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Three Dimensional Thermal Stress Analysis of Rectangular Plate Using Integral Transform Method

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Abstract

This paper is concerned with inverse thermoelastic problem in which we need to determine the temperature distribution, displacement function and thermal stresses of a thin rectangular plate when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem.

Keywords: Three dimensional rectangular plate, inverse problem, Integral transform, three dimensional problem

1 INTRODUCTION

Adams and **Bert** [1] studied thermoelastic vibrations of a laminated rectangular plate subjected to a thermal shock. **Ghume and Khobragade** [5] discussed deflection of a thick rectangular plate. **Grysa** and **Kozlowski** [7, 8] studied one dimensional problems of temperature and heat flux determination at the surfaces of a thermoelastic slab. **Ishihara, Noda** and **Tanigawa** [10] discussed Theoretical analysis of thermoelastic plastic deformation of a circular plate due to partially distributed heat supply. **Jadhav and Khobragade** [11] studied an inverse thermoelastic problem of a thin finite rectangular plate due to internal heat source. **Khobragade; Payal Hiranwar; Roy and Lalsingh Khalsa** [12] discussed thermal deflection of a thick clamped rectangular plate. **Khobragade and Wankhede** [13] studied an inverse unsteady-state thermoelastic problem of a thin rectangular plate. **Lamba and Khobragade** [14] discussed thermoelastic problem of a thin rectangular plate due to partially distributed heat supply. **Noda; Ashida** and **Tsuji** [16] studied an inverse transient thermoelastic problem for a transversely isotropic body. **Noda; Tanigawa; Kawamura** and **Ishihara** [18] discussed Theoretical analysis of thermoelastic-plastic deformation of a circular plate due to a partially distributed heat supply.

Recently **Patil and Khobragade** [21] studied direct thermoelastic problem of heat conduction with internal heat generation and partially distributed heat supply in rectangular plate. **Roy ; Bagade and Khobragade**: [22] discussed thermal stresses of a semi infinite rectangular beam. **Roy and Khobragade** [23] studied transient thermoelastic problem of an infinite rectangular slab. **Roychoudhary** [24] discussed thermoelastic vibrations of a simply supported rectangular plate produced by temperature prescribed on the faces. **Sabherwal** [25] studied an inverse problem of transient heat conduction. **Sharma; Sharma** and **Sharma** [26] discussed behavior of thermoelastic thick plate under lateral loads. **Sugano; Kimura; Sato** and **Sumi** [29] studied three-dimensional analysis of transient thermal stresses in a non homogenous plate. **Sutar and Khobragade** [30] discussed an inverse thermoelastic problem of heat conduction with internal heat generation for the rectangular plate. **Tanigawa** and **Komatsubara [31]** studied thermal stress analysis of a rectangular plate and its thermal stress intensity factor for compressive stress field. **Tanigawa; Matsumoto** and **Akai** [32] discussed optimization of material composition to minimize thermal stresses in non homogeneous plate subjected to unsteady heat supply. **Wankhede** [33] studied the quasi- static thermal stresses in a circular plate.

In this paper, an attempt has been made to solve two problems of thermoelasticity. In the first problem, an attempt is made to determine the temperature distribution, unknown temperature gradient, thermal stresses and deflection of the plate occupying the space *D:* $\{(x, y, z) \in R^3 : -a \le x \le a, 0 \le y \le b, 0 \le z \le h\}$ with the known boundary conditions.

In the second problem, an attempt is made to determine the temperature distribution, unknown temperature gradient, thermal stresses and deflection of the plate occupying the space *D:* $\{(x, y, z) \in R^3 : 0 \le x \le a, 0 \le y \le b, 0 \le z \le h\}$ with the known boundary conditions.

Finite Marchi-Fasulo transform, Finite Fourier cosine transform and Fourier sine transform techniques are used to find the solution of the problem. Numerical estimate for the expressions have been obtained and depicted graphically.

2 STATEMENT OF THE PROBLEM

Consider a thin isotropic rectangular plate occupying the space *D.* The temperature of the plate at time t satisfying the differential equation as **Tanigawa et al. [31]** is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{g(x, y, z)}{k} = 0 \tag{2.1}$$

where k is the thermal diffusivity of the material of the plate,

subject to the boundary conditions**:**

$$\left[T(x,y,z)+k_1\frac{\partial T(x,y,z)}{\partial x}\right]_{x=-a}=f_1(y,z) \tag{2.2}$$

$$\left[T(x,y,z)+k_2\frac{\partial T(x,y,z)}{\partial x}\right]_{x=a}=f_2(y,z) \tag{2.3}$$

$$\left[\frac{\partial T(x,y,z)}{\partial y}\right]_{y=0}=g_1(x,z) \tag{2.4}$$

$$\left[\frac{\partial T(x,y,z)}{\partial y}\right]_{y=b}=g_2(x,z) \tag{2.5}$$

$$\left[T(x,y,z)\right]_{z=0}=\left(\frac{-Q_0}{\lambda}f_3(x,y)\right) \tag{2.6}$$

$$\left[T(x,y,z)\right]_{z=\xi}=f_4(x,y) \tag{2.7}$$

$$\left[T(x,y,z)\right]_{z=h}=H(x,y)\ \text{(unknown)} \tag{2.8}$$

The displacement components u_x and u_y u_z in the x and y and z directions respectively as **Tanigawa et al. [31]** are

$$u_x=\int_{-a}^{a}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}-\nu\frac{\partial^2 U}{\partial x^2}\right)+\lambda T\right]dx \tag{2.9}$$

$$u_y=\int_{0}^{b}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial z^2}+\frac{\partial^2 U}{\partial x^2}-\nu\frac{\partial^2 U}{\partial y^2}\right)+\lambda T\right]dy \tag{2.10}$$

$$u_z=\int_{0}^{h}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}-\nu\frac{\partial^2 U}{\partial z^2}\right)+\lambda T\right]dz \tag{2.11}$$

where E, ν and λ are the young's modulus, Poisson's ratio and the linear coefficient of the thermal expansion of the material of the beam respectively and U (x,y,z) is the Airy's stress function which satisfy the differential equation as **Tanigawa et al. [31]** is

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)^2 U(x,y,z)=-\lambda E\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\times T(x,y,z) \tag{2.12}$$

The stress components in terms of U(x, y, z) **Noda et al**. **[145]** are given by

$$\sigma_{xx} = \left[\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}\right] \tag{2.13}$$

$$\sigma_{yy} = \left[\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2}\right] \tag{2.14}$$

$$\sigma_{zz} = \left[\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right] \tag{2.15}$$

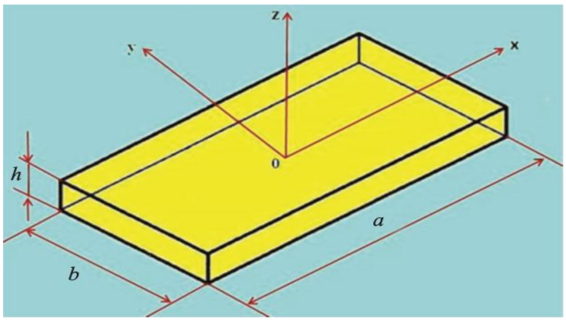

Figure 1: Geometry of the problem

Equations (2.1) to (2.15) constitute the mathematical formulation of the problem under consideration.

3 SOLUTION OF THE PROBLEM

Applying Marchi-Fasulo, transform and finite Fourier cosine Transform and Fourier sine Transform to the equations (2.1) one obtains.

$$\frac{\partial^2 \overline{T}^*}{\partial z^2} - p^2 \overline{T}^* = \Pi \tag{3.1}$$

where, $p^2 = \lambda_n^2 + \frac{m^2\pi^2}{b^2}$ (3.2)

and $\Pi = \frac{P_n(-a)}{k_1} f_2^* - \frac{P_n(a)}{k_2} f_2^* - (-1)^m \overline{g}_2 + \overline{g}_1 - \frac{\overline{g}^*}{k}$ (3.3)

solution of equation (3.1) is second order differential equation whose solution is given by

$$\overline{T}^* = Ae^{pz} + Be^{-pz} + F(z) \tag{3.4}$$

where $F(z)$ is the P.I.

where $$A = \frac{\frac{Q_0}{\lambda}\bar{f}_3^{*} e^{-p\xi} + \bar{f}_4^{*} + F(0)e^{-p\xi} - F(\xi)}{2\sinh(p\xi)} \tag{3.5}$$

$$B = \frac{-\frac{Q_0}{\lambda}\bar{f}_3^{*} e^{p\xi} - \bar{f}_4^{*} - F(0)e^{p\xi} + F(\xi)}{2\sinh(p\xi)} \tag{3.6}$$

$$\bar{T}^{*} = \frac{\left(\frac{Q_0}{\lambda}\bar{f}_3^{*} + F(0)\right)\sinh\left(p(z-\xi)\right) + \left(\bar{f}_4^{*} - F(\xi)\right)\sinh(pz)}{2\sinh(p\xi)} + F(z) \tag{3.7}$$

Applying inverse Fourier cosine transform and inverse Marchi-Fasulo transform on equation (3.7) we get

$$T = \frac{2}{\pi}\sum_{m,h=1}^{\infty}\frac{P_n(x)}{\lambda_n}\cos\left(\frac{m\pi y}{b}\right)$$
$$\left\{\frac{\left(\frac{Q_0}{\lambda}\bar{f}_3^{*} + F(0)\right)\sinh\left(p(z-\xi)\right) + \left(\bar{f}_4^{*} - F(\xi)\right)\sinh(pz)}{\sinh(p\xi)} + F(z)\right\} \tag{3.8}$$

$$H(x,y) = \frac{2}{\pi}\sum_{m,h=1}^{\infty}\frac{P_n(x)}{\lambda_n}\cos\left(\frac{m\pi y}{b}\right)\left\{\frac{\left(\frac{Q_0}{\lambda}\bar{f}_3^{*} + F(0)\right)\sinh\left(p(h-\xi)\right) + \left(\bar{f}_4^{*} - F(\xi)\right)\sinh(ph)}{\sinh(p\xi)} + F(h)\right\} \tag{3.9}$$

Substituting the value of temperature distribution T from equation (3.8) in equation (2.12) one obtains the expression for displacement function U as

$$U = \frac{-2\lambda E}{b}\sum_{m,n=1}^{\infty}\frac{P_n(x)}{\lambda_n}\cos\left(\frac{m\pi y}{b}\right)$$
$$\left\{\frac{\left(\frac{Q_0}{\lambda}\bar{f}_3^{*} + F(0)\right)\sinh\left(p(z-\xi)\right) + \left(\bar{f}_4^{*} - F(\xi)\right)\sinh(pz)}{\sinh(p\xi)} + F(z)\right\} \tag{3.10}$$

$$u_x = \frac{-2\lambda}{\pi}\int_{-a}^{a}\sum_{m,n=1}^{\infty}\frac{1}{\lambda_n}\cos\left(\frac{m\pi y}{b}\right)$$

$$\left\{\frac{\left(\frac{Q_0}{\lambda}\bar{f}_3^{\,*}+F(0)\right)\sinh\left(p(z-\xi)\right)+\left(\bar{f}_4^{\,*}-F(\xi)\right)\sinh(pz)}{\sinh(p\xi)}\right.$$

$$\left[\left(p^2+\frac{m^2\pi^2}{b^2}\right)P_n(x)+\nu\, P_n''(x)\right]$$

$$\left.+\left[\left(\frac{m^2\pi^2}{b^2}+1\right)P_n(x)+\nu\, P_n''(x)\right]F(z)-P_n(x)F''(z)\right\}dx \tag{3.11}$$

$$u_y=\frac{-2\lambda}{b}\int_0^b\sum_{m,n=1}^{\infty}\frac{1}{\lambda_n}\cos\left(\frac{m\pi y}{b}\right)$$

$$\left\{\frac{\left(\frac{Q_0}{\lambda}\bar{f}_3^{\,*}+F(0)\right)\sinh\left(p(z-\xi)\right)+\left(\bar{f}_4^{\,*}-F(\xi)\right)\sinh(pz)}{\sinh(p\xi)}\right.$$

$$\left[\left(\frac{m^2\pi^2}{b^2}P^2-1\right)P_n(x)+P_n''(x)\right]$$

$$\left.+\left[\left(\frac{m^2\pi^2}{b^2}-1\right)P_n(x)+P_n''(x)\right]F(z)+P_n(x)F''(z)\right\}dy \tag{3.12}$$

$$u_z=\frac{-2\lambda}{b}\int_0^h\sum_{m,n=1}^{\infty}\frac{1}{\lambda_n}\cos\left(\frac{m\pi y}{b}\right)$$

$$\left\{\frac{\left(\frac{Q_0}{\lambda}\bar{f}_3^{\,*}+F(0)\right)\sinh\left(p(z-\xi)\right)+\left(\bar{f}_4^{\,*}-F(\xi)\right)\sinh(pz)}{\sinh(p\xi)}\right.$$

$$\left[\left(\nu p^2+1-\frac{m^2\pi^2}{b^2}\right)P_n(x)+P_n''(x)\right]$$

$$\left.+\left[\left(1-\frac{m^2\pi^2}{b^2}\right)P_n(x)+P_n''(x)\right]F(z)-\nu P_n(x)F''(z)\right\}dz \tag{3.13}$$

Substituting the value of displacement function U from equation (3.9) in equations (2.13) – (3.15) one obtains the expression for stress components as

$$\sigma_{xx} = \frac{2\lambda E}{\pi}\sum_{m,n=1}^{\infty}\frac{P_n(x)}{\lambda_n}\cos\left(\frac{m\pi y}{b}\right)$$

$$\left\{\frac{\left(\frac{Q_0}{\lambda}\bar{f}_3^{\,*}+F(0)\right)\sinh\left(p(z-\xi)\right)+\left(\bar{f}_4^{\,*}-F(\xi)\right)\sinh(pz)}{\sinh(p\xi)}\right.$$

$$\left[\frac{m^2\pi^2}{b^2}+p^2\right]-\frac{m^2\pi^2}{b^2}F(z)-F''(z) \tag{3.14}$$

$$\sigma_{yy} = \frac{-2\lambda E}{\pi}\sum\frac{1}{\lambda_n}\cos\left(\frac{m\pi y}{b}\right)\left\{\frac{\left(\frac{Q_0}{\lambda}\bar{f}_3^{\,*}+F(0)\right)\sinh\left(p(z-\xi)\right)+\left(\bar{f}_4^{\,*}-F(\xi)\right)\sinh(pz)}{\sinh(p\xi)}\right.$$

$$\left(-P^2P_n(x)+P_n''(x)\right)+P_n(x)F''(z)+P_n''(x)F(z)\} \tag{3.15}$$

$$\sigma_{zz} = \frac{-2\lambda E}{\pi}\sum_{m,n=1}^{\infty}\frac{1}{\lambda_n}\cos\left(\frac{m\pi y}{b}\right)$$

$$\left\{\frac{\left(\frac{Q_0}{\lambda}\bar{f}_3^{\,*}+F(0)\right)\sinh\left(p(z-\xi)\right)+\left(\bar{f}_4^{\,**}-F(\xi)\right)\sinh(pz)}{\sinh(p\xi)}+F(z)\right.$$

$$\left[P_n''(x)-\frac{m^2\pi^2}{b^2}P_n(x)\right] \tag{3.16}$$

4 SPECIAL CASE

Set $f_3(x,y)=(x^2-ax)(y^2-by)=f_4(x,y)$, (4.1)

Applying finite Fourier cosine transform and sine transform to the equation (4.1) we get

$$\bar{f}_3^{\,*}(m,n) = \left[\frac{-8b^3}{n\pi}\cos(n\pi - 1) + \frac{b^5 \cos p\pi}{p\pi}\right] \times \left[\frac{2b^3}{n^2\pi^2} + \frac{b^3}{n^2\pi^2}(1 - \cos n\pi)\right] \tag{4.2}$$

Substituting the above value to the equations (3.8) we obtain

$$T = \frac{2}{\pi}\sum_{m,h=1}^{\infty}\frac{P_n(x)}{\lambda_n}\cos\left(\frac{m\pi y}{b}\right) \left\{\frac{\left(\frac{Q_0}{\lambda}\bar{f}_3^{\,*} + F(0)\right)\sinh\left(p(z-\xi)\right) + \left(\bar{f}_4^{\,*} - F(\xi)\right)\sinh(pz)}{\sinh(p\xi)} + F(z)\right\} \tag{4.3}$$

5. NUMERICAL RESULTS

Set $a = 1$, $b = 2$, $h = 2$, $\xi = 1.5$ and $k = 0.86$ in equations (4.3) we get

$$T = \frac{2}{\pi}\sum_{m,h=1}^{\infty}\frac{P_n(x)}{\lambda_n}\cos\left(\frac{m\pi y}{b}\right) \left\{\frac{\left(\frac{Q_0}{\lambda}\bar{f}_3^{\,*} + F(0)\right)\sinh\left(p(z-\xi)\right) + \left(\bar{f}_4^{\,*} - F(\xi)\right)\sinh(pz)}{\sinh(p\xi)} + F(z)\right\} \tag{5.1}$$

6. STATEMENT OF THE PROBLEM-II

Consider a thin isotropic rectangular plate occupying the space *D*. The differential equation satisfied by the deflection $\omega(x,y,t)$ as **Khobragade et al**. **[5]** is

$$D\nabla^4\omega(x,y,t) = \frac{-\nabla^2 M_T(x,y,t)}{1-\nu} \tag{6.1}$$

where,

ν is the Poisson's ratio of the plate material ,

M_T denote the thermal momentum of the plate and

D denote the flexural rigidity,

$$\nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2}$$

The resultant thermal momentum M_T is defined as

$$M_T(x, y, t) = \alpha E \int_0^h zT(x, y, z, t)dz \tag{6.2}$$

where α, *E* are the linear coefficient of thermal expansion of the material, and Young's modulus respectively.

Since the edge of the rectangular plate is fixed and clamped,

$$\omega = \frac{\partial^2 \omega}{\partial x^2} = \frac{\partial^2 \omega}{\partial y^2} = 0 \text{ at x} = a \text{ and y = b} \tag{6.3}$$

The temperature of the plate at time t satisfying the differential equation as **Tanigawa et al. [31]** is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{g(x, y, z, t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{6.4}$$

where *k* is the thermal diffusivity of the material of the plate, subject to the initial and boundary conditions:

$$T(x, y, z, 0) = 0 \tag{6.5}$$

$$\left[T(x, y, z, t) + k_1 \frac{\partial T(x, y, z, t)}{\partial x}\right]_{x=-a} = f_1(y, z, t) \tag{6.6}$$

$$\left[T(x, y, z, t) + k_2 \frac{\partial T(x, y, z, t)}{\partial x}\right]_{x=a} = f_2(y, z, t) \tag{6.7}$$

$$\left[\frac{\partial T(x, y, z, t)}{\partial y}\right]_{y=0} = g_1(x, z, t) \tag{6.8}$$

$$\left[\frac{\partial T(x, y, z, t)}{\partial y}\right]_{y=b} = g_2(x, z, t) \tag{6.9}$$

$$\left[T(x, y, z, t)\right]_{z=0} = 0 \tag{6.10}$$

$$\left[T(x,y,z,t)\right]_{z=\xi} = g_3(x,y,t) \tag{6.11}$$

$$\left[T(x,y,z,t)\right]_{z=h} = H(x,y,t) \quad \text{(unknown)} \tag{6.12}$$

The displacement components u_x and u_y u_z in the x and y and z directions respectively as **Tanigawa et al. [31]** are

$$u_x = \int_{-a}^{a} \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} - \nu \frac{\partial^2 U}{\partial x^2} \right) + \lambda T \right] dx \tag{6.13}$$

$$u_y = \int_{0}^{b} \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2} - \nu \frac{\partial^2 U}{\partial y^2} \right) + \lambda T \right] dy \tag{6.14}$$

$$u_z = \int_{0}^{h} \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - \nu \frac{\partial^2 U}{\partial z^2} \right) + \lambda T \right] dz \tag{6.15}$$

where E, ν and λ are the young's modulus, Poisson's ratio and the linear coefficient of the thermal expansion of the material of the beam respectively and U (x,y,z,t) is the Airy's stress functions which satisfy the differential equation as **Tanigawa et al. [31]** is

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)^2 U(x,y,z,t) = -\lambda E \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \times T(x,y,z,t) \tag{6.16}$$

The stress components in terms of U(x, y, z, t) **Tanigawa et al. [31]** are given by

$$\sigma_{xx} = \left[\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} \right] \tag{6.17}$$

$$\sigma_{yy} = \left[\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2} \right] \tag{6.18}$$

$$\sigma_{zz} = \left[\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right] \tag{6.19}$$

Equations (6.1) to (6.19) constitute the mathematical formulation of the problem under consideration.

7. SOLUTION OF THE PROBLEM

Applying Marchi-Fasulo, transform and finite Fourier cosine Transform and Fourier sine Transform to the equations (6.4) one obtains.

$$\frac{d\overline{\overline{T}}^{*}}{dt}+\alpha q^{2}\overline{\overline{T}}^{*}=\Pi \tag{7.1}$$

Where $q^{2}=\mu_{n}^{2}+\dfrac{m^{2}\pi^{2}}{b^{2}}+\dfrac{p^{2}\pi^{2}}{\xi^{2}}$

and

$$\Pi=\alpha\left[\frac{p_{n}(a)}{k_{2}}\bar{f}_{2}^{*}-\frac{p_{n}(-a)}{k_{1}}\bar{f}_{1}^{*}+(-1)^{m}\bar{g}_{2}^{*}-\bar{g}_{1}^{*}+(-1)^{p+1}\left(\frac{p\pi}{\xi}\right)\overline{\overline{g}}_{3}+\frac{\overline{\overline{g}}^{*}}{k}\right].$$

Solution of equation (7.1) is given by

$$\overline{\overline{T}}^{*}=\left(\int_{0}^{t}\Pi e^{-\alpha q^{2}(t-t')}dt'\right) \tag{7.2}$$

Applying inversion of Fourier sine, Fourier cosine and Marchi-Fasulo transform to the equation (7.2) we get the temperature distribution and unknown temperature gradient as

$$T(x,y,z,t)=\frac{4}{b\xi}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{P_{n}(x)}{\mu_{n}}\cos\left(\frac{m\pi y}{b}\right)\times\sin\left(\frac{p\pi z}{\xi}\right)$$
$$\times\int_{0}^{t}\Pi e^{-\alpha q^{2}(t-t')}dt' \tag{7.3}$$

$$f(x,y,t)=\frac{4}{b\xi}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{P_{n}(x)}{\mu_{n}}\cos\left(\frac{m\pi y}{b}\right)\times\sin\left(\frac{p\pi h}{\xi}\right)$$
$$\times\int_{0}^{t}\Pi e^{-\alpha q^{2}(t-t')}dt' \tag{7.4}$$

where *p, m, n* are the positive integers

Using equation (7.3) in equation (7.16) we get

$$U(x,y,z,t) = \frac{-4(1+\nu)a_t}{q^2 b\xi} \sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{P_n(x)}{\mu_n} \cos\left(\frac{m\pi y}{b}\right) \times \sin\left(\frac{p\pi z}{\xi}\right)$$

$$\times \int_0^t \prod e^{-\alpha q^2 (t-t')} dt' \tag{7.5}$$

Using equation (7.5) in equations (6.13) to (6.15) we get

$$u_x = \frac{4}{b\xi} \int_{-a}^{a} \sum_{m,n,p} \left\{ \frac{(1+\upsilon)a_t}{E\mu_n q^2} \left[\left(\frac{m^2\pi^2}{b^2} - \frac{p^2\pi^2}{\xi^2} \right) P_n(x) - \upsilon P_n''(x) \right] + \frac{P_n(x)}{\mu_n} \right\}$$

$$\times \cos\left(\frac{m\pi y}{b}\right) \times \sin\left(\frac{p\pi z}{\xi}\right) \times \left(\int_0^t \prod e^{-\alpha q^2 (t-t')} dt' \right) dx \tag{7.6}$$

$$u_y = \frac{4}{b\xi} \int_0^b \sum_{m,n,p} \left\{ \frac{(1+\upsilon)a_t}{E\mu_n q^2} \left[\left(\frac{p^2\pi^2}{\xi^2} - \frac{\upsilon m^2\pi^2}{b^2} \right) P_n(x) - \upsilon P_n''(x) \right] + \frac{\lambda P_n(x)}{\mu_n} \right\}$$

$$\times \cos\left(\frac{m\pi y}{b}\right) \times \sin\left(\frac{p\pi z}{\xi}\right) \times \left(\int_0^t \prod e^{-\alpha q^2 (t-t')} dt' \right) dy \tag{7.7}$$

$$u_z = \frac{4}{b\xi} \int_0^{\xi} \sum_{m,n,p} \left\{ \frac{(1+\upsilon)a_t}{E\mu_n q^2} \left[\left(\frac{m^2\pi^2}{b^2} - \frac{\upsilon p^2\pi^2}{\xi^2} \right) P_n(x) - P_n''(x) \right] + \frac{\lambda P_n(x)}{\mu_n} \right\}$$

$$\times \cos\left(\frac{m\pi y}{b}\right) \times \sin\left(\frac{p\pi z}{\xi}\right) \times \left(\int_0^t \prod e^{-\alpha q^2 (t-t')} dt' \right) dz \tag{7.8}$$

Using equation (7.5) in equations (62.17) to (6.19) we get

$$\sigma_{xx} = \frac{4\pi^2(1+\nu)a_t}{q^2 b^3 \xi^3} \sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{(\xi^2 m^2 + p^2 b^2)}{\mu_n} P_n(x)$$

$$\times \cos\left(\frac{m\pi y}{b}\right) \times \sin\left(\frac{p\pi z}{\xi}\right) \times \int_0^t \prod e^{-\alpha q^2 (t-t')} dt' \tag{7.9}$$

$$\sigma_{yy} = \frac{-4(1+\nu)a_t}{q^2 b \xi^3} \sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{\left(\xi^2 P_n''(x) - p^2\pi^2 P_n(x)\right)}{\mu_n}$$

$$\times \cos\left(\frac{m\pi y}{b}\right) \times \sin\left(\frac{p\pi z}{\xi}\right) \times \int_0^t \prod e^{-\alpha q^2 (t-t')} dt' \tag{7.10}$$

$$\sigma_{zz} = \frac{-4(1+\nu)a_t}{q^2 b^3 \xi} \sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{\left(b^2 P_n''(x) - m^2\pi^2 P_n(x)\right)}{\mu_n}$$

$$\times \cos\left(\frac{m\pi y}{b}\right) \times \sin\left(\frac{p\pi z}{\xi}\right) \times \int_0^t \prod e^{-\alpha q^2 (t-t')} dt' \tag{7.11}$$

8. SPECIAL CASE

Set $g_3(x, y, t) = (1 - e^{-t})(x^2 - ax)(y^2 - by)$,
(8.1)

Applying finite Fourier cosine transform and sine transform to the equation (8.1) we get

$$\bar{g}_3^{\,*}(x, y, t) = (1 - \bar{e}^{t})\left[\frac{-8b^3}{n\pi}\cos(n\pi - 1) + \frac{b^5 \cos p\pi}{p\pi}\right]$$

$$\times \left[\frac{2b^3}{n^2\pi^2} + \frac{b^3}{n^2\pi^2}(1 - \cos n\pi)\right] \tag{8.2}$$

Substituting the above value to the equations (7.3) – (7.4) we obtain

$$T(x, y, z, t) = \frac{4}{b\xi} \sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{P_n(x)}{\mu_n} \cos\left(\frac{m\pi y}{b}\right) \times \sin\left(\frac{p\pi z}{\xi}\right)$$

$$\times \int_0^t \prod e^{-\alpha q^2 (t-t')} dt' \tag{8.3}$$

$$H(x,y,t) = \frac{4}{b\xi}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{P_n(x)}{\mu_n}\cos\left(\frac{m\pi y}{b}\right) \times \sin\left(\frac{p\pi h}{\xi}\right)$$

$$\times \int_0^t \prod e^{-\alpha q^2 (t-t')} dt' \tag{8.4}$$

9. NUMERICAL RESULTS

Set $a = 2$, $b = 2$, $h = 2$, $t = 1\text{sec}$ $\xi = 1.5$ and $k = 0.86$ in equations (8.3) to (8.4) we get

$$T(x,y,z,t) = 1.33\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{P_n(x)}{\mu_n}\cos(1.57my)\times\sin(2.09pz)$$

$$\times \sin\left(\frac{p\pi z}{\xi}\right)$$

$$\times \int_0^t \prod e^{-\alpha q^2 (t-t')} dt' \tag{9.1}$$

$$H(x,y,t) = 1.33\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{P_n(x)}{\mu_n}\cos(1.57my)\sin(2.09mz)$$

$$\times \int_0^t \prod e^{-\alpha q^2 (t-t')} dt' \tag{9.2}$$

10. CONCLUSION

In both the problems, the temperature distribution, unknown temperature gradient, displacement function, thermal stresses and thermal deflection of a three dimensional rectangular plate have been derived, with the aid of finite Marchi-Fasulo transform, finite Fourier cosine transform and Fourier sine transform techniques when the stated boundary conditions are known. The results are obtained in the form of infinite series. The series solutions converge provided if we take sufficient number of terms in the series. The expressions are represented graphically. The results that are obtained can be applied to the design of useful structures or machines in engineering applications.

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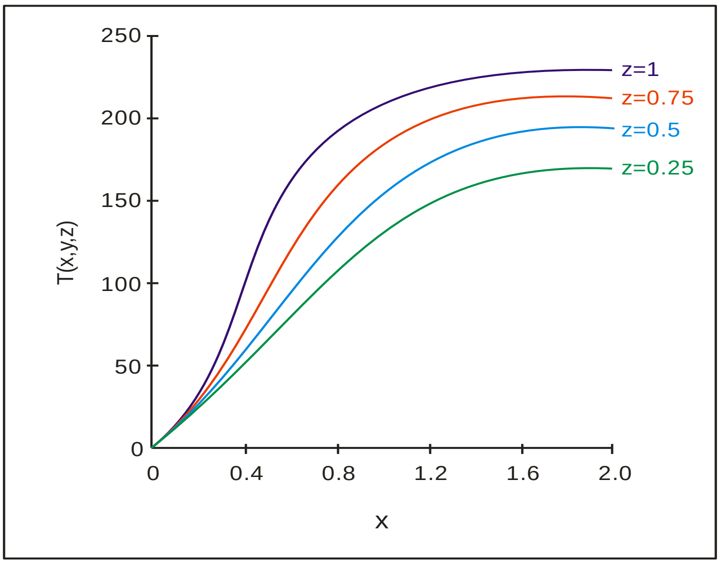
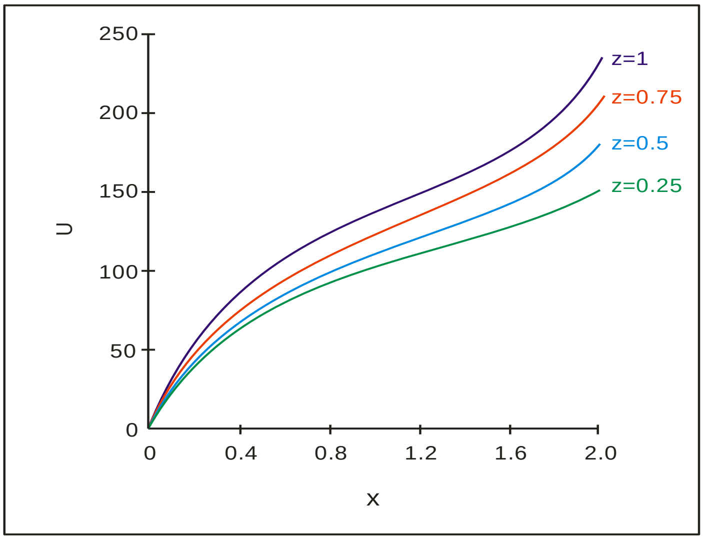
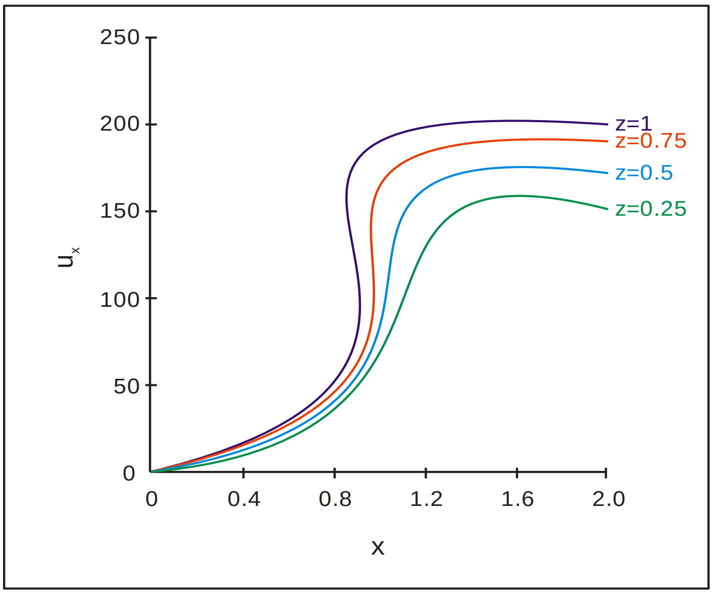


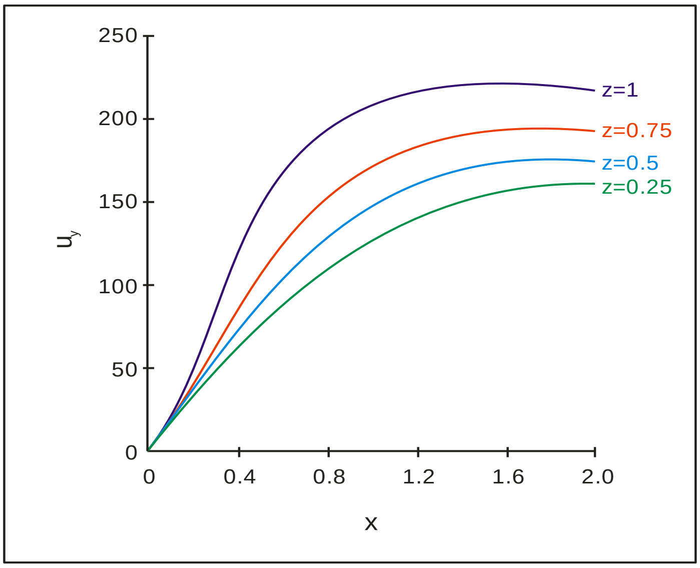
Graph 1: Temperature distribution versus x.



Graph 2: Airy's stress function versus x.



Graph 3: Displacement component versus x.



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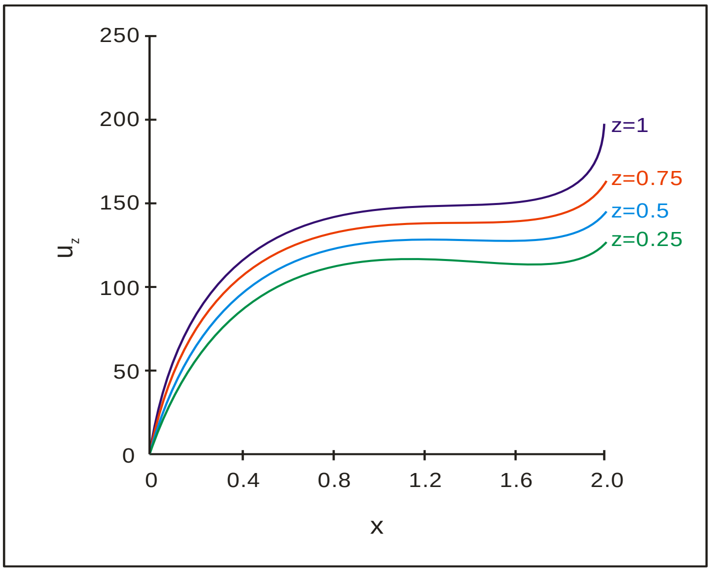
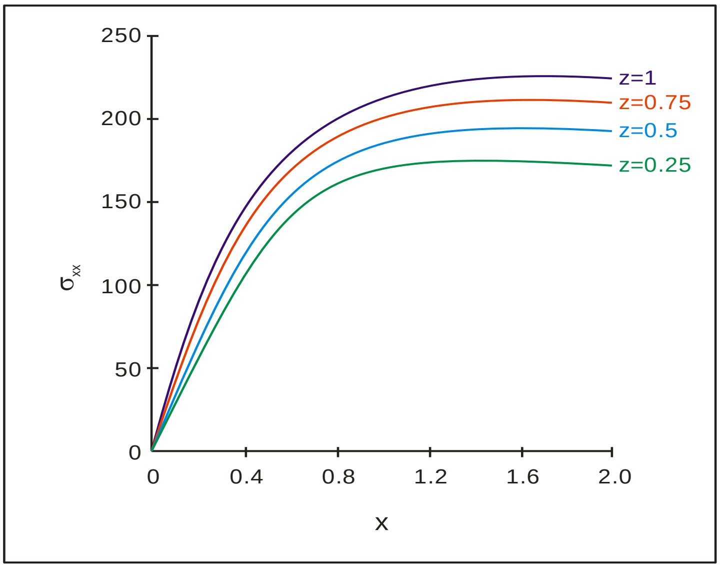
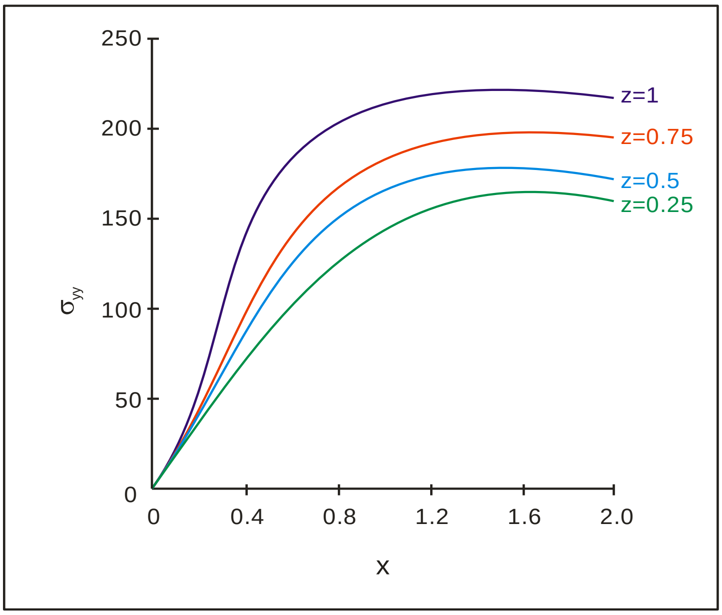


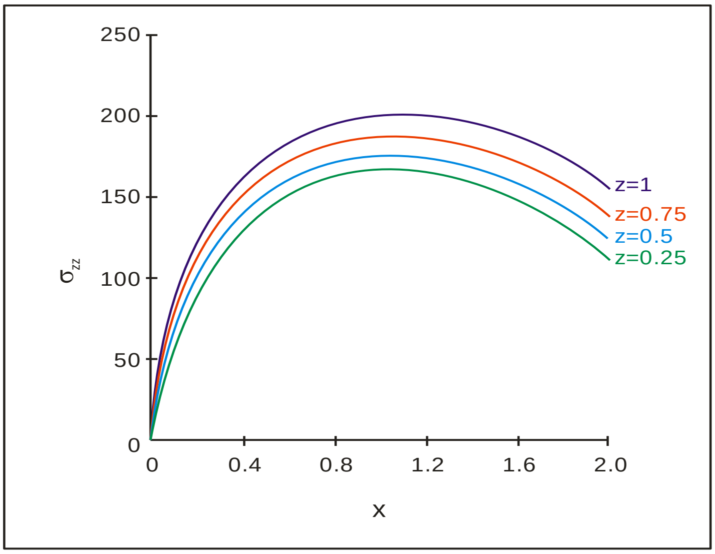
Graph 5: Displacement component versus x.



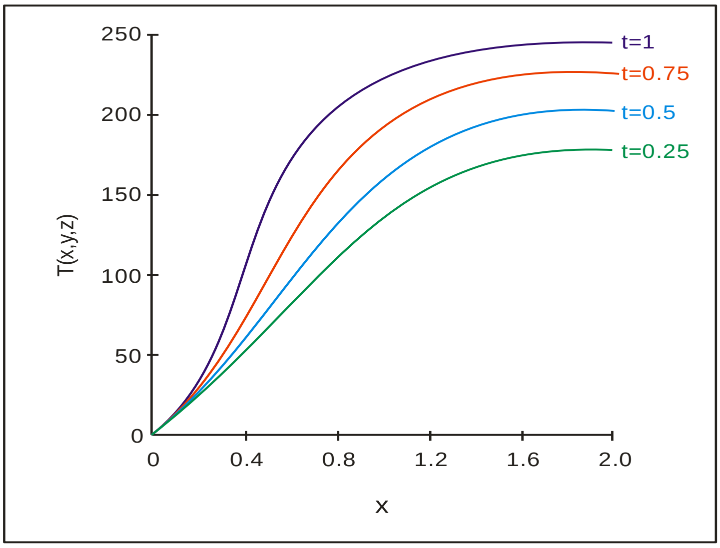
Graph 6: Stress component versus x.



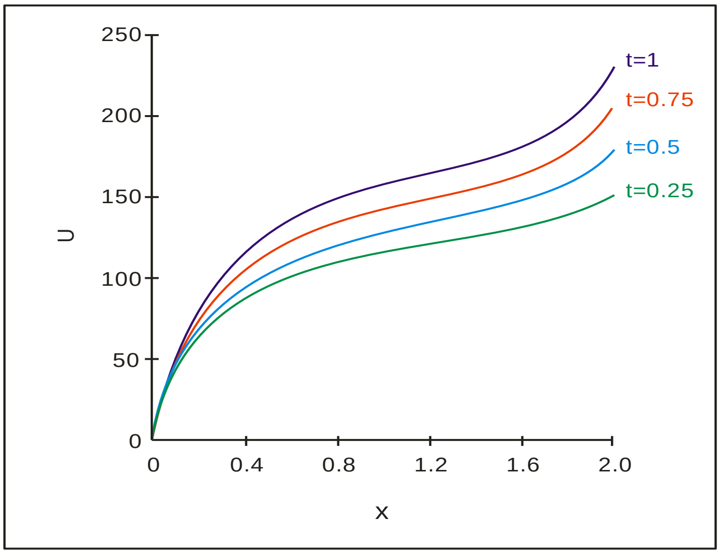
Graph 7: Stress component versus x.



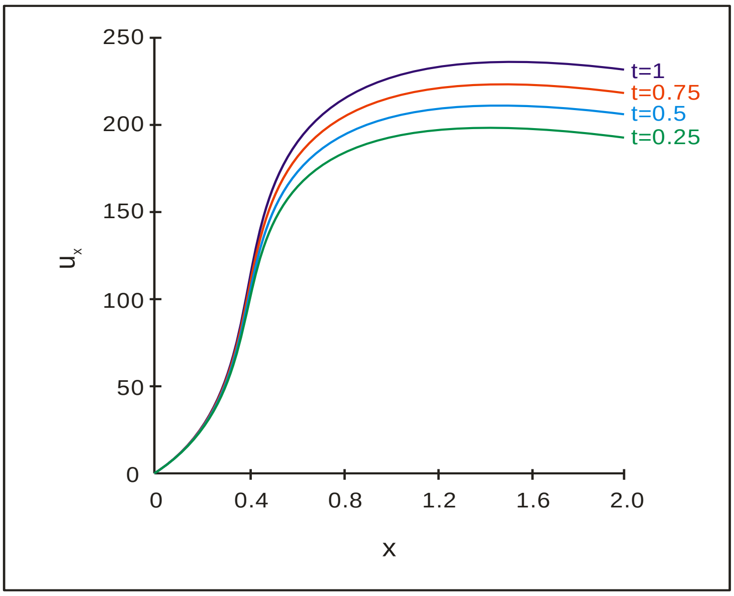
Graph 8: Stress component versus x.



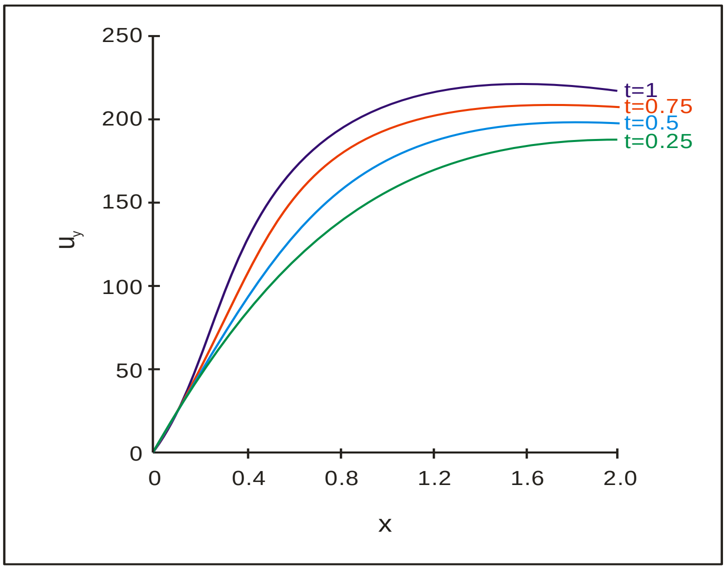
Graph 9: Temperature distribution versus x.



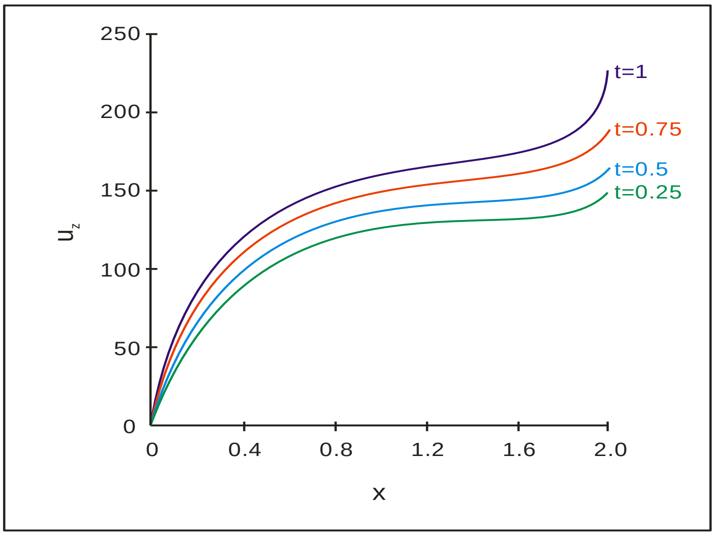
Graph 10 : Airy's stress function versus x.



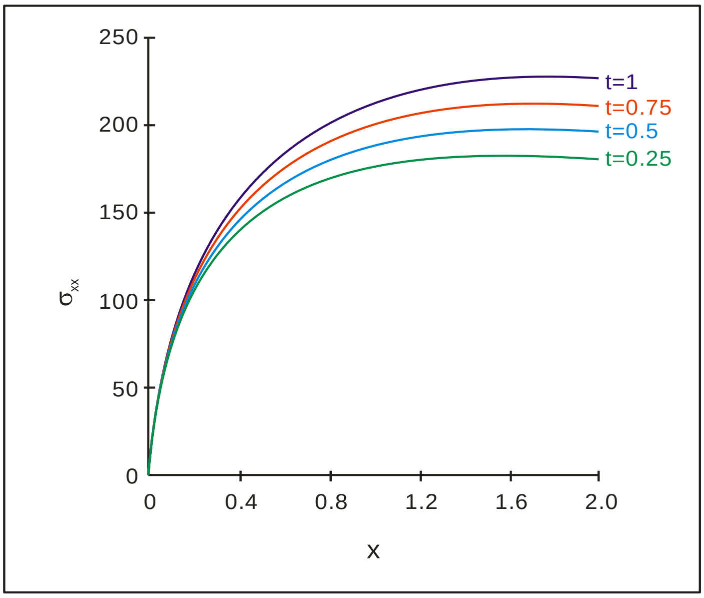
Graph 11: Displacement component versus x.



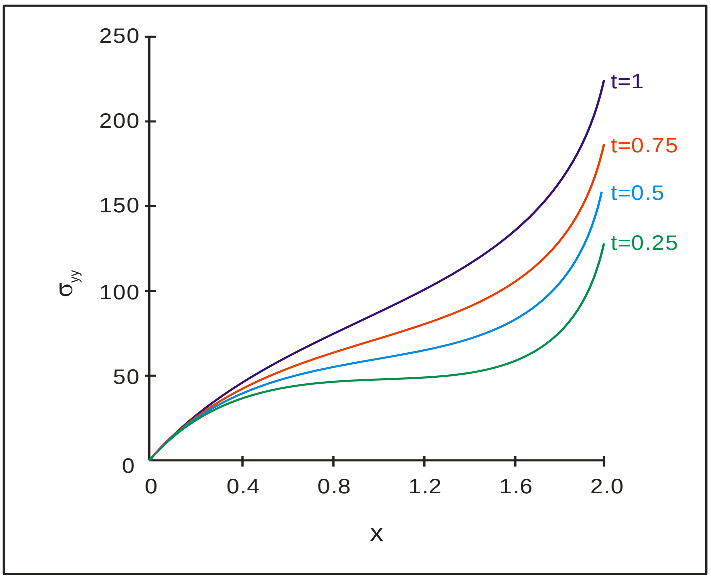
Graph 12: Displacement components versus x.



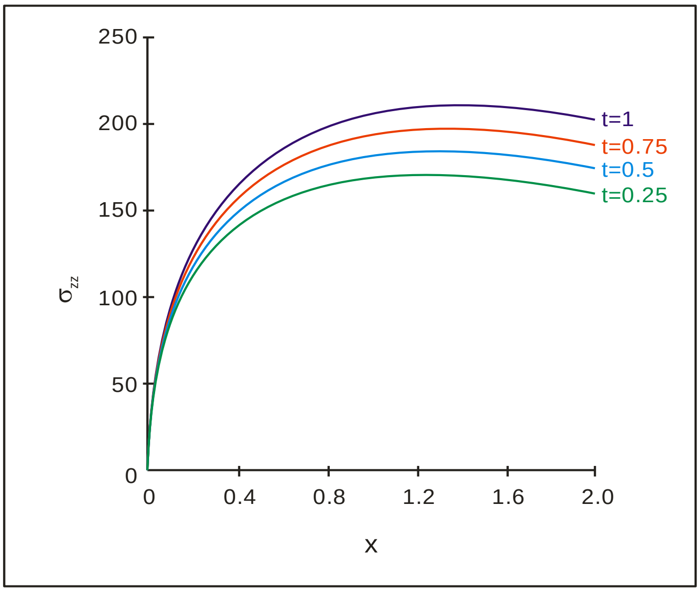
Graph 13: Displacement components versus x.



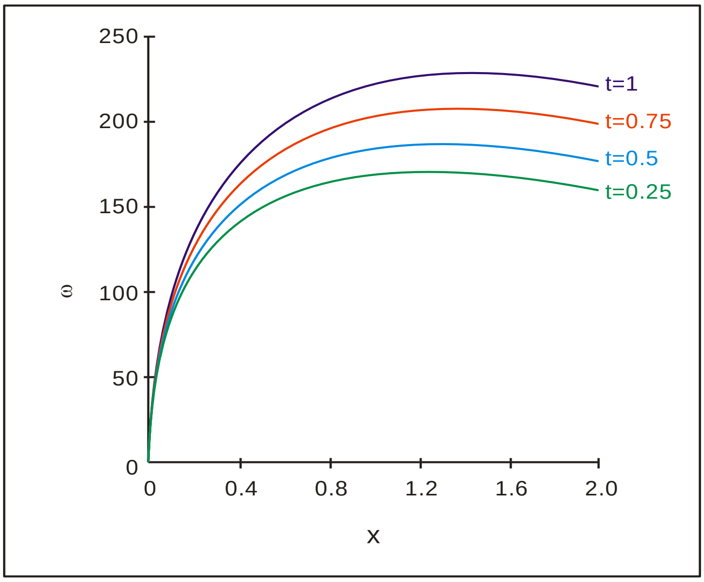
Graph 14: Stress component versus x.



Graph 15: Stress component versus x.



Graph 16: Stress component versus x.



Graph 17: Deflection versus x.

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