Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 10 (2017), pp. 7377-7385

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# Super Lehmer-3 Mean Number of Graphs

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**Abstract**

Let G be a graph and f:V(G)→{1,2,….n} be a function such that the labels of the edge uv is defined by f(e)= $\left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil$ (or) $\left\lfloor \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rfloor$, and {(f(V(G)∪(f(e)/eЄE(G)}⊆{1,2,…..n}. If n is the smallest positive integer which satisfies these conditions along with the condition that both the vertex and edge labels are distinct and there is no common vertex and edge labels, then 'n' is called the Super Lehmer-3 mean number of graph G and can be denoted as $S_{l\text{-}3m}(G)$

**Keywords:** Lehmer-3 mean graph, Super Lehmer-3 mean graph , Super Lehmer-3 mean number of a graph, Path, Cycle, Comb, Kite, Crown etc..

## 1. INTRODUCTION

A graph considered here are finite, undirected and simple. The vertex set and edge set of a graph are denoted by V(G) and E(G) respectively. For standerd terminology and notations we follow Harrary[1]. S Somasundaram & S.S Sandhya introduced the concept of Harmonic Mean Labeling of Graphs in [2] and its basic results was proved in [3] and [4].We will provide a brief summary of other informations which are necessary for our present investigation.

### Definition 1.1

A graph G=(V,E) with P vertices and q edges is called **Lehmer -3 mean graph**,if it is possible to label vertices x ЄV with distinct labels f(x) from 1,2,3,…………q+1 in such a way that when each edge e=uv is labeled with f(e=uv)=$\left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil$ (or) $\left\lfloor \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rfloor$ , then the edge labels are distinct. In this case "f " is called Lehmer -3 mean labeling of G.

**Definition 1.2**

Let f:V(G)→{1,2,…..p+q} be an injective function .For a vertex labeling "f " the induced edge labeling f(e=uv) is defined by f(e)= $\left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil$ (or) $\left\lfloor \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rfloor$, then f is called Super Lehmer -3 mean labeling ,if {f (V(G))} ∪ {f(e)/e Є E(G)} ={1,2,3,…..p+q}, A graph which admits Super Lehmer -3 Mean labeling is called **Super Lehmer -3 Mean graph**

**Definition 1.3**

Let G be a graph and f:V(G)→{1,2,….n} be a function such thet the labels of the edge uv is defined by f(e)= $\left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil$ (or) $\left\lfloor \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rfloor$, and {(f(V(G)∪(f(e)/eЄE(G)}⊆{1,2,…..n}. If n is the smallest positive integer which satisfies these conditions along with the condition that both the vertex and edge labels are distinct and there is no common vertex and edge labels, then 'n' is called the **Super Lehmer-3 mean number** of graph G and can be denoted as $S_{l\text{-}3m}(G)$

**2. MAIN RESULTS**

**Theorem 2.1**

$S_{l\text{-}3m}(P_n)$=2n

**Proof:-**

Let $u_1,u_2,\ldots u_n$ be the vertices of a path $P_n$. We define a function f:V($P_n$)→{1,2,….n} by

f($u_i$)=2i , 2≤i≤n

f($u_n$)=2n

Thus the edge labels are all distinct and f(V(G)∩f(E(G))=ϕ.

Hence $S_{l\text{-}3m}(P_n)$=2n

**Example 2.2**

Super Lehmer-3 mean number of $P_6$ is $S_{l\text{-}3m}(P_6)=2n=2x6=12$ is given below.

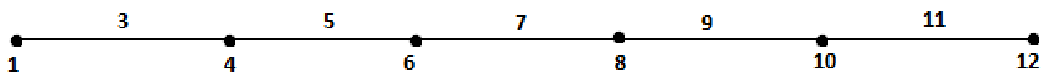


Figure-1

**Theorem 2.3**

$S_{l\text{-}3m}(P_n \odot K_1)=4n$

**Proof :-**

Let $P_n \odot K_1$ be a graph obtained from a path $u_1,u_2,\ldots u_n$ of n vertices by joining a pendant vertex $v_i$ to $u_i$ where $1 \leq i \leq n$. Define a function $f:V(P_n \odot K_1) \rightarrow \{1,2,\ldots .n\}$by

$f(u_1)=1$

$f(u_i)=4i-2 \quad ; 2 \leq i \leq n$

$f(v_i)=4i \quad ;1 \leq i \leq n$

Thus the vertex labels and the induced edge labels are distinct and further $f(V(G) \cap f(E(G))=\phi$. Hence $S_{l\text{-}3m}(G)=4n$.

**Example 2.4**

$S_{l\text{-}3m}(P_6 \odot K_1)=4x6=24$ labeling pattern is given below.

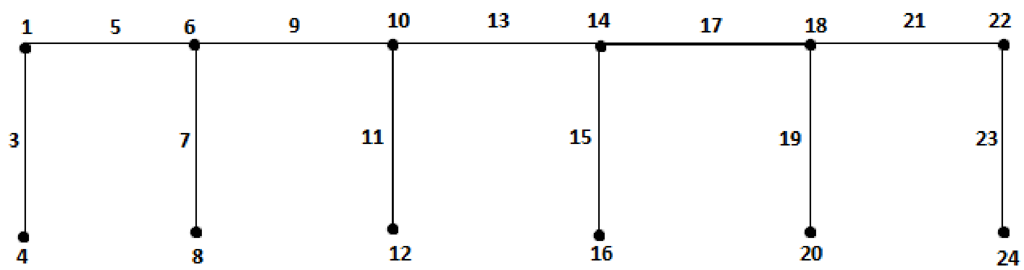


Figure-2

**Theorem 2.5**

$S_{l\text{-}3m}(L_n)=2n+(3n-1)$

**Proof :-**

Let $L_n$ be a $P_n x P_2$ graph . The vertices of $L_n$ be $u_1,u_2,\ldots .u_n$ and $v_1,v_2,\ldots .v_n$.Here $v_i$

starts from the opposite direction of $u_i$. Define a function $f:V(L_n)\rightarrow\{1,2,\ldots.n\}$ by

$f(u_1)=1$

$f(u_i)=2i,\ 2\leq i\leq n$

$f(v_i)=2n+(3i-1)\ ;\ 1\leq i\leq n-1$

$f(v_n)=2n+(3n-1)$

Hence the vertex and edge labels are distinct and $f(V(G)\cap f(E(G))=\phi$.

Hence $S_{l\text{-}3m}(L_n)=2n+(3n-1)$.

**Example 2.6**

Super Lehmer 3 mean number of $L_5$ is $S_{l\text{-}3m}(L_n)=2x5+(3x5-1)=10+(15-1)=10+14=24$ is given below.

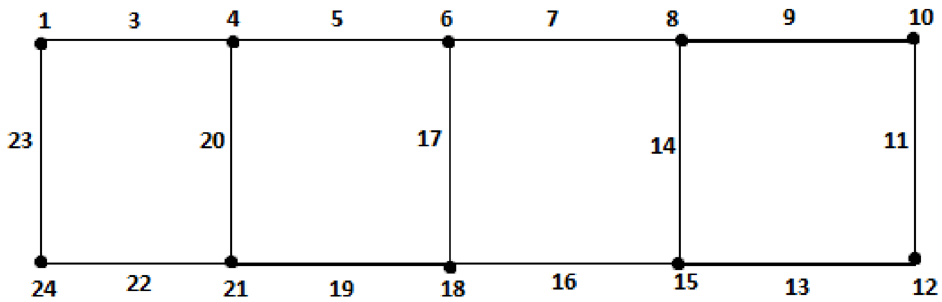


Figure-3

**Theorem 2.7**

Let G be a graph obtained by attaching $C_3$ at the end vertex of a path $P_n$.

$S_{l\text{-}3m}(G)=2n+5$

**Proof :-**

Let G be a graph such that the vertices of the path $P_n$ be $u_1,u_2,\ldots.u_n$ and thet of the cycle $C_3$ be $u_nvw$. Define a function $f:V(G)\rightarrow\{1,2,\ldots.n\}$ by

$f(u_1)=1$

$f(u_i)=2i\quad ;\ 2\leq i\leq n$

$f(v)=2n+2$

$f(w)=2n+5$

Clearly the vertex labels and edge labels are distinct and no vertex and edge labels are equal. Hence $S_{l\text{-}3m}(G)=2n+5$.

**Example 2.8**

Super Lehmer-3 mean number of G when n=4 is $S_{l\text{-}3m}(G)$=2n+5=2x4+5=8+5=13 is

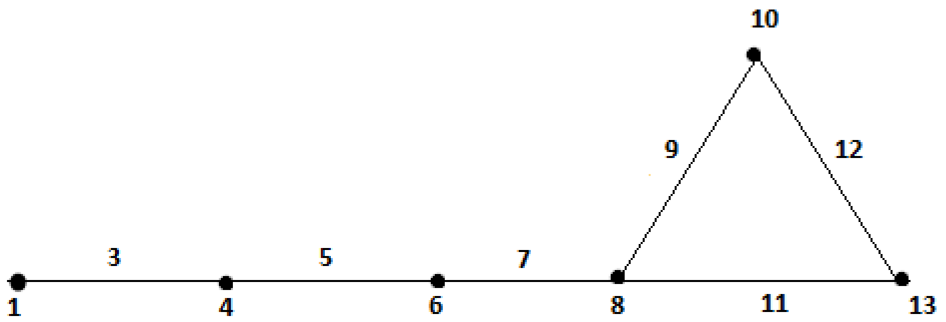


Figure-4

**Theorem 2.9**

$S_{l\text{-}3m}(C_n \odot K_1)=4n+1$ for $n \geq 4$.

**Proof :-**

Let $C_n \odot k_1$ be a graph of vertices $u_1, u_2, \ldots u_n$ and $v_1, v_2, \ldots . v_n$ and we call it as crown. Define a function $f:V(C_n \odot K_1) \rightarrow \{1,2,\ldots.n\}$by

$f(u_1)=1$

$f(u_i)=4i-2 \quad ; 2 \leq i \leq n-1$

$f(u_n)=4n-1$

$f(v_i)=4i \quad ;1 \leq i \leq n-1$

$f(v_n)=4n+1$

Thus $f(V(G) \cap f(E(G))=\phi$. Therefore no vertices and edge are in common.

Hence $S_{l\text{-}3m}(C_n \odot K_1)=4n+1$.

**Example 2.10**

Super Lehmer-3 mean number of $C_6 \odot k_1$ is $S_{l\text{-}3m}(C_6 \odot K_1)$=4x6+1=24+1=25 labeling pattern is given below.

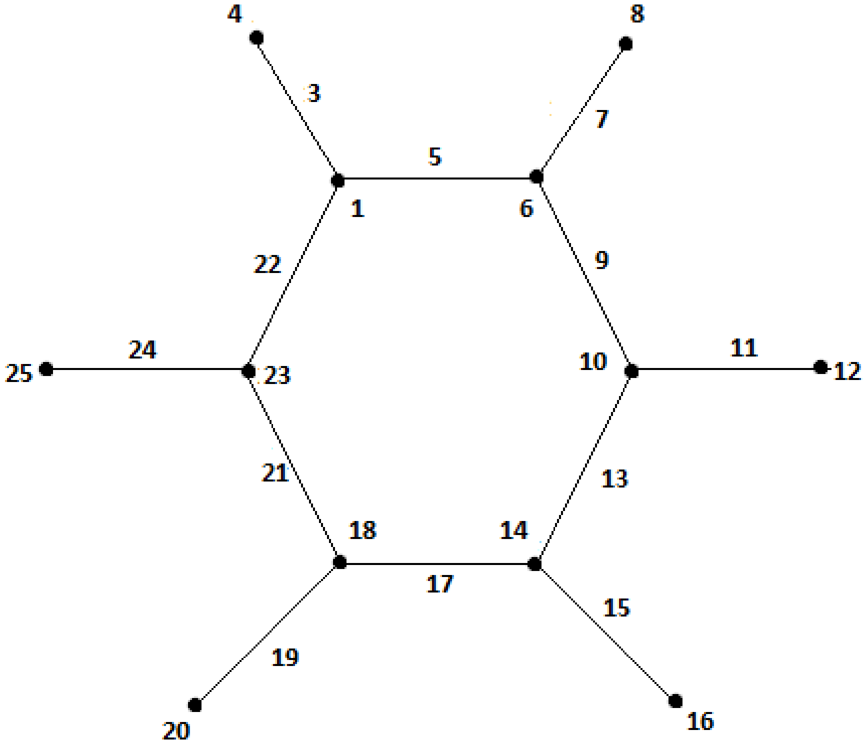


Figure -5

**Theorem 2.11**

$S_{l-3m}(K_{1,n})$=2n+2

**Proof:-**

Let u,$v_1$,$v_2$,….$v_n$ be the vertices of the graph $K_{1,n}$ . Let us define a function f:V($K_{1,n}$)→{1,2,….n}by

f($u_1$)=1

f($v_i$)=2i+2 ; 1≤i≤n-1

f($v_n$ )=2n+2

Thus there is no vertex labels and edge labels in common.

Hence $S_{l-3m}(K_{1,n})$=2n+2.

**Example 2.12**

Super Lehmer-3 mean number labeling pattern of $K_{1,9}$ is $S_{l\text{-}3m}(K_{1,9})$=2x9+2=18+2=20 is

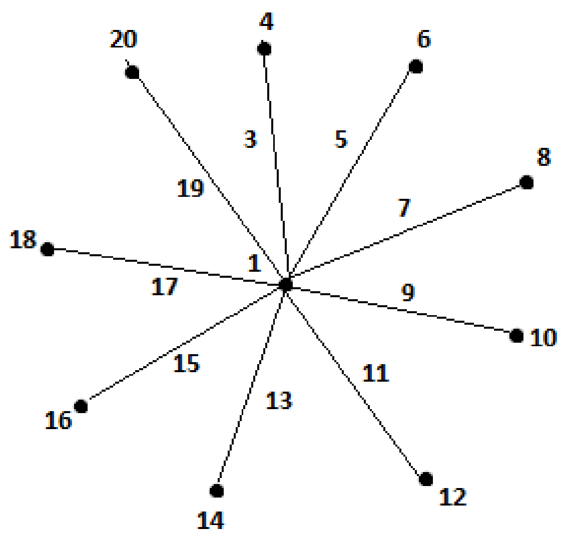


Figure -6

**Theorem 2.13**

Let G be a graph obtained by identifying $K_{1,n}$ at one end vertex of a path $S_{l\text{-}3m}(G)$=2m+2n.

**Proof:-**

Let G be a graph of vertices $u_1,u_2,\dots.u_m$, $v_1,v_2,\dots v_n$. We define a function $f:V(G)\rightarrow\{1,2,\dots.n\}$by $f(u_1)=1$

$f(u_i)=2i$ ; $2\leq i\leq m+1$

$f(v_j)=2m+2j$ ; $2\leq j\leq n-1$

$f(v_n)=2m+2n$

Thus $f(E(G))\cap f(V(G))=\phi$. Here no vertices and edges are in common .

Hence $S_{l\text{-}3m}(G)=2m+2n$.

**Example 2.14**

A graph obtained by identifying $K_{1,6}$ to a path $P_4$ is $S_{l\text{-}3m}(G)$=2m+2n=2x6+2x4= 12+8 =20 is given below.

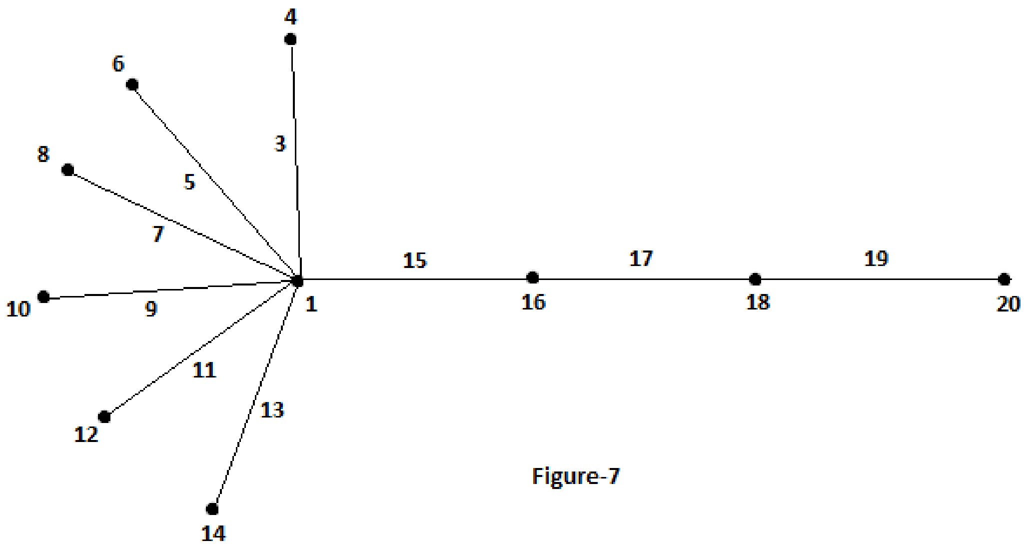


Figure-7

**Theorem 2.15**

$S_{l\text{-}3m}(C_n)=2n+1$ ; $n \geq 6$.

**Proof:-**

Let $C_n$ be a cycle of n vertices $u_1,u_2,\ldots u_n$. We define a function $f:V(C_n)\rightarrow\{1,2,\ldots.n\}$by

$f(u_1)=1$

$f(u_i)=2i$ ; $2\leq i\leq n-1$

$f(u_n)=2n+1$

It can be verified that the vertex and edge labels are distinct and $f(V(G)\cap f(E(G))=\phi$.

Thus $S_{l\text{-}3m}(C_n)=2n+1$.

**Example 2.16**

Super Lehmer-3 mean number of $C_7$ is $S_{l\text{-}3m}(C_7)$=2n+1=2x7+1=14+1=15

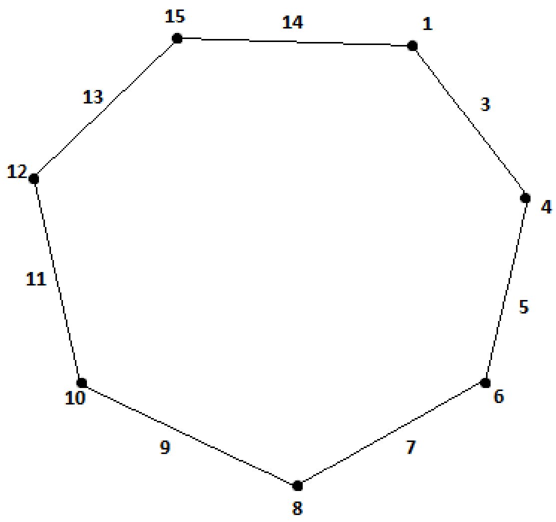


Figure-8

**Remark 2.17**

The graph which are Super Lehmer-3 mean satisfies Super Lehmer-3 mean number.

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