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Double Layered Fuzzy Planar Graph

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Abstract

Fuzzy planar graph is a very important subclass of fuzzy graph. In this paper, we define a new fuzzy graph double layered fuzzy planar graph $(DLFPG)$and weak double layered fuzzy planar graph. And we discuss the basic properties of it.

Keywords: Fuzzy graphs, fuzzy planar graph, double layered fuzzy planar graph, weak double layered fuzzy planar graph, fuzzy faces.

1. INTRODUCTION

Fuzzy set theory was introduced by Zadeh in 1965[10]. Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975[1]. Though introduced recently, it has been growing fast and has numerous applications in various fields. During the same time Yeh and Bang[2] have also introduced various concepts in connectedness with fuzzy graph. Nagoorgani and Malarvizhi[3] have defined different types of fuzzy graphs and discussed its relationships with isomerism in fuzzy graphs. Abdul-jabbar and Naoom[8] introduced the concept of fuzzy planar graph. Also, Nirmala andDhanabal [7] defined special fuzzy planar graph. A.Pal, S.Samanta and M.Pal[4] have defined fuzzy planar graph in a different concept where crossing of edge are allowed. Pathinathan and JesinthaRosline[6] have introduced double layered fuzzy graph. The Intuitionistic double layered fuzzy graph is given by JesinthaRoseline and Pathinathan[9]. In this paper we define Double Layered Fuzzy Planar Graph $(DLFPG)$ and we discuss some properties.

Section two contains the basic definitions in fuzzy graph and fuzzy multigraph, in section three we introduce a new fuzzy graph called double layered fuzzy planar graph, section four presents the theoretical concept of double layered fuzzy planar graph and finally we give conclusion on $DLFPG$.

2. PRELIMINARIES

A graph can be drawn in many different ways. A graph may or may not be drawn on a plane without crossing of edges. A drawing of a geometric representation of graph on any surface such that no edges intersect is called embedding [12]. A graph G is planar if it can be drawn in the plane with its edges only intersecting at vertices of G. So the graph is non – planar if it cannot be drawn without crossing. Several definitions of strong edge are available in literature. Among them the definition of [11] is more suitable for our purpose. The definition is given below: For the fuzzy graph $\in= (V, \sigma, \mu)$ an edge (x, y) is called strong [11] if $\frac{1}{2}\min\{\sigma(a), \sigma(b)\} \leq \mu(a,b)$ and weak otherwise. A multilation is a graph that may contain multiple edges between any two vertices, but it does not contain any self loops.

Definition 2.1 [1] A fuzzy graph $G = (V, \sigma, \mu)$ is a non-empty set V together with a pair of functions $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ such that for all $a, b \in V$, $\mu(a,b) \leq \sigma(a) \wedge \sigma(b)$, where $\sigma(a), \sigma(b)$ and $\mu(a,b)$ represent the membership values of the vertex a and of the edge (a,b) in G respectively.

Definition 2.2 [4] Let V be a non-empty set and $\sigma : V \to [0,1]$ be a mapping. Also let $E = \{((a,b),(a,b)\mu^i), i = 1,2,.....P_{ab} \mid (a,b) \in V \times V\}$ be a fuzzy multiset of $V \times V$ such that $(a,b)\mu^i \leq \{\sigma(a) \wedge \sigma(b)\}$ for all $i = 1,2,.....p_{ab}$, where $p_{ab} = \max\{i \mid (a,b)\mu^i \neq 0\}$. Then $\Psi = (V, \sigma, E)$ is denoted as fuzzy multigraph where $\sigma(a)$ and $(a,b)\mu^i$ represent the membership value of the vertex a and the membership value of the edge (a,b) in Ψ respectively.

Definition 2.3 [4]Let Ψ be a fuzzy multigraph and for a certain geometrical representation $P_1, P_2, ...P_N$ be the point of intersections between the edges Ψ is said to be fuzzy planar graph with fuzzy planarity value f , where

$$f = \frac{1}{1+\{I_{P_1} + I_{P_2} + ... + I_{P_N}\}}$$

It is obvious that f is bounded and the range of f is $0< f \leq 1$.

3. DOUBLE LAYERED FUZZY PLANAR GRAPH(DLFPG)

Definition 3.1. Let $\Psi = (V, \sigma, E)$ be a fuzzy multigraph with the underlying crisp multigraph $\Psi^* = (V, \sigma^*, E^*)$ .The vertex set of $DL(\Psi)$ be $\sigma^* \cup E^*$. The geometrical representation $DLP_1, DLP_2, ..., DLP_N$ be the points of intersections between the edges $DL(\Psi) = (V, \sigma_{DL}, E_{DL})$ is said to be double layered fuzzy planar graph (DLFPG) with double layered fuzzy planarity value f_{DL} , where

$$f_{DL} = \frac{1}{1+\{I_{DLP_1} + I_{DLP_2} + ... + I_{DLP_N}\}}.$$

It is obvious that f_{DL} is bounded and the range of f_{DL} is $0 < f_{DL} \leq 1$.

Algorithm for double layered fuzzy planar graph:

Step1: We consider the fuzzy planar graph.

Step2: We choose the edges of the fuzzy planar graph to consider new vertex of the fuzzy planar graph.We get new vertex set of the double layer fuzzy planar graph $(V + E)$.

Step3: In the given graph, draw edges for the adjacent vertex and also for the adjacent edges.

Step4: Here we will get a new graph which has many intersecting edges. Plot the intersecting point as $DLP_1, DLP_2, ..., DLP_N$.

Step5: Evaluate the value for the intersecting point using the formula $DLI_{(a,b)} = \frac{(a,b)\mu^i}{(\sigma(a) \wedge \sigma(b))}$ and $I_{DLP_k} = \frac{DLI_{(a,b)} + DLI_{(c,d)}}{2}$ where $k = 1,2,......, N$

Step6: If the double layered planarity value is $0 < f_{DL} \leq 1$ then the graph is double layered fuzzy planar graph.

Remark: 3.1.1. We only consider minimal intersecting points of the double layered fuzzy planar graph.

Example:3.1.2. Consider the fuzzy planar graph Ψ, whose crisp graph Ψ^* is a cycle with n=4 vertices.

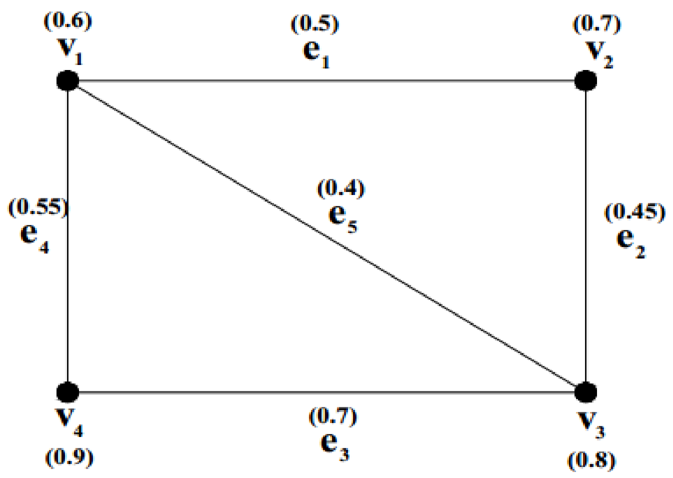

Figure1: A Fuzzy Planar graph $\Psi = (V, \sigma, E)$

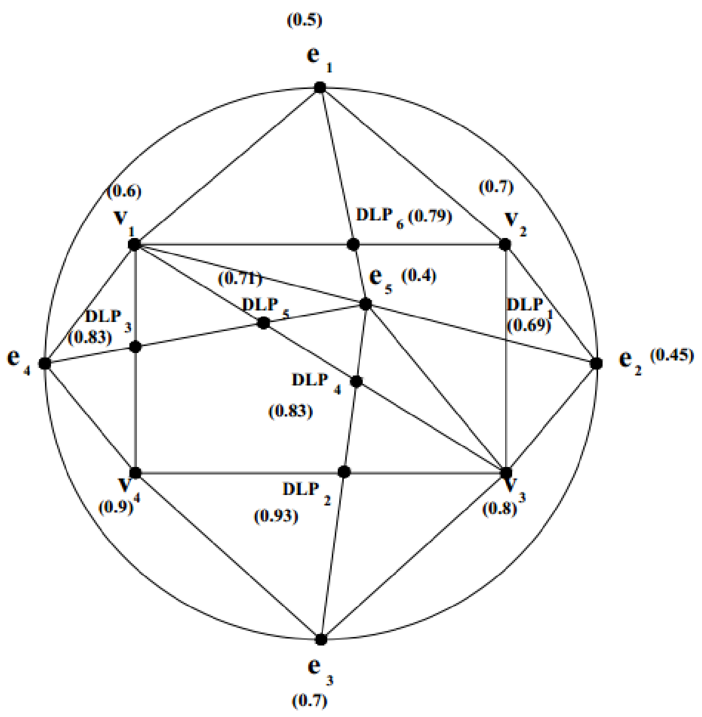

Figure2: Double layered fuzzy planar graph $DL(\Psi) = (V, \sigma_{DL}, E_{DL})$

Here we calculate the intersecting value at the intersecting point between two edges.Two edges (b,c) and (e_2,e_5) are intersected where $\sigma(a)$=(0.7), $\sigma(b)$=(0.8), $E(e_2)$=(0.45), $E(e_5)$=(0.4), $E(b,c)$=0.5, $E(e_2)\wedge E(e_5)$=0.3(see figure 2). Strength of the edge (b,c) is $\frac{0.45}{0.7}$=0.57 i.e.) $DLI_{(b,c)}$=0.64 and that of (e_2,e_5) is $\frac{0.3}{0.4}$ = 0.75i.e.) $DLI_{(e_2,e_5)}$=0.75. Thus the intersecting value at the point is

$\frac{0.65+0.75}{2}$=0.695.$\therefore$ $DLP_1 = 0.695$. Similarly, we can find

$DLP_2 = 0.93$
$DLP_3 = 0.83$
$DLP_4 = 0.83$
$DLP_5 = 0.17$
$DLP_6 = 0.79$

Consider the fuzzy planar graph with n=5 vertices.

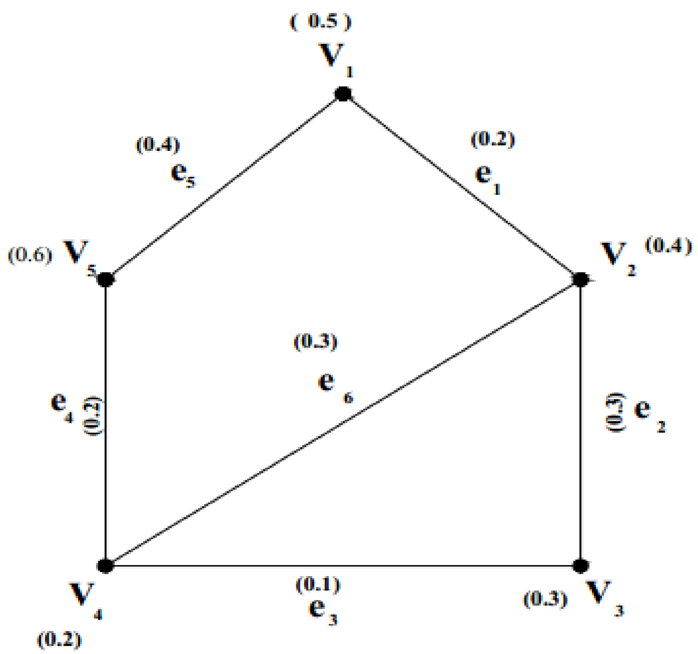

Figure3: A Fuzzy Planar graph $\Psi = (V, \sigma, E)$

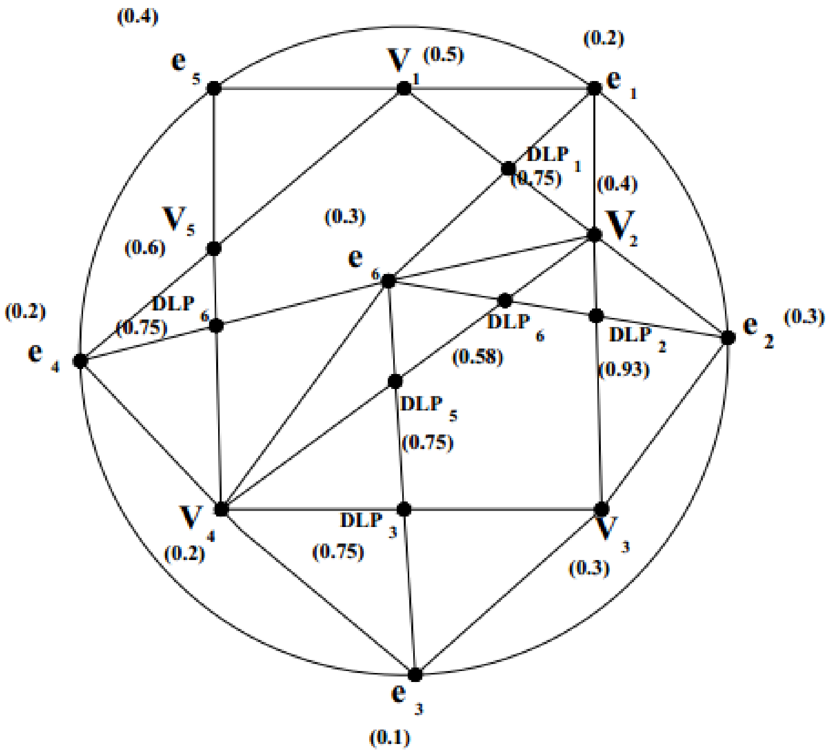

Figure4: Double layered fuzzy planar graph $DL(\Psi) = (V, \sigma_{DL}, E_{DL})$

Consider the fuzzy planar graph with n=6 vertices.

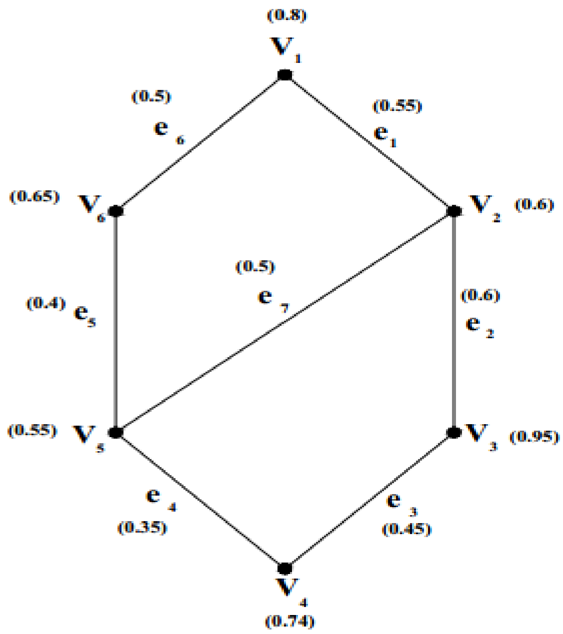

Figure5: A Fuzzy Planar graph $\Psi = (V, \sigma, E)$

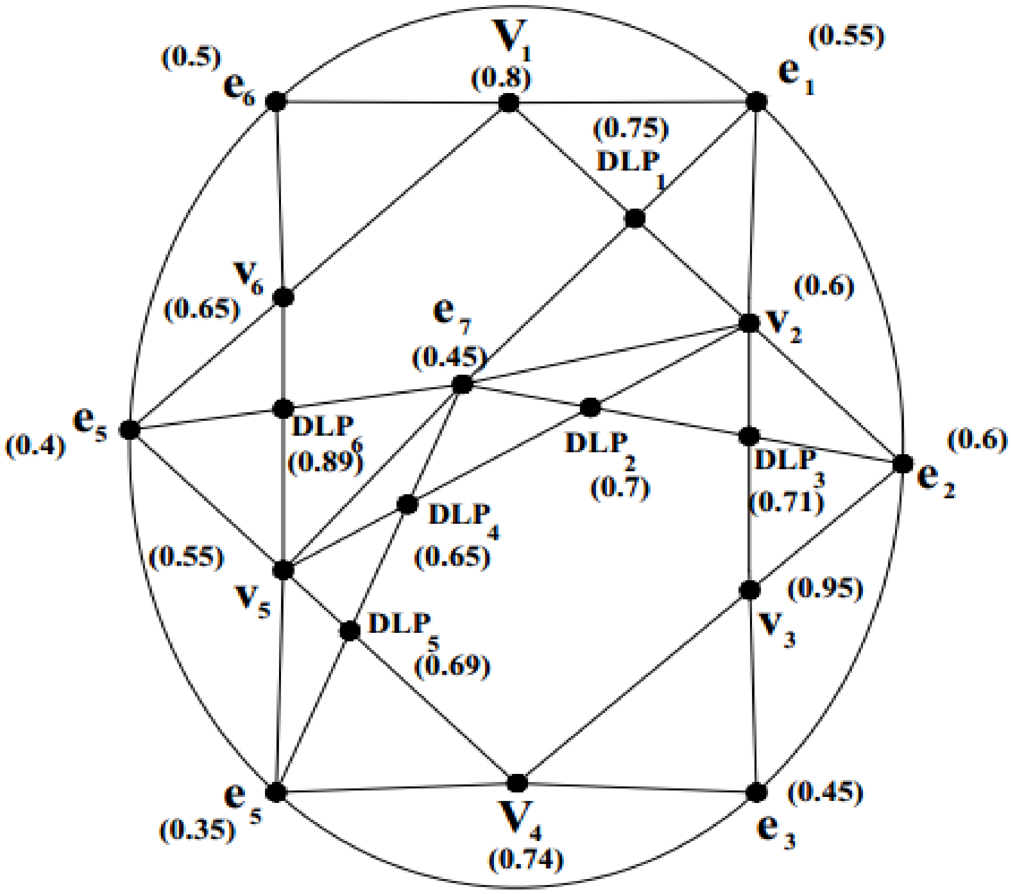

Figure6: Double layered fuzzy planar graph $DL(\Psi) = (V, \sigma_{DL}, E_{DL})$

Definition:3.2.A double layered fuzzy planar graph $DL(\Psi)$ is called weak double layered fuzzy planar graph if the double layered fuzzy planarity value of the graph is less than or equal to 0.5.

Definition: 3.3. Let $DL(\Psi) = (V, \sigma_{DL}, E_{DL})$ be a double layered fuzzy planar graph and $E_{DL} = \{((a,b),(a,b)\mu^i, i = 1,2,....., P_{ab} \mid (ab) \in V \times V\}$ $P_{ab} = \max\{i \mid (a,b)\mu^i \neq 0\}$. A fuzzy face of $DL(\Psi)$ is a region bounded by the set of fuzzy edges $E'_{DL} \subset E_{DL}$ of a geometric representation of $DL(\Psi)$. The membership value of the fuzzy face is

$$\min\left\{\frac{(a,b)\mu^i}{\{\sigma(a) \wedge \sigma(b)\}}, i = 1,2,....., P_{ab} \mid (a,b) \in E'_{DL}\right\}$$

A fuzzy face is called weak fuzzy face if its membership value is < 0.5 and otherwise strong face. Every double layered fuzzy planar graph has an infinite region which is called outer fuzzy face. Other faces are called inner fuzzy faces.

4. THEORETICAL CONCEPTS

Theorem: 4.1. Let $DL(\Psi)$ be a weak double layered fuzzy planar graph. The number of point of intersections between strong edges in $DL(\Psi)$ is $DLP_1, DLP_2, ..., DLP_N$.

Proof: Let $DL(\Psi) = (V, \sigma_{DL}, E_{DL})$ be a weak double layered fuzzy planar graph.

Let, if possible, $DL(\Psi)$ has one point of intersections between two strong edges in $DL(\Psi)$.

For any strong edge $((V_1,V_2),(V_1,V_2)\mu^i)$, $(V_1,V_2)\mu^i \geq \frac{1}{2}\{\sigma(V_1) \wedge \sigma(V_2)\}$. So $DLI_{(V_1,V_2)} \geq 0.5$

Thus for two intersecting strong edges $((V_1,V_2),(V_1,V_2)\mu^i)$, and $((V_3,V_4),(V_3,V_4)\mu^j)$,

$\frac{DLI_{(V_1,V_2)} + DLI_{(V_3,V_4)}}{2} \geq 0.5$. That is $I_{DLP_1} \geq 0.5$, Then $1+0.5 \geq 1.5$.

Therefore $f_{DL} = \frac{1}{1+I_{DLP_1}} > 0.5$.

It contradicts the fact that the fuzzy graph is a weak double layered fuzzy planar graph.

So number of point of intersections between strong edges cannot be one. It is clear that if the number of point of intersection of strong edges in one then the double layered fuzzy planarity value $f_{DL} \geq 0.5$. Similarly, if the number of point of intersections of strong edges increases, the double layered fuzzy planarity value decreases.

Any $DLFPG$ without any crossing between edges is a strong $DLFPG$. But $DLFPG$ contains large number of intersecting points. So, the $DLFPG$ is a weak double layered fuzzy planar graph.

Thus we conclude that the number of point of intersections between strong edges in $DL(\Psi)$ is $DLP_1, DLP_2, ..., DLP_N$.

Theorem: 4.2. Let $DL(\Psi)$ be a double layered fuzzy planar graph. Then $DL(\Psi)$ is not strong double layered fuzzy planar graph.

Proof: Let $DL(\Psi) = (V, \sigma_{DL}, E_{DL})$ be a weak double layered fuzzy planar graph.

Let, if possible, $DL(\Psi)$ has at least two intersecting points DLP_1 and DLP_2.

For any strong edge $((V_1,V_2),(V_1,V_2)\mu^i)$, $(V_1,V_2)\mu^i \geq \frac{1}{2}\{\sigma(V_1)\wedge\sigma(V_2)\}$. So $DLI_{(V_1,V_2)}$ ≥0.5

Thus for two intersecting strong edges $((V_1,V_2),(V_1,V_2)\mu^i)$, and $((V_3,V_4),(V_3,V_4)\mu^j)$

$\frac{DLI_{(V_1,V_2)} + DLI_{(V_3,V_4)}}{2}$ ≥0.5. That is I_{DLP_1} ≥0.5, similarly I_{DLP_2} ≥0.5

Then $1 + I_{DLP_1} + I_{DLP_2} \geq 2$.

Therefore $f_{DL} = \frac{1}{1 + I_{DLP_1} + I_{DLP_2}}$ ≤ 0.5. It is clearly that double layered fuzzy planar is not strong double layered fuzzy planar graph.

Theorem:4.3. Let $DL(\Psi)$ be a double layered fuzzy planar graph with double layered planarity value 0< f_{DL} ≤1. Then $DL(\Psi)$ has strong and weak fuzzy faces.

Proof: Let $DL(\Psi)$ be a double layered fuzzy planar graph. Let F_1 and F_2 be two fuzzy faces,

F_1 is bounded by the edges, $((V_1,V_2),(V_1,V_2)\mu^i)$, $((V_3,V_4),(V_3,V_4)\mu^j)$ and $((V_4,V_5),(V_4,V_5)\mu^k)$ Then,

$Min\left\{\frac{(V_1,V_2)\mu^i}{\sigma(V_1)\wedge\sigma(V_2)}, \frac{(V_3,V_4)\mu^j}{\sigma(V_3)\wedge\sigma(V_4)}, \frac{(V_5,V_6)\mu^k}{\sigma(V_5)\wedge\sigma(V_6)}\right\}$ >0.5 i.e.) its membership value is 0.5.

Similarly, F_2 is fuzzy bounded face with membership value 0.33.

Every strong fuzzy face has membership value is greater than 0.5. So, every edge of a strong fuzzy face is a strong fuzzy edge, and every weak fuzzy face has membership value is less than 0.5. So, $DL(\Psi)$ has strong and weak fuzzy faces.

Example:4.3.1. In figure 7, $F_1, F_2, ..., F_{21}$ are twenty one fuzzy faces.

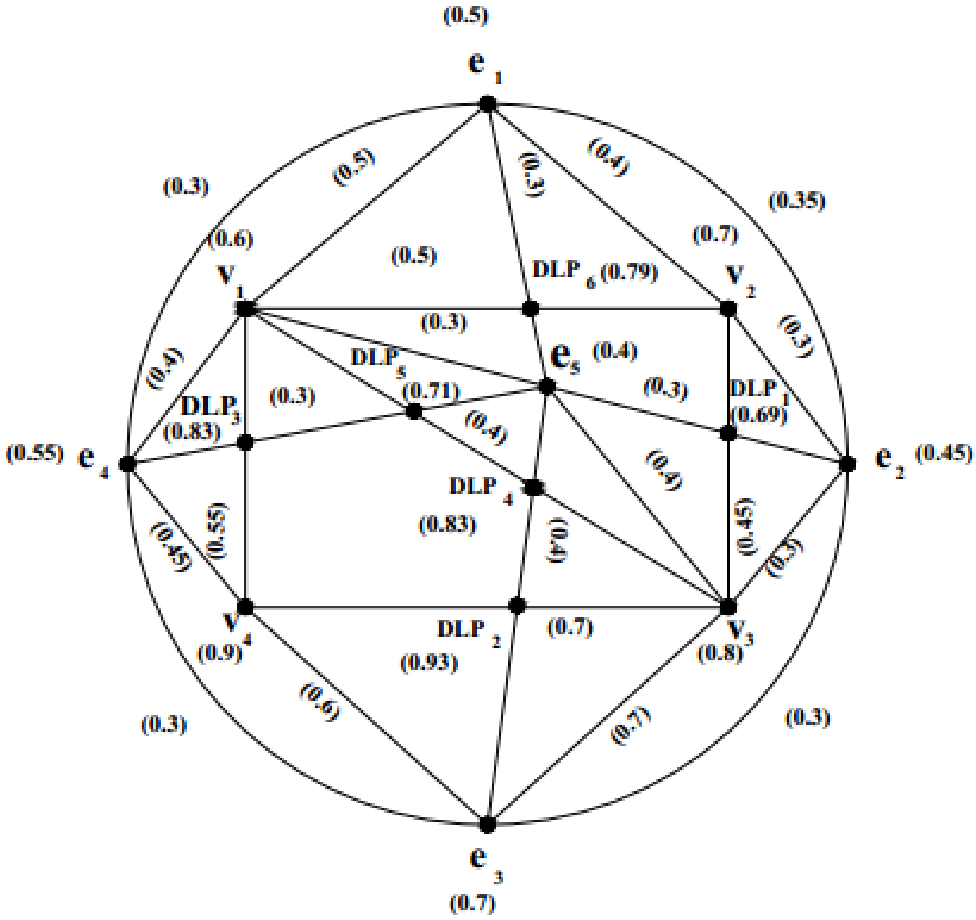

Figure7: Double layered fuzzy planar graph $DL(\Psi) = (V, \sigma_{DL}, E_{DL})$

F_1 is bounded by the edges $((e_1, b), 0.4), ((b, e_2), 0.3), ((e_1, e_2), 0.35)$ the membership value of the fuzzy face is,

$\min\left\{ \dfrac{(a,b)\mu^i}{\sigma(a) \wedge \sigma(b)}, i = 1,2......, p_{ab} \mid (a,b) \in E' \right\}$. i.e.) $\min(0.8, 0.67, 0.78) = 0.67$, its membership value is 0.67.

So, F_1 is a strong fuzzy face. Similarly, we can find $F_2, F_3, ..., F_{21}$.Here F_{20} is a fuzzy bounded face and F_{21} is the outer fuzzy face with membership value 0.33 obviously, F_{20} and F_{21} are weak fuzzy faces.

CONCLUSION

This study describes the double layered fuzzy planar graph and its properties. We have defined a new term called double layered fuzzy planarity value of a fuzzy graph. If the double layered fuzzy planarity value of a fuzzy graph is one then no edge crosses other edge. This is a very important concept of fuzzy graph theory. Double layered fuzzy planarity value is less than 0.5 because this graph contain large number of intersecting points. So double layered fuzzy planar graph is defined as weak double layered fuzzy planar graph.

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