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Harmonic Mean Technique to Solve Multi Objective Fuzzy Linear Fractional Programming Problems

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Abstract

A new technique is used to solve multi objective fuzzy linear fractional programming problem. We convert the multi objective fuzzy linear fractional programming problem (MOFLFPP) into multi objective linear fractional programming problem (MOLFPP) using graded mean integration representation (GMIR) method. The MOLFPP is transformed to a single objective linear fractional programming problem (SOLFPP) by using Harmonic Mean technique and solved by simplex method. A numerical example is given to show the efficiency of the proposed technique.

Keywords: Multi objective fuzzy linear fractional programming problem, Harmonic Mean technique, Graded Mean Integration Representation.

AMS Subject Classification: 90C32, 03E72

1. INTRODUCTION

Linear Fractional Programming is a mathematical programming problem in which the objective function to be optimized is a ratio of two linear functions subject to a set of constraints. Many researchers suggested several techniques to solve the linear fractional programming problems. The concept of a fuzzy decision making was first proposed by Bellman and Zadeh [1]. Geeta Modi et. al. [2] presented a new approach based on simplex method and dual simplex method for solving all integer linear fractional programming problem. Hasan and Sumi Acharjee [3] introduced a computer-oriented technique using MATHEMATICA for solving linear fractional programming problem by converting it into a single linear programming problem.

Herry Suprajitno [4] presented multiobjective linear programming problems with interval numbers as coefficients and value of its variables and he solved the problem by modified simplex method. Kanti Swarup [5] gave an algorithm for the solution of LFPP. Muruganandam and Ambika [6] proposed LU Decomposition method to solve fuzzy linear fractional programming problem. Saad and Hafez [7] presented an algorithm for solving bi-level integer linear fractional programming problem based on fuzzy approach. Seshan and Tikekar [8] proposed two algorithms to solve integer linear fractional programming problems. Ranking of fuzzy numbers plays an important role in fuzzy optimization. Chen and Hsieh [9] proposed graded mean integration representation for representing generalized fuzzy numbers. Sulaiman and Mustafa [10] suggested a new technique to transform multi-objective linear programming problem to a single objective linear programming problem by using harmonic mean for values of objective functions and an algorithm is given for its solution. Sulaiman et. al [11] used a new transformation technique for solving multi-objective linear fractional programming problem to single-objective linear fractional programming problem through a new method using arithmetic average and new arithmetic average technique. Then the problem is solved by modified simplex method.

Youness et. al [12] presented an algorithm to solve a bi-level multi-objective fractional integer programming problem involving fuzzy numbers in the right-hand side of the constraints. The suggested algorithm combines the method of Taylor series together with the Kuhn Tucker conditions to solve fuzzy bi-level multi-objective fractional integer programming problem. Then Gomory cuts were added till the integer solution is obtained.

This paper is organized as follows. Section 2 gives the preliminaries of trapezoidal fuzzy number and ranking method of fuzzy numbers. Section 3 gives the mathematical formulation of multi objective fuzzy linear fractional programming problem. Section 4 describes the algorithm of the proposed method.

2. PRELIMINARIES

Trapezoidal fuzzy number

A trapezoidal fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4)$ with membership function μ(x) given by

$$\mu(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, & a_1\le x\le a_2\\ 1, & a_2\le x\le a_3\\ \dfrac{a_4-x}{a_4-a_3}, & a_3\le x\le a_4\\ 0, & otherwise\end{cases}$$

Ranking of fuzzy numbers

Let F(R) denote the set of all trapezoidal fuzzy numbers. Let us define a ranking function $\Re : F(R) \to R$ which maps all trapezoidal fuzzy numbers into R.

If $\tilde{a}=(a_1,a_2,a_3,a_4)$ is a trapezoidal fuzzy number, then the Graded Mean Integration Representation (GMIR) method to defuzzify the fuzzy number is given by

$$\Re(\tilde{a})=\frac{\int_0^1 \frac{h(a_1+a_4+(a_2-a_1-a_4+a_3)h)}{2}\,dh}{\int_0^1 h\,dh}$$

$$\Re(\tilde{a})=\frac{a_1+2a_2+2a_3+a_4}{6} \tag{2.1}$$

3. MATHEMATICAL FORMULATION MULTI OBJECTIVE FUZZY LINEAR FRACTIONAL PROGRAMMING PROBLEM

Maximize $\tilde{Z}_i=\dfrac{\tilde{c}_i^{\,T}x+\tilde{\alpha}_i}{\tilde{d}_i^{\,T}x+\tilde{\beta}_i}$ $\quad i=1,2,\ldots\ldots,k$ (3.1)

Minimize $\tilde{Z}_i=\dfrac{\tilde{c}_i^{\,T}x+\tilde{\alpha}_i}{\tilde{d}_i^{\,T}x+\tilde{\beta}_i}$ $\quad i=k+1,k+2,\ldots\ldots,n$ (3.2)

Subject to

$$\tilde{A}x\begin{bmatrix}\le\\=\\\ge\end{bmatrix}\tilde{b} \tag{3.3}$$

$$x\ge 0$$

where x is an n-dimensional vector of decision variables, and $\tilde{c}_i, \tilde{d}_i$ are $n \times 1$ vectors, $\tilde{A}$ is an $m \times n$ constraint fuzzy matrix, $\tilde{b}$ is an m- dimensional fuzzy vector, $\tilde{\alpha}_i$ and $\tilde{\beta}_i$ are scalars.

4. PROPOSED METHOD

Harmonic Mean: Harmonic mean of a set of observations is defined as the reciprocal of the arithmetic average of the given values.

If $x_1, x_2, ..., x_n$ are the n observations, then $HM = \dfrac{n}{\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)}$

We find the optimum solution of each of the individual objective functions (3.1) and (3.2) subject to the constraints (3.3) using simplex method.

$$\left.\begin{array}{l} Max.Z_1 = f_1 \\ Max.Z_2 = f_2 \\ .... \\ .... \\ Max.Z_k = f_k \\ Min.Z_{k+1} = f_{k+1} \\ .... \\ .... \\ Min.Z_n = f_n \end{array}\right\} \tag{4.1}$$

where $f_1, f_2, ..., f_n$ are the optimum values of the objective functions.

The multi-objective linear fractional programming problem can be transformed to single objective linear fractional programming problem function as below.

$$Maximize\, Z = \sum_{i=1}^{k}(Max.Z_i)/HM1 - \sum_{i=k+1}^{n}(Min.Z_i)/HM2 \tag{4.2}$$

Subject to the same set of constraints (3.3) where $HM1 = k / \sum_{i=1}^{k} (1/f_i)$ and $HM2 = (n-k) / \sum_{i=k+1}^{n} (1/f_i)$ (4.3)

HM1 is the harmonic mean of the optimum values of the maximized objective functions and HM2 is that of the minimized objective functions. The values of HM1 and HM2 may be positive or negative. If they are negative, we consider the absolute values of HM1 and HM2.

Solution Procedure

(i) Convert all the constraints of ≥ type into ≤ type by multiplying the constraints by -1.

(ii) Write the problem in Standard form by introducing slack variables.

(iii) Solve the probem by Simplex method. If infeasible optimum solution occurs, then use the dual simplex method and find the optimum values of the objective functions.

(iv) Find HM1 and HM2 as defined by the equation (4.3)

(v) Convert the MOLFPP into SOLFPP using (4.2)

(vi) Solve the SOLFPP using Simplex Method.

5. NUMERICAL EXAMPLE

Consider the following MOFLFPP

Maximize $Z_1 = \dfrac{(2,4,6,8)x_1 + (5,7,9,11)x_2}{(0,1,3,4)x_1 + (0,1,1,2)x_2 - (1,1,1,1)}$

Maximize $Z_2 = \dfrac{(4,7,9,12)x_1 + (3,4,6,7)x_2}{(2,3,5,6)x_1 + (0,1,3,4)x_2 - (1,2,2,3)}$

Minimize $Z_3 = \dfrac{(0,1,3,4)x_1 - (5,9,11,15)x_2}{(3,5,7,9)x_1 + (1,2,4,5)x_2 - (0,1,5,6)}$

Minimize $Z_4 = \dfrac{-(1,3,5,7)x_1 - (2,5,7,10)x_2}{(-1,1,3,5)x_1 + (1,1,1,1)x_2 - (0,1,1,2)}$

Subject to

$$(0,1,3,4)x_1 + (1,3,7,9)x_2 \geq (6,8,12,14)$$

$$(0,3,5,8)x_1 + (-1,2,4,7)x_2 \leq (12,18,22,28)$$

$$-(1,1,1,1)x_1 + (0,1,1,2)x_2 \leq (0,1,3,4)$$

$$(1,1,1,1)x_1 \leq (-1,1,3,5)$$

$$(-1,1,1,3)x_2 \geq (1,2,6,7)$$

$$x_1, x_2 \geq 0$$

Using the ranking technique of fuzzy numbers (2.1), the above MOFLFPP can be converted to the following MOLFPP

$$\text{Maximize } Z_1 = \frac{5x_1 + 8x_2}{2x_1 + x_2 - 1} \qquad (5.1)$$

$$\text{Maximize } Z_1 = \frac{8x_1 + 5x_2}{4x_1 + 2x_2 - 2} \qquad (5.2)$$

$$\text{Minimize } Z_3 = \frac{2x_1 - 10x_2}{6x_1 + 3x_2 - 3} \qquad (5.3)$$

$$\text{Minimize } Z_4 = \frac{-4x_1 - 6x_2}{2x_1 + x_2 - 1} \qquad (5.4)$$

Subject to

$$2x_1 + 5x_2 \geq 10$$

$$4x_1 + 3x_2 \leq 20 \qquad (5.5)$$

$$-x_1 + x_2 \leq 2$$

$$x_1 \leq 2$$

$$x_2 \geq 4$$

$$x_1, x_2 \geq 0$$

We find the optimum value of each of the individual objective function by simplex method. The initial iteration for the first objective function (5.1) subject to (5.5) is shown in the following table.

			c_j	5	8	0	0	0	0	0
			d_j	2	1	0	0	0	0	0
d_B	c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5	y_6	y_7
0	0	y_3	-10	-2	-5	1	0	0	0	0
0	0	y_4	20	4	3	0	1	0	0	0
0	0	y_5	2	-1	1	0	0	1	0	0
0	0	y_6	2	1	0	0	0	0	1	0
0	0	y_7	-4	0	-1	0	0	0	0	1
$z_1 = c_B x_B + \alpha = 0$		$z_j^{(2)} - c_j = c_B y_j - c_j$		-5	-8	0	0	0	0	0
$z_2 = d_B x_B + \beta = -1$		$z_j^{(2)} - d_j = d_B y_j - d_j$		-2	-1	0	0	0	0	0
		$\Delta_j = z_2\left(z_j^{(1)} - c_j\right) - z_1\left(z_j^{(2)} - d_j\right)$		5	8	0	0	0	0	0

Since all $\Delta_j \geq 0$ and $y_3 = -10$, $y_7 = -4$, the solution is optimum but infeasible. Use Dual Simplex method to obtain optimum feasible solution.

$\min\{x_{Bi}, x_{Bi} < 0\} = \min\{-10,-4\} = -10$; y_3 leaves the basis.

$\max\left\{\frac{\Delta_j}{y_{3j}}, y_{3j} < 0\right\} = \max\left\{\frac{5}{-2}, \frac{8}{-5}\right\} = \frac{8}{-5}$; y_2 enters the basis. Proceeding the iteration process in this way, we get the solutions for (5.1), (5.2), (5.3) (5.4) using LINGO 13.0 package and are given as

$$f_1 = 6, f_2 = \frac{18}{7}, f_3 = \frac{-12}{7}, f_4 = \frac{-32}{7}$$

Using Harmonic Mean technique, we get $HM1 = \frac{18}{5}$ and $HM2 = \frac{192}{77}$

Using the equation (4.2), The MOLFPP can be transformed to

Maximize $Z = \frac{1105x_1 + 1918x_2}{576x_1 + 288x_2 - 288}$

Subject to

$2x_1 + 5x_2 \geq 10$

$4x_1 + 3x_2 \leq 20$

$-x_1 + x_2 \leq 2$

$x_1 \leq 2$

$x_2 \leq 4$

$x_1, x_2 \geq 0$

Solving by Simplex Method using LINGO 13.0, we get

Max $Z = 4.9018, x_1 = 2, x_2 = 4$

CONCLUSION

We proposed a new technique to solve MOFLFPP. First the MOFLFPP is converted into MOLFPP using GMIR method. The optimum values of each of the individual objective functions are found by using simplex method. Then the MOLFPP is transformed to a single objective LFPP using harmonic mean technique and then it is solved by simplex method using LINGO 13.0 version. A numerical example is presented to show the effectiveness of the suggested method.

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