. Then since $\{U_\lambda\}_{\lambda\in\Lambda}$ is an intuitionistic $\widehat{\mathcal{B}}$ structure point-finite there exists a largest element ξ of Λ such that $x \in U_\xi$. If $\xi \geq \mu$ then $x \in \cup_{\lambda\geq\mu}(U_\lambda)$, if $\xi < \mu$ then $x \in \cup_{\lambda\leq\xi}(V_\lambda) \subseteq \cup_{\lambda<\mu}(V_\lambda)$. Hence $\cup_{\lambda<\mu}(V_\lambda) \cup \cup_{\lambda\geq\mu}(U_\lambda) = X_\sim$.

Thus U_μ contains the complement of $\cup_{\lambda<\mu}(V_\lambda)\cup\cup_{\lambda>\mu}(U_\lambda)$. Since X is intuitionistic $\widehat{\mathcal{B}}$ structure normal there exists an intuitionistic $\widehat{\mathcal{B}}$ structure open set V_μ where $V_\mu = \langle x, V_\mu^1, V_\lambda^2\rangle$ such that $\overline{\cup_{\lambda<\mu}V_\lambda \cup \cup_{\lambda>\mu}U_\lambda} \subseteq V_\mu \subseteq I\widehat{\mathcal{B}}cl\{V_\mu\} \subseteq U_\mu$. Thus $I\widehat{\mathcal{B}}cl\{V_\mu\} \subseteq U_\mu$ and $\cup_{\lambda\leq\mu}V_\lambda \cup \cup_{\lambda>\mu}U_\lambda = X_\sim$. The construction of an intuitionistic $\widehat{\mathcal{B}}$ structure shrinking of $\{U_\lambda\}_{\lambda\in\Lambda}$ is completed by transfinite induction. ■

Proposition 3.34. Let $(X, \mathcal{B})$ be an intuitionistic $\widehat{\mathcal{B}}$ structure space. Let $A = \langle x, A^1, A^2\rangle$ be an intuitionistic set in X. If each intuitionistic $\widehat{\mathcal{B}}$ structure point-finite open covering of X is an intuitionistic $\widehat{\mathcal{B}}$ structure shrinkable then each finite intuitionistic $\widehat{\mathcal{B}}$ structure open covering of X has an intuitionistic $\widehat{\mathcal{B}}$ structure locally finite closed refinement.

Proof. The proof is obvious. ■

Proposition 3.35. If $f : (X, T) \to (Y, S)$ is an intuitionistic $\widehat{\mathcal{B}}$ structure continuous open surjection and X is an intuitionistic $\widehat{\mathcal{B}}$ structure normal space, then Y is an intuitionistic $\widehat{\mathcal{B}}$ structure normal space.

Proof. Let $A = \langle x, A^1, A^2\rangle$ and $B = \langle x, B^1, B^2\rangle$ be any two intuitionistic $\widehat{\mathcal{B}}$ structure closed sets in (Y, S) such that $A \subset \overline{B}$. Then $f^{-1}(A) \subset f^{-1}(\overline{B})$ is an intuitionistic $\widehat{\mathcal{B}}$ structure open set in (X, T). Since (X, T) is an intuitionistic $\widehat{\mathcal{B}}$ structure normal space then there exists an intuitionistic $\widehat{\mathcal{B}}$ structure open set $U = \langle x, U^1, U^2\rangle$ such that $f^{-1}(A) \subset U \subset I\widehat{\mathcal{B}}cl(U) \subset f^{-1}(\overline{B})$, $f(f^{-1}(A)) \subset f(U) \subset f(I\widehat{\mathcal{B}}cl(U)) \subset f(f^{-1}(\overline{B}))$. Therefore $A \subset f(U) \subset I\widehat{\mathcal{B}}clf(U) \subset \overline{B}$. Then (Y, S) is an intuitionistic $\widehat{\mathcal{B}}$ structure normal space. ■

Remark 3.36. Let $\{f_\lambda\}_{\lambda\in\Lambda}$ be a family of intuitionistic real valued functions on an intuitionistic topological space X and for each λ let $S_\lambda = \langle x, S_\lambda^1, S_\lambda^2\rangle$ where $S_\lambda^1 = \{x \in X/f_\lambda(x) \neq 0\}$ and $S_\lambda^2 = X - S_\lambda^1$. If $\{S_\lambda\}_{\lambda\in\Lambda}$ is an intuitionistic $\widehat{\mathcal{B}}$ structure point finite then for each x in X we can define $f(x) = \sum_\lambda f_\lambda(x)$, for the sum has only finitely many zero terms.

Notation 3.37. Let $(X, \mathcal{B})$ be an intuitionistic $\widehat{\mathcal{B}}$ structure space. Let $A = \langle x, A^1, A^2\rangle$ be any intuitionistic set in X. A is said to be empty if $A = \phi_\sim$ and A is said to be non-empty if $A \neq \phi_\sim$.

Definition 3.38. Let X be any set. The families $\{A_\lambda\}_{\lambda\in\Lambda}$ and $\{B_\lambda\}_{\lambda\in\Lambda}$ of intuitionistic sets of X are said to be an intuitionistic $\widehat{\mathcal{B}}$ structure similar if for each finite set γ of Λ the intuitionistic $\widehat{\mathcal{B}}$ structure sets $\cap_{\lambda\in\gamma}A_\lambda$ and $\cap_{\lambda\in\gamma}B_\lambda$ are either both empty or both non-empty.

Proposition 3.39. Let $\{U_\lambda\}_{\lambda\in\Lambda}$ be an intuitionistic $\widehat{\mathcal{B}}$ structure locally finite family of intuitionistic $\widehat{\mathcal{B}}$ structure open sets of an intuitionistic $\widehat{\mathcal{B}}$ structure normal space X and let $\{F_\lambda\}_{\lambda\in\Lambda}$ where $F_\lambda = \langle x, F_\lambda^1, F_\lambda^2\rangle$ be a family of intuitionistic $\widehat{\mathcal{B}}$ structure closed sets such that $F_\lambda \subseteq U_\lambda$ for each λ. Then there exists a family $\{G_\lambda\}_{\lambda\in\Lambda}$ where $G_\lambda = \langle x, G_\lambda^1, G_\lambda^2\rangle$

of intuitionistic $\widehat{\mathcal{B}}$ structure open sets such that $F_\lambda \subseteq G_\lambda \subseteq I\widehat{\mathcal{B}}cl\{G_\lambda\} \subseteq U_\lambda$ and the families $\{F_\lambda\}_{\lambda\in\Lambda}$ and $I\widehat{\mathcal{B}}cl\{G_\lambda\}_{\lambda\in\Lambda}$ are intuitionistic $\widehat{\mathcal{B}}$ structure similar.

Proof. Let Λ be well-ordered with a last element. By transfinite induction we shall construct a family $\{G_\lambda\}_{\lambda\in\Lambda}$ of an intuitionistic $\widehat{\mathcal{B}}$ structure open sets such that $F_\lambda \subseteq G_\lambda \subseteq I\widehat{\mathcal{B}}cl\{G_\lambda\} \subseteq U_\lambda$, and for each element ν of Λ the family $\{K^\nu_\lambda\}_{\lambda\in\Lambda}$, given by

$$K^\nu_\lambda = \begin{cases} I\widehat{\mathcal{B}}cl\{G_\lambda\} & \text{if } \lambda \le \nu \\ F_\lambda & \text{if } \lambda > \nu \end{cases}$$

is $\widehat{\mathcal{B}}$ structure similar to $\{F_\lambda\}_{\lambda\in\Lambda}$.

Suppose that $\mu \in \Lambda$ and that G_λ has been defined for $\lambda < \mu$ such that for each $\nu < \mu$ the family $\{K^\nu_\lambda\}_{\lambda\in\Lambda}$ is similar to $\{F_\lambda\}_{\lambda\in\Lambda}$. Let $\{L_\lambda\}_{\lambda\in\Lambda}$ be the family given by

$$L_\lambda = \begin{cases} I\widehat{\mathcal{B}}cl\{G_\lambda\} & \text{if } \lambda < \mu \\ F_\lambda & \text{if } \lambda \ge \mu \end{cases}$$

Then $\{L_\lambda\}_{\lambda\in\Lambda}$ is similar to $\{F_\lambda\}_{\lambda\in\Lambda}$. For suppose that $\lambda_1, ..., \lambda_r \in \Lambda$ and $\lambda_1 < \lambda_2 < .... < \lambda_j < \mu \le \lambda_{j+1} < .... < \lambda_r$. Then $\cap_{i=1}^r L_{\lambda_i} = \cap_{i=1}^r K^{\lambda_j}_{\lambda_i}$, so that $\cap_{i=1}^r L_{\lambda_i} = \phi_\sim$ if and only if $\cap_{i=1}^r F_{\lambda_i} = \phi_\sim$, since $\cap_{i=1}^r K^{\lambda_j}_{\lambda_i}$ is similar to $\{F_\lambda\}_{\lambda\in\Lambda}$. Since $L_\lambda \subseteq U_\lambda$ for each λ the family $\{L_\lambda\}_{\lambda\in\Lambda}$ is $\widehat{\mathcal{B}}$ structure locally finite. Thus if Γ is the set of finite sets of Λ and for each γ in Γ, $E_\gamma = \cap_{\lambda\in\gamma} L_\lambda$, then $\{E_\gamma\}_{\gamma\in\Gamma}$ is an intuitionistic $\widehat{\mathcal{B}}$ structure locally finite family of intuitionistic $\widehat{\mathcal{B}}$ structure closed sets. Hence $E = \cup\{E_\gamma / E_\gamma \cap F_\mu = \phi_\sim\}$ is an intuitionistic $\widehat{\mathcal{B}}$ structure closed set with $E \cap F_\mu = \phi_\sim$. Therefore there exists an intuitionistic $\widehat{\mathcal{B}}$ structure open set G_μ such that $F_\mu \subseteq G_\mu \subseteq I\widehat{\mathcal{B}}cl\{G_\mu\} \subseteq U_\mu$ and $I\widehat{\mathcal{B}}cl\{G_\lambda\} \cap E = \phi_\sim$.

Now the intuitionistic $\widehat{\mathcal{B}}$ structure open sets G_λ are defined for $\lambda \le \mu$ and it remains to show that the family $\{K^\mu_\lambda\}_{\lambda\in\Lambda}$ is similar to $\{F_\lambda\}_{\lambda\in\Lambda}$. It is sufficient to show that the families $\{K^\mu_\lambda\}_{\lambda\in\Lambda}$ and $\{L_\lambda\}_{\lambda\in\Lambda}$ are similar. Suppose that $\lambda_1, ..., \lambda_r \in \Lambda$ and that $\cap_{i=1}^r L_{\lambda_i} = \phi_\sim$. It must be shown that $\cap_{i=1}^r \{K^\mu_\lambda\}_{\lambda\in\Lambda} = \phi_\sim$. suppose that $\lambda_1 < .... < \lambda_j \le \mu < \lambda_{j+1} < ... < \lambda_r$. If $\lambda \ne \mu = \phi_\sim$, there is nothing to prove. If $\lambda = \mu$, then $L_{\lambda_1} \cap ..... \cap L_{\lambda_{j-1}} \cap F_\mu \cap L_{\lambda_{j+1}} \cap .... \cap L_{\lambda_r} = \phi_\sim$. Hence by the construction $L_{\lambda_1} \cap ..... \cap L_{\lambda_{j-1}} \cap I\widehat{\mathcal{B}}cl\{G_\mu\} \cap L_{\lambda_{j+1}} \cap .... \cap L_{\lambda_r} = \phi_\sim$. Thus $\cap_{i=1}^r K_{\lambda_i^\mu} = \phi_\sim$. ■

Proposition 3.40. The following statements about an intuitionistic $\widehat{\mathcal{B}}$ structure space $(X, \mathcal{B})$ are equivalent:

(i) $(X, \mathcal{B})$ is intuitionistic $\widehat{\mathcal{B}}$ structure normal;

(ii) For each finite family $\{F_1, ..., F_k\}$ of intuitionistic $\widehat{\mathcal{B}}$ structure closed sets of X there exists a family $\{G_1, ..., G_k\}$ of intuitionistic $\widehat{\mathcal{B}}$ structure open sets of $(X, \mathcal{B})$ such that each $F_i \subseteq G_i$ and the families $\{F_1, ..., F_k\}$ and $\{I\widehat{\mathcal{B}}cl\{G_1\}, ..., I\widehat{\mathcal{B}}cl\{G_k\}\}$ are intuitionistic $\widehat{\mathcal{B}}$ structure similar

(iii) For each pair F_1, F_2 of intuitionistic $\widehat{\mathcal{B}}$ structure closed sets with $F_1 \cap F_2$ of X there exists a pair G_1, G_2 of intuitionistic $\widehat{\mathcal{B}}$ structure open sets such that $F_i \subseteq G_i$ for $i = 1, 2$ and the intuitionistic $\widehat{\mathcal{B}}$ structure sets $I\widehat{\mathcal{B}}cl\{G_1\} \cap I\widehat{\mathcal{B}}cl\{G_2\} = \phi_\sim$.

Proof. The implication (i)⇒ (ii) holds, for if $\{F_1, \ldots.., F_k\}$ is a family of intuitionistic $\widehat{\mathcal{B}}$ structure closed sets in an intuitionistic $\widehat{\mathcal{B}}$ structure normal space $(X, \mathcal{B})$, then let us take $U_i = X_\sim$ for $i = 1, \ldots., k$ and apply the preceding proposition. The implications (ii) ⇒ (iii) and (iii) ⇒(i). ■

4. Properties of intuitionistic $\widehat{\mathcal{B}}$ structure total normality

In this section, the concepts of intuitionistic $\widehat{\mathcal{B}}$ structure normally situated, intuitionistic $\widehat{\mathcal{B}}$ structure totally normal spaces and intuitionistic $\widehat{\mathcal{B}}$ structure regular spaces are introduced and studied. Some interesting properties are discussed.

Definition 4.1. Let $(X, \mathcal{B})$ be an intuitionistic $\widehat{\mathcal{B}}$ structure space. An intuitionistic set $A = \langle x, A^1, A^2\rangle$ is said to be intuitionistic $\widehat{\mathcal{B}}$ structure open F_σ if it is both intuitionistic $\widehat{\mathcal{B}}$ structure open set and intuitionistic $\widehat{\mathcal{B}}$ F_σ-set.

Definition 4.2. Let $(X, \mathcal{B})$ be an intuitionistic $\widehat{\mathcal{B}}$ structure space. An intuitionistic set $M = \langle x, M^1, M^2\rangle$ is said to be intuitionistic $\widehat{\mathcal{B}}$ structure generalized F_σ if for each intuitionistic $\widehat{\mathcal{B}}$ structure open set $U = \langle x, U^1, U^2\rangle$ such that $M \subseteq U$ there exists an intuitionistic $\widehat{\mathcal{B}}$ structure F_σ set $F = \langle x, F^1, F^2\rangle$ in $(X, \mathcal{B})$ such that $M \subseteq F \subseteq U$.

Definition 4.3. Let $(X, \mathcal{B})$ be an intuitionistic $\widehat{\mathcal{B}}$ structure space. An intuitionistic set $M = \langle x, M^1, M^2\rangle$ is said to be intuitionistic $\widehat{\mathcal{B}}$ structure normally situated in $(X, \mathcal{B})$ if for each intuitionistic $\widehat{\mathcal{B}}$ structure open set $U = \langle x, U^1, U^2\rangle$ such that $M \subseteq U$ there exists an intuitionistic $\widehat{\mathcal{B}}$ structure open set $G = \langle x, G^1, G^2\rangle$ such that $M \subseteq G \subseteq U$ and $G = \cup_{\lambda\in\Lambda} G_\lambda$ where $\{G_\lambda\}_{\lambda\in\underline{\Lambda}}$ is a family of intuitionistic $\widehat{\mathcal{B}}$ structure open F_σ sets in $(X, \mathcal{B})$ which is intuitionistic $\widehat{\mathcal{B}}$ locally finite in G, of intuitionistic $\widehat{\mathcal{B}}$ structure open F_σ-sets of $(X, \mathcal{B})$.

Remark 4.4. If $A = \langle x, A^1, A^2\rangle$ is an intuitionistic $\widehat{\mathcal{B}}$ structure normal space X and $U = \langle x, U^1, U^2\rangle$ is an intuitionistic $\widehat{\mathcal{B}}$ structure open set such that $A \subseteq U$, then by Proposition 3.1.1., there exists an intuitionistic $\widehat{\mathcal{B}}$ structure open F_σ set $H = \langle x, H^1, H^2\rangle$, such that $A \subseteq H \subseteq U$.

Proposition 4.5. If $M = \langle x, M^1, M^2\rangle$ is an intuitionistic $\widehat{\mathcal{B}}$ generalized F_σ set in an intuitionistic $\widehat{\mathcal{B}}$ structure normal space $(X, \mathcal{B})$, then M is intuitionistic $\widehat{\mathcal{B}}$ structure normally situated in $(X, \mathcal{B})$.

Proof. If M is an intuitionistic $\widehat{\mathcal{B}}$ generalized F_σ set in an intuitionistic $\widehat{\mathcal{B}}$ structure normal space $(X, \mathcal{B})$ and $M \subseteq U$, where $U = \langle x, U^1, U^2\rangle$ is an intuitionistic $\widehat{\mathcal{B}}$ structure open set, then there exists a family $\{A_i\}_{i\in N}$ of intuitionistic $\widehat{\mathcal{B}}$ structure closed sets such that $M \subseteq \cup_{i\in N} A_i \subseteq U$.

But since $(X, \mathcal{B})$ is an intuitionistic $\widehat{\mathcal{B}}$ structure normal space, $A_i \subseteq H_i \subseteq U$ for each i, where H_i is an intuitionistic $\widehat{\mathcal{B}}$ structure open F_σ set and $M \subseteq \cup H_i \subseteq U$. Hence M is intuitionistic $\widehat{\mathcal{B}}$ structure normally situated in $(X, \mathcal{B})$. ■

Definition 4.6. An intuitionistic $\widehat{\mathcal{B}}$ structure normal space $(X, \mathcal{B})$ is said to be intuitionistic $\widehat{\mathcal{B}}$ structure totally normal if every intuitionistic set in X is intuitionistic $\widehat{\mathcal{B}}$ normally situated in $(X, \mathcal{B})$.

Example 4.7. Let $X = \{a, b\}$. Then the intuitionistic sets A, B, C, D, E and F of X are defined by $A = \langle x, \{a\}, \{\phi\}\rangle$, $B = \langle x, \{\phi\}, \{a\}\rangle$, $C = \langle x, \{a\}, \{b\}\rangle$, $D = \langle x, \{b\}, \{a\}\rangle$, $E = \langle x, \{a, b\}, \{\phi\}\rangle$ and $F = \langle x, \{\phi\}, \{a, b\}\rangle$. Then the family $\widehat{\mathcal{B}} = \{X_\sim, \phi_\sim, A, B, C, D, E, F\}$ is an intuitionistic topology on X. Clearly $(X, \mathcal{B})$ is an intuitionistic $\widehat{\mathcal{B}}$ structure normal space. Now, $A = \cup\{B, C, F\}$ is an intuitionistic $\widehat{\mathcal{B}}$ structure normally situated set. Now, $(X, \mathcal{B})$ is an intuitionistic $\widehat{\mathcal{B}}$ normal space and B intuitionistic $\widehat{\mathcal{B}}$ open set such that $B \subseteq A \subseteq E$. Hence $(X, \widehat{\mathcal{B}})$ is an intuitionistic $\widehat{\mathcal{B}}$ structure totally normal space.

Definition 4.8. Let $(X, \mathcal{B})$ be an intuitionistic $\widehat{\mathcal{B}}$ structure space. Let $x_\sim$ be intuitionistic $\widehat{\mathcal{B}}$ structure closed in X for each $x \in X$. Then $(X, \mathcal{B})$ is said to be an intuitionistic $\widehat{\mathcal{B}}$ structure regular space if for each point x and each intuitionistic $\widehat{\mathcal{B}}$ closed set $F = \langle x, F^1, F^2\rangle$, $x \notin F$ there exist intuitionistic $\widehat{\mathcal{B}}$ open sets $U = \langle x, U^1, U^2\rangle$ and $V = \langle x, V^1, V^2\rangle$ such that $x \in U$ and $F \subseteq V$.

Notation 4.9. Let $(X, \mathcal{B})$ be an intuitionistic $\widehat{\mathcal{B}}$ structure space and let $A = \langle x, A^1, A^2\rangle$ be an intuitionistic set of X.

(i) $x_\sim \in A$ means $x_\sim \in A^1$ and $x_\sim \notin A^2$.

Proposition 4.10. Let $(X, \mathcal{B})$ be intuitionistic $\widehat{\mathcal{B}}$ structure totally normal space. Let $x_\sim$ be intuitionistic $\widehat{\mathcal{B}}$ structure closed in X for each $x \in X$. Then $(X, \mathcal{B})$ intuitionistic $\widehat{\mathcal{B}}$ structure regular space.

Proof. Let $(X, \mathcal{B})$ be an intuitionistic $\widehat{\mathcal{B}}$ structure totally normal space space. If $G = \langle x, G^1, G^2\rangle$ is an intuitionistic $\widehat{\mathcal{B}}$ structure open set and $x_\sim \in G$ then $x_\sim \in U \subseteq G$, where $U = \langle x, U^1, U^2\rangle$ is an intuitionistic $\widehat{\mathcal{B}}$ structure open F_σ-set. Hence there exists an intuitionistic $\widehat{\mathcal{B}}$ structure closed set $F = \langle x, F^1, F^2\rangle$ such that $x_\sim \in F \subseteq G$. Since $(X, \mathcal{B})$ is intuitionistic $\widehat{\mathcal{B}}$ structure normal there exists an intuitionistic $\widehat{\mathcal{B}}$ structure open set $V = \langle x, V^1, V^2\rangle$ such that $F \subseteq V \subseteq \widehat{\mathcal{B}}cl(V) \subseteq G$. Then $x_\sim \in V \subseteq \widehat{\mathcal{B}}cl(V) \subseteq G$. Hence $(X, \mathcal{B})$ is an intuitionistic $\widehat{\mathcal{B}}$ structure regular space. ■

Proposition 4.11. If an intuitionistic $\widehat{\mathcal{B}}$ structure open set $G = \langle x, G^1, G^2\rangle$ is intuitionistic $\widehat{\mathcal{B}}$ structure normally situated in an intuitionistic $\widehat{\mathcal{B}}$ structure normal space X, then for each positive integer i there is a family $\{W_{i\lambda}\}_{\lambda\in\Lambda}$, where $W_{i\lambda} = \langle x, W^1_{i\lambda}, W^2_{i\lambda}\rangle$, intuitionistic $\widehat{\mathcal{B}}$ structure locally finite in G, of disjoint intuitionistic $\widehat{\mathcal{B}}$ structure open sets and a family $\{F_{i\lambda}\}_{\lambda\in\Lambda}$, where $F_{i\lambda} = \langle x, F^1_{i\lambda}, F^2_{i\lambda}\rangle$ of intuitionistic $\widehat{\mathcal{B}}$ structure closed sets such that $F_{i\lambda} \subseteq W_{i\lambda} \subseteq G$ and $\cup_{i\in N} \cup_{\lambda\in\Lambda} F_{i\lambda} = G$.

Proof. If $G = \langle x, G^1, G^2 \rangle$ is an intuitionistic $\widehat{\mathcal{B}}$ structure normally situated open set of an intuitionistic $\widehat{\mathcal{B}}$ structure normal space X then $G = \cup_{\lambda \in \Lambda} G_\lambda$, where $\{G_\lambda = \langle x, G^1_\lambda, G^2_\lambda \rangle\}$ is intuitionistic $\widehat{\mathcal{B}}$ structure locally finite in G and each G_λ is an intuitionistic $\widehat{\mathcal{B}}$ structure open F_σ set of X. By Proposition 3.1.3., G_λ is an intuitionistic $\widehat{\mathcal{B}}$ structure cozero set so that for each λ there exists an intuitionistic $\widehat{\mathcal{B}}$ structure continuous function $f_\lambda : X \to I$ such that $f_\lambda(x) > 0$ if and only if $x \in G_\lambda$. Let Λ be well ordered and for each positive integer i and each $\lambda \in \Lambda$, let $W_{i\lambda} = \{x \in X/f_\lambda(x) > 1/(i+1), f_\mu(x) < 1/(i+1)$ for all $\mu < \lambda\}$.

If $x_0 \in W_{i\lambda}$, then $x_0 \in G$ so that there is an intuitionistic $\widehat{\mathcal{B}}$ structure open neighbourhood N of x_0 such that the intuitionistic set $\gamma = \{\mu \in \Lambda/\mu < \lambda$ and $N \cap G_\mu \neq \phi_\sim\}$ is finite. If $\mu \in \gamma$, let $N_\mu = \{x \in X/f_\mu(x) < 1/(i+1)\}$ and let $M = \{x \in X/f_\lambda(x) > 1/(i+1)\}$.

If $P = N \cap M \cap \cap_{\mu \in \gamma} N_\mu$, then P is an intuitionistic $\widehat{\mathcal{B}}$ structure open set and $x_0 \in P \subseteq W_{i\lambda}$. Thus $W_{i\lambda}$ is intuitionistic $\widehat{\mathcal{B}}$ structure open and since $W_{i\lambda} \subseteq G$, for each positive integer i the family $\{W_{i\lambda}\}_\lambda \in \Lambda$ is intuitionistic $\widehat{\mathcal{B}}$ structure locally finite in G. Clearly if $\lambda \neq \mu$ then $W_{i\lambda} \cap W_{i\mu} = \phi_\sim$. For each integer i and each λ in Λ let $F_{i\lambda} = \{x \in X/x \notin \cup_{\mu<\lambda} G_\mu$ and $f_\lambda(x) \geq 1/i\}$.

Then $F_{i\lambda}$ is intuitionistic $\widehat{\mathcal{B}}$ structure closed and $F_{i\lambda} \subseteq W_{i\lambda}$. If $x \in G$ there exists λ_0 in Λ such that $x \in G_{\lambda_0}$ and $x \notin G_\mu$ for $\mu < \lambda_0$. Then $f_{\lambda_0}(x) > 0$ and hence $f_{\lambda_0}(x) \geq 1/i$ for some i. Since $x \notin \cup_{\mu<\lambda_0} G_\mu$, it follows that $x \in F_{i\lambda_0}$. Thus $\cup_{i \in N} \cup_{\lambda \in \Lambda} F_{i\lambda_0} = G$. ■

References

[1] D. Coker, A note on intuitionistic sets and intuitionistic points, *TURKISH. J. math.*, 20(3)(1996), 343–351.

[2] D. Coker, An Introduction to Intuitionistic Topological Spaces, *Preliminary Report*. Ankara, 1995.

[3] A.R. Pears, Dimension Theory of General Spaces, Cambridge University Press, Cambridge, 1975.

[4] P. Saranya, Decompositions of Intuitionistic B-open sets, *International Journal of Mathematical Sciences and Engineering Applications*, 2(2014), 121–131.