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Thermoelastic Response of a Thick Circular Plate due to Heat Generation and its Thermal Stresses

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Abstract

In this paper, an attempt has been made to study thermoelastic response of a thick circular plate occupying the space $D: 0 \le r \le a,\ -h \le z \le h,$ due to heat generation with radiation type boundary conditions. Here we apply integral transform techniques to find the thermoelastic solution.

Keywords: Thermo elastic problem, thick Circular Plate, Thermal Stresses, integral transform.

1. INTRODUCTION

Gahane T T; Khalsa L H and Khobragade N W [1] discussed Thermal Stresses in A Thick Circular Plate With Internal Heat Sources, Journal of Statistics and Mathematics. **Ghume R S; Ashwini Mahakalkar and Khobragade N W** [2] studied Thermoelastic solution of a thin circular plate due to partially distributed heat supply, Int. J. of Engg. And Information Technology, vol. 3, Issue 6, pp.314-317, **2013**. **Hamna Parveen and Khobragade N W [3]** discussed Thermal Stresses Of A Thick Circular Plate Due To Heat Generation. Further **Hamna Parveen; N K Lamba and Khobragade N W [4]** studied Thermal Stresses Of A Circular Disk With Internal Heat Sources, Journal of Statistics and Mathematics. **Hamna Parveen; Navneet Kumar and Khobragade N W [5]** studied Thermal deflection of a thin circular plate with radiation. **Khobragade N W; Khalsa L H; Gahane T T and Pathak A C [6]** discussed Transient Thermo elastic Problem of a Circular Plate With Heat Generation. **Khobragade N W[7]** discussed Thermal stresses of a thin circular plate with internal heat source and **Khobragade N W [8]** studied Thermoelastic analysis of a thick circular plate. **Lamba N K and Khobragade N W [9]** discussed Analytical Thermal Stress Analysis in a thin circular plate due to diametrical compression. **Noda N; Ashida F** and **Tsuji T [12]** studied An inverse transient thermoelastic problem for a transversely isotropic body. **Noda N; Tanigawa Y; Kawamura R** and **Ishihara M [13]** discussed Theoretical analysis of thermoelastic-plastic deformation of a circular plate due to a partially distributed heat supply.

Nowacki W[14] studied the state of stress in thick circular plate due to temperature field. **Sabherwal K C [16]** discussed An inverse problem of transient heat conduction. **Sharma J N; Sharma P K** and **Sharma R L [17]** studied Behavior of thermoelastic thick plate under lateral loads. **Sugano Y; Kimura N; Sato K** and **Sumi N** [20] discussed Three-dimensional analysis of transient thermal stresses in a non homogenous plate. **Varghese V and Khobragade N W[21]** studied Alternative Solution of a Transient Heat Conduction in a Circular Plate with Radiation and **Wankhede P C [22]** discussed the quasi- static thermal stresses in a circular plate.

This paper is concerned with steady-state thermoelastic problem and transient thermoelastic problem of a thick circular plate occupying the space $D: 0 \le r \le a,\ -h \le z \le h,$ due to heat generation with radiation type boundary conditions.

2. STATEMENT OF THE PROBLEM-I

Consider thick circular plate of thickness $2h$ occupying the space $D: 0 \le r \le a,\ -h \le z \le h,$ the material is homogenous and isotropic. The differential equation governing the displacement potential function $\phi(r,z,t)$ as **Nowacki** [**14**] is

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = \left(\frac{1+\nu}{1-\nu}\right)\alpha_t T \tag{2.1}$$

where ν and a_t are Poisson's ratio and linear coefficient of thermal expansion of the material of the plate and T is the temperature of the plate satisfying the differential equation as **Noda et al.** [**11**] is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \chi(r,z) = 0 \tag{2.2}$$

Subject to the boundary conditions

$$M_r(T,0,1,a) = g(z) \qquad -h \le z \le h, \tag{2.3}$$

$$M_z(T,1,k_1,h) = f_1(r) \qquad 0 \le r \le a$$

$$M_z(T,1,k_2,-h) = -\frac{Q_0}{\lambda} f_2(r) \quad 0 \le r \le a \tag{2.4}$$

where k is thermal diffusivity of material of the plate.

The displacement function in the cylindrical coordinate system are represented by Love's function as **Khobragade** [**8**] are

$$u_r = \frac{\partial \phi}{\partial r} - \frac{\partial^2 L}{\partial r \partial z} \tag{2.5}$$

$$u_z = \frac{\partial \phi}{\partial z} + 2(1-\nu)\nabla^2 L - \frac{\partial^2 L}{\partial z^2} \tag{2.6}$$

The Love's function [**10**] must satisfy

$$\nabla^2 \nabla^2 L = 0 \tag{2.7}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

The component of stresses are represented by the thermoelastic displacement potential ϕ and Love's function L as **Noda et al.** [**11**] are

$$\sigma_{rr} = 2G\left\{\left(\frac{\partial^2 \phi}{\partial r^2} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left(\nu \nabla^2 L - \frac{\partial^2 L}{\partial r^2}\right)\right\} \tag{2.8}$$

$$\sigma_{\theta\theta} = 2G\left\{\left(\frac{1}{r}\frac{\partial \phi}{\partial r} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left(\nu \nabla^2 L - \frac{1}{r}\frac{\partial^2 L}{\partial r^2}\right)\right\} \tag{2.9}$$

$$\sigma_{zz} = 2G\left\{\left(\frac{\partial^2 \phi}{\partial z^2} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left\{\left((z-\nu)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right)\right\}\right\} \tag{2.10}$$

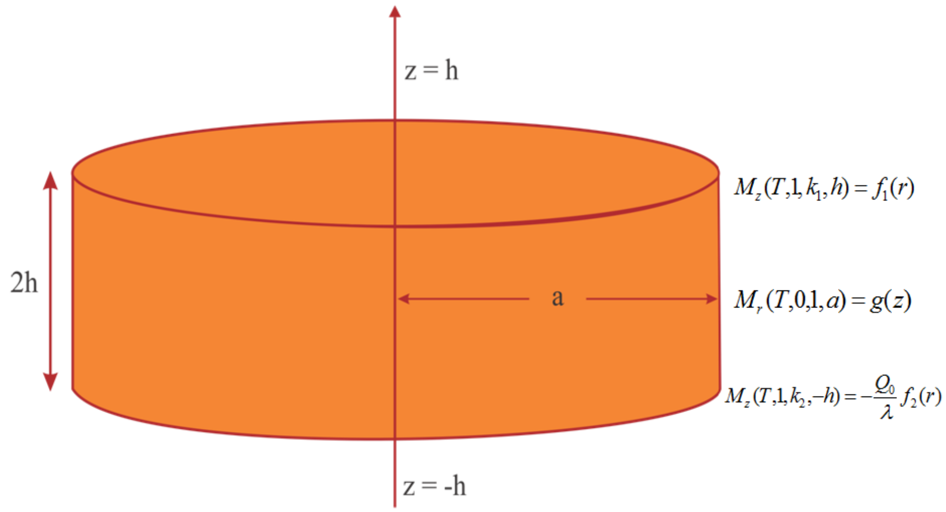

Fig. 1: Shows the geometry of the problem

$$\sigma_{rz} = 2G\left\{\frac{\partial^2\phi}{\partial r\partial z} + \frac{\partial}{\partial r}\left\{\left((1-\nu)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right)\right\}\right\} \tag{2.11}$$

For traction free surface stress function $\sigma_z = \sigma_{r\theta} = 0$ at $z = \pm h$ for thick plate.

Equations (2.1) to (2.11) constitute the mathematical formulation of the problem under consideration.

3. SOLUTION OF THE PROBLEM

Applying Marchi-Fasulo transform to the equation (2.2), we get

$$\frac{\partial^2 \overline{T}}{\partial r^2} + \frac{1}{r}\frac{\partial \overline{T}}{\partial r} - \lambda_n^2\, \overline{T} = \psi \tag{3.1}$$

where, $\psi = \dfrac{-P_n(h)}{k_1} f_1(r) \dfrac{-P_n(-h)}{k_2}\dfrac{Q_0}{\lambda} f_2(r)$ (3.2)

Equation (3.1) is a Bessel's equation whose solution gives

$$\overline{T} = AI_0(\lambda_n r) + BK_0(\lambda_n r) + \overline{F}(r) \tag{3.3}$$

Where $\overline{F}(r)$ is the P.I.

As $r \to 0, K_0 \to \infty$, But $\overline{T}$ is finite

$$\therefore B = 0 \tag{3.4}$$

$$A = \frac{\overline{g} \;\, - \overline{F'}(a)}{I_0'(\lambda_n a)} \tag{3.5}$$

$$\therefore \overline{T} = \frac{\overline{g} \;\, - \overline{F'}(a)}{I_0'(\lambda_n a)} I_0(\lambda_n r) + \overline{F}(r) \tag{3.6}$$

Applying inverse Marchi-Fasulo transform to the equation (3.6) we get

$$T = \sum_{n=1}^{\infty}\left[\frac{P_n(z)}{\lambda_n}\frac{\overline{g} - \overline{F'}(a)}{I_0'(\lambda_n a)} I_0(\lambda_n r) + \overline{F}(r)\right] \tag{3.7}$$

and

$$\phi = \frac{r^2}{4}\left(\frac{1+\nu}{1-\nu}\right) a_t \sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\left[\frac{\overline{g} \;\, - \overline{F'}(a)}{I_0'(\lambda_n a)} I_0(\lambda_n r) + \overline{F}(r)\right] \tag{3.8}$$

We assume L as

$$L = \frac{r^2}{4}\left(\frac{1+\nu}{1-\nu}\right)a_t \sum_{n=1}^{\infty} \frac{P_n(z)}{\lambda_n}\, \frac{\bar{g} - F'(a)}{I_0'(\lambda_n a)} I_0(\lambda_n r) \tag{3.9}$$

4. DETERMINATION OF DISPLACEMENT FUNCTION

Substituting equations (3.8) and (3.9) in equations (3.1) - (3.2) we get

$$u_r = \frac{a_t}{4}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{1}{\lambda_n}\ \left\{\frac{\bar{g} - \overline{F'}(a)}{I_0'(\lambda_n a)}\left[r^2 I_0'(\lambda_n r) + 2r I_0(\lambda_n r)\right]\right.$$

$$\left.\left(P_n(z) - P_n'(z)\right) + P_n(z)\overline{F'}(r)\right\} \tag{4.1}$$

$$u_z = \frac{r^2}{4}\left(\frac{1+\nu}{1-\nu}\right) a_t \sum_{n=1}^{\infty}\frac{\bar{g} - \overline{F'}(a)}{I_0'(\lambda_n a)} I_0(\lambda_n r)\left[(1-2\nu)P_n''(z) + P_n'(z)\right] + \frac{P_n'(z)\overline{F}(r)}{\lambda_n}$$

$$+\frac{1+\nu}{2}\, a_t \sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\left(\frac{\bar{g} - \overline{F'}(a)}{I_0'(\lambda_n a)}\right)\left[r^2 I_0''(\lambda_n r) + 5r I_0' + 4 I_0(\lambda_n r)\right] \tag{4.2}$$

5. DETERMINATION OF STRESS FUNCTIONS

Substituting equations (3.8) and (3.9) in equations (2.8) – (2.11) we get

$$\sigma_{rr} = \frac{G a_t}{2}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{\bar{g} - \overline{F'}(a)}{\lambda_n I_0'(\lambda_n a)}$$

$$\left\{P_n'''(z)\nu\, r^2 I_0(\lambda_n r) - P_n''(z)\left[r^2 I_0(\lambda_n r) + \frac{r^2 I_0'(\lambda_n r)}{\bar{g} - F'(a)}\overline{F}(r)\right]\right.$$

$$+ P_n'(z)\left[(\nu - 1)\, r^2 I_0''(\lambda_n r) + (5\nu\, r - 4r) I_0'(\lambda_n r) + 2(2\nu - 1) I_0(\lambda_n r)\right]$$

$$- P_n(z)\left[r I_0'(\lambda_n r) + 2 I_0(\lambda_n r)\right] \tag{5.1}$$

$$\sigma_{\theta\theta} = \frac{G a_t}{2}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{\bar{g} - \overline{F'}(a)}{\lambda_n I_0'(\lambda_n a)}$$

$$\left\{P_n'''(z)\left[\nu\, r^2 I_0(\lambda_n r)\right] - P_n''(z)\left[r^2 I_0(\lambda_n r) + \frac{r^2 I_0'(\lambda_n r)\overline{F}(r)}{\bar{g} - \overline{F'}(a)}\right]\right.$$

$$+ P_n'(z)\left[\left(\nu\, r^2 - r\right) I_0''(\lambda_n r) + \left(5\nu\, r - 4\right) I_0'(\lambda_n r) + \left(4\nu - \frac{2}{r}\right) I_0(\lambda_n r)\right]$$

$$- P_n(z)\left[r^2 I_0''(\lambda_n r) + 4\, r I_0'(\lambda_n r) + 2 I_0(\lambda_n r)\right)$$

$$+\frac{I_0'(\lambda_n a)}{\overline{g}-F(a)}\left(r^2F''(r)+4r\overline{F'}(r)+2\overline{F}(r)\right)\Bigg]\Bigg\} \tag{5.2}$$

$$\sigma_{zz}=\frac{Ga_t}{2}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{\overline{g}-\overline{F'}(a)}{\lambda_n I_0'(\lambda_n a)}$$

$$\Big\{P_n'''(z)(1-\nu)r^2I_0(\lambda_n r)+P_n'(z)(2-\nu)\left[r^2I_0''(\lambda_n r)+5rI_0'(\lambda_n r)+4I_0(\lambda_n r)\right]$$

$$+P_n'(z)\left[\left(\nu\, r^2-r\right)I_0''(\lambda_n r)+\left(5\nu\, r-4\right)I_0'(\lambda_n r)+\left(4\nu-\frac{2}{r}\right)I_0(\lambda_n r)\right]$$

$$-P_n(z)\left[r^2I_0''(\lambda_n r)+5\,rI_0'(\lambda_n r)+4I_0(\lambda_n r)\right.$$

$$\left.+\frac{I_0'(\lambda_n a)}{\overline{g}-F'(a)}\left(r^2F''(r)+5rF'(r)+4\overline{F}(r)\right)\right]\Bigg\} \tag{5.3}$$

$$\sigma_{rz}=\frac{Ga_t}{2}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\frac{\overline{g}-\overline{F'}(a)}{\lambda_n I_0'(\lambda_n a)}\Big\{P_n''(z)\nu\, r\left[\,r^2I_0'(\lambda_n r)+2I_0'(\lambda_n r)\right]$$

$$\Big\{P_n''(z)\nu\, r\left[r^2I_0'(\lambda_n r)+2I_0'(\lambda_n r)\right]$$

$$-P_n'(z)r\left[\,rI_0'(\lambda_n r)+2I_0(\lambda_n r)+\frac{I_0'(\lambda_n a)}{\overline{g}-\overline{F'}(a)}\left(r^2\overline{F''}(r)+2\overline{F'}(r)\right)\right]$$

$$-P_n(z)(1-\nu)\left[\,r^2I_0'''(\lambda_n r)+7\,rI_0''(\lambda_n r)+10I_0'(\lambda_n r)+\frac{2}{r}I_0(\lambda_n r)\right]\Bigg\} \tag{5.4}$$

6. SPECIAL CASE

Set $F(r,z)=z^2(1-r^2)$ (6.1)

Applying Marchi-Fasulo transform, we obtain

$$\overline{F}(r,n)=(1-r^2)\int_{-h}^{h}z^2P_n(z)dz$$

$$\overline{F}(r,n)=(1-r^2)\Phi_n\left[\frac{2h^2\sin(a_nh)}{a_n}+\frac{4h\cos(a_nh)}{a_n^{\;2}}-\frac{4\sin(a_nh)}{a_n^{\;3}}\right] \tag{6.2}$$

Where

$P_n(z)=Q_n\cos(a_nz)-W_n\sin(a_nz)$,

$Q_n=a_n\left(\alpha_1+\alpha_2\right)\cos(a_nh)+(\beta_1-\beta_2)\sin(a_nh)$

$$W_n = (\beta_1 - \beta_2)\cos(a_n h) + a_n(\alpha_1 - \alpha_2)\sin(a_n h)$$

Again on applying Hankel transform, we obtain

$$\overline{F}^*(m,n) = \prod_n \left[\frac{a}{\xi_m} J_1(a\xi_m) - \frac{a(a^2\xi_m^2 - 4)}{\xi_m^3} J_1(a\xi_m) - \frac{2a^2}{\xi_m^2} J_0(a\xi_m) \right] \tag{6.3}$$

Where

$$\Pi_n = \Phi_n \left[\frac{2h^2 \sin(a_n h)}{a_n} + \frac{4h\cos(a_n h)}{{a_n}^2} - \frac{4\sin(a_n h)}{{a_n}^3} \right]$$

And

$$\Phi_n = a_n(\alpha_1 + \alpha_2)\cos(a_n h) + (\beta_1 - \beta_2)\sin(a_n h).$$

Using equation (6.3) in equation (3.7), one obtains

$$T = \sum_{n=1}^{\infty} \left[\frac{P_n(z)}{\lambda_n} \frac{\bar{g} - \overline{F'}(a)}{I_0'(\lambda_n a)} I_0(\lambda_n r + (1 - r^2)\Phi_n \left[\frac{2h^2 \sin(a_n h)}{a_n} + \frac{4h\cos(a_n h)}{{a_n}^2} - \frac{4\sin(a_n h)}{{a_n}^3} \right] \right] \tag{6.4}$$

7. NUMERICAL RESULTS

Set $a = 2, k = 15.9 \times 10^6, t = 1$ second in equation (6.4) we get

$$T = \sum_{n=1}^{\infty} \left[\frac{P_n(z)}{\lambda_n} \frac{\bar{g} - \overline{F'}(a)}{I_0'(\lambda_n a)} I_0(\lambda_n r + (1 - r^2)\Phi_n \left[\frac{2h^2 \sin(a_n h)}{a_n} + \frac{4h\cos(a_n h)}{{a_n}^2} - \frac{4\sin(a_n h)}{{a_n}^3} \right] \right] \tag{7.1}$$

8. STATEMENT OF THE PROBLEM-II

Consider thick circular plate of thickness $2h$ occupying the space $D: 0 \le r \le a,\ -h \le z \le h,$ the material is homogenous and isotropic. The differential equation governing the displacement potential function $\phi(r,z,t)$ as **Nowacki** [**14**] is

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = \left(\frac{1+\nu}{1-\nu} \right) \alpha_t T \tag{8.1}$$

where ν and a_t are Poisson's ratio and linear coefficient of thermal expansion of the material of the plate and T is the temperature of the plate satisfying the differential equation as is

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}+\chi(r,z,t)=\frac{1}{k}\frac{\partial T}{\partial t} \tag{8.2}$$

Subject to initial condition

$$M_t(T,1,0,0)=F(r,z) \quad 0\le r\le a,-h\le z\le h. \tag{8.3}$$

The boundary conditions are

$$M_r(T,0,1,a)=g(z_, t) \qquad -h\le z\le h,\ t>0 \tag{8.4}$$

$$M_z(T,1,k_1,h)=f_1(r_, t) \qquad 0\le r\le a,\ t>0$$

$$M_z(T,1,k_2,-h)=-\frac{Q_0}{\lambda}f_2(r_, t)\ \ 0\le r\le a,\ t>0 \tag{8.5}$$

where k is thermal diffusivity of material of the plate.

The displacement function in the cylindrical coordinate system are represented by Love's function as **Khobragade** [**8**] are

$$u_r=\frac{\partial\phi}{\partial r}-\frac{\partial^2 L}{\partial r\partial z} \tag{8.6}$$

$$u_z=\frac{\partial\phi}{\partial z}+2(1-\nu)\nabla^2 L-\frac{\partial^2 L}{\partial z^2} \tag{8.7}$$

Love's function L [**10**] must satisfy

$$\nabla^2\nabla^2 L=0 \tag{8.8}$$

where

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}$$

The component of stresses are represented by the thermoelastic displacement potential ϕ and Love's function L as **Noda et al.** [**11**] are

$$\sigma_{rr}=2G\left\{\left(\frac{\partial^2\phi}{\partial r^2}-\nabla^2\phi\right)+\frac{\partial}{\partial z}\left(\nu\nabla^2 L-\frac{\partial^2 L}{\partial r^2}\right)\right\} \tag{8.9}$$

$$\sigma_{\theta\theta}=2G\left\{\left(\frac{1}{r}\frac{\partial\phi}{\partial r}-\nabla^2\phi\right)+\frac{\partial}{\partial z}\left(\nu\nabla^2 L-\frac{1}{r}\frac{\partial^2 L}{\partial r^2}\right)\right\} \tag{8.10}$$

$$\sigma_{zz}=2G\left\{\left(\frac{\partial^2\phi}{\partial z^2}-\nabla^2\phi\right)+\frac{\partial}{\partial z}\left\{\left((2-\nu)\nabla^2 L-\frac{\partial^2 L}{\partial z^2}\right)\right\}\right\} \tag{8.11}$$

$$\sigma_{rz} = 2G\left\{\frac{\partial^2 \phi}{\partial r \partial z} + \frac{\partial}{\partial r}\left\{\left((1-\nu)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right)\right\}\right\} \tag{8.12}$$

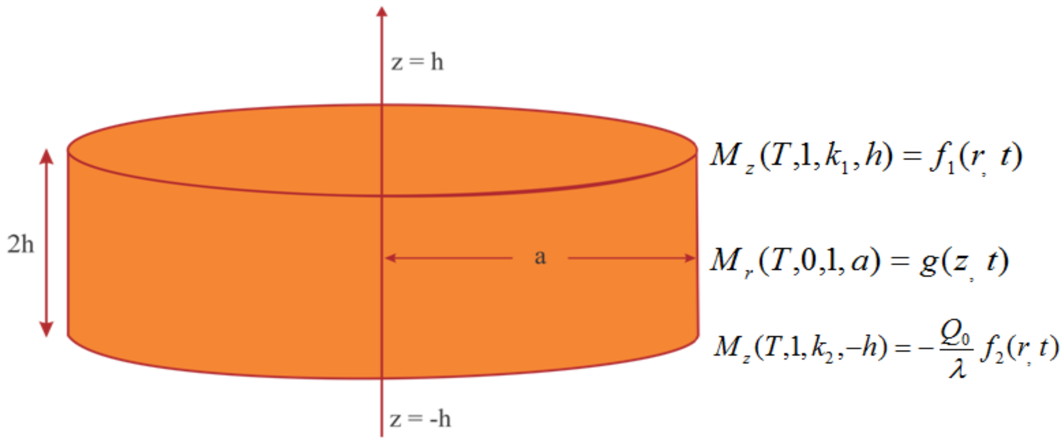

Fig. 1: Shows the geometry of the problem

For traction free surface stress function

$\sigma_z = \sigma_{r\theta} = 0$ at $z = \pm h$ for thick plate.

Equations (8.1) to (8.12) constitute the mathematical formulation of the problem under consideration.

9. SOLUTION OF THE PROBLEM

Applying Hankel transform to the equation (8.2), we get

$$-\xi_m^2 \overline{T}(\xi_m, z, t) + \frac{d^2 \overline{T}}{dz^2}(\xi_m, z, t) + \overline{\chi}(\xi_m, z, t) = \frac{1}{k}\frac{d\overline{T}}{dt} \tag{9.1}$$

Again applying Marchi-Fasulo transform to above equation, we obtain

$$\frac{d\overline{T}^*}{dt} + kp^2 \overline{T}^* = \Pi \tag{9.2}$$

where

$$p^2 = \xi_m^2 + a_n^2$$

Equation (9.2) is a linear equation whose solution is given by

$$\overline{T}^{*}(\xi_m, n, t) = e^{-kP^2 t}\int_0^t \prod e^{kp^2 t'} dt^1 + Ce^{-kp^2 t} \tag{9.3}$$

Using (8.3), we get

$$C = F^{*}(m, n)$$

Thus we have

$$\overline{T}^{*}(\xi_m, n, t) = e^{-kp^2 t}\left[\int_0^t \prod e^{kp^2 t^1} dt^1 + \overline{F}^{*}(m, n)\right] \tag{9.4}$$

Applying inversion of Marchi-Fasulo transform and Hankel transform to the differential equation (9.4), we get

$$T(r, z, t) = \frac{2}{a^2}\sum_m \sum_n \frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n(z)}{\lambda_n} e^{-kp^2 t} \times \left[\int_0^t \prod e^{kp^2 t'} dt' + \overline{F}^{*}(m, n)\right] \tag{9.5}$$

This is the desired solution of the given problem.

Temperature Distribution (Cooling Process)

T' is the temperature(cooling) of the plate satisfying the differential equation as **Noda et al.** [**11**] is

$$\frac{\partial^2 T'}{\partial r^2} + \frac{1}{r}\frac{\partial T'}{\partial r} + \frac{\partial^2 T'}{\partial z^2} = \frac{1}{k}\frac{\partial T'}{\partial t} \tag{9.6}$$

Subject to initial condition

$$T'(r, z, t)\big|_{t=t_0} = T(r, z, t)\big|_{t=t_0} = T(r, z, t_0) = G(r, z) \tag{9.7}$$

The boundary conditions are

$$M_r(T', 0, 1, a) = g_1(z_, t), \quad -h \le z \le h,\ t > 0 \tag{9.8}$$

$$\left.\begin{aligned} M_z(T', 1, k_1, h) &= f_3(r_, t) \\ M_z(T', 1, k_2, -h) &= f_4(r_, t) \end{aligned}\right\}, \quad 0 \le r \le a,\ t > 0 \tag{9.9}$$

10. SOLUTION OF THE PROBLEM

Applying Hankel transform and Marchi- Fasulo transform one obtains

$$\frac{d\overline{T}'^{*}}{dt} + kp^2 \overline{T'}^{*} = \Pi \tag{10.1}$$

where

$$p^2 = \xi_m^2 + \lambda_n^2$$

and $\Pi = k\left(\frac{P_n(h)}{k_1}\overline{f_3} - \frac{P_n(-h)}{k_2}\overline{f_4} \right)$

Equation (10.1) is a linear equation whose solution is given by

$$\overline{T'}^{*}(\xi_m, n, t) = \Psi + \int_0^t \prod e^{-kp^2(t-t')} dt^1 \tag{10.2}$$

where

$$\Psi = \overline{G}^{*} - \int_0^{t_0} \prod e^{-kp^2(t_0 - t)} dt$$

Applying inversion of Marchi-Fasulo transform and Hankel transform to the differential equation (10.2), we get

$$T'(r,z,t) = \frac{2}{a^2} \sum_m \sum_n \frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2} \frac{P_n(z)}{\lambda_n} e^{-kp^2 t} \times \left(\Psi + \int_0^t \prod e^{-kp^2(t-t')} dt' \right) \tag{10.3}$$

This is the desired solution of the given problem.

Let us assume Love's function L which satisfy condition (8.10) as

$$L(r,z) = \frac{2}{a^2} \sum_m \sum_n \frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2} \frac{P_n(z)}{\lambda_n} \Omega \tag{10.4}$$

where

$$\Omega = e^{-kp^2 t} \left[\int_0^t \prod e^{kp^2 t'} dt' + \overline{F}^{*}(m,n) \right]$$

Using (8.1) and (9.5), we get displacement potential $\phi\, as$

$$\phi = A\sum_m \sum_n \frac{J_0(r\xi_m)}{\left[J_1(a\xi_m)\right]^2}\left[\frac{P_n'\ (z)}{\lambda_n}\Omega + B(t)\right] \tag{10.5}$$

where

$$A = \left(\frac{1+\nu}{1-\nu}\right)\frac{2\alpha_t}{a^2}$$

$$B(t) = \int e^{-kp^2 t}\left(\int_0^t \prod\ e^{-kp^2 t'}dt' + \overline{F}^*(m,n)\right)dt$$

11. DETERMINATION OF DISPLACEMENT FUNCTION

Substituting equations (10.4) and (10.5) in equation (82.6), (8.7) we get

$$u_r = A\sum_m \sum_n \frac{\xi_m J_1(r\xi_m)}{\left[J_1(a\xi_m)\right]^2}\left[\frac{P_n'\ (z)}{\lambda_n}\prod + B(t)\right]$$

$$-\frac{2}{a^2}\sum_m \sum_n \frac{\xi_m J_1(r\xi_m)}{\left[J_1(a\xi_m)\right]^2}\frac{P_n'\ (z)}{\lambda_n}\Omega \tag{11.1}$$

$$u_z = A\sum_m \sum_n \frac{J_0(r\xi_m)}{\left[J_1(a\xi_m)\right]^2}\left[\frac{P_n''\ (z)}{\lambda_n}\Omega + B(t)\right]$$

$$+2(1-\nu)\left[\frac{2}{a^2}\sum_m \sum_n \frac{\xi_m^2[J_1'\ (r\xi_m) + J_1(r\xi_m)]}{\left[J_1(a\xi_m)\right]^2}\frac{P_n(z)}{\lambda_n}\Omega\right]$$

$$+\frac{2(1-2\nu)}{a^2}\sum\sum \frac{J_0(r\xi_m)}{J_1(a\xi_m)^2}\frac{P_n''\ (z)}{\lambda_n}\Omega \tag{11.2}$$

Substituting equations (10.4) and (10.5) in equations (8.9) to (8.12), we obtain

$$\sigma_{rr} = 2G\left\{\frac{2(\nu-1)}{a^2}\sum_m \sum_n \frac{\xi_m^2 J_1'\ (r\xi_m)}{\left[J_1(a\xi_m)\right]^2}\frac{P_n'\ (z)}{\lambda_n}\Omega\right.$$

$$+\frac{2}{a^2 r}\sum_m \sum_n \frac{\xi_m J_1(r\xi_m)}{\left[J_1(a\xi_m)\right]^2}\frac{P_n'\ (z)}{\lambda_n}\Omega + \frac{2}{a^2}\ \sum_m \sum_n \frac{J_0(r\xi_m)}{\left[J_1(a\xi_m)\right]^2}\frac{P_n'\ (z)}{\lambda_n}\Omega$$

$$-\frac{A}{r}\sum_m\sum_n\frac{\xi_m J_1(r\xi_m)}{J_1(a\xi_m)^2}\left[\frac{P_n'(z)}{\lambda_n}\Omega+B(t)\right]$$

$$-A\sum_m\sum_n\frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2}\left[\frac{P_n'(z)}{\lambda_n}\Omega+B(t)\right]\Bigg\}\tag{11.3}$$

$$\sigma_{\theta\theta}=2G\left\{\frac{2\nu}{a^2}\sum_m\sum_n\frac{\xi_m^2 J_1^1(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n^1(z)}{\lambda_n}\Omega\right.$$

$$+\frac{2(\nu-1)}{a^2}\sum\sum\frac{\xi_m J_1(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n^1(z)}{\lambda_n}\mathrm{A}\Omega$$

$$+\frac{2\nu}{a^2}\sum_m\sum_n\frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n^{111}(z)}{\lambda_n}\Omega$$

$$-A\sum_m\sum_n\frac{J_0(r\xi_m)}{J_1(a\xi_m)^2}\left[\frac{P_n^1(z)}{\lambda_n}\Omega+B(t)\right]$$

$$-A\sum_m\sum_n\xi_m^2\frac{J_1^1(r\xi_m)}{[J_1(a\xi_m)]^2}\left[\frac{P_n^1(z)}{\lambda_n}\Omega+B(t)\right]\Bigg\}\tag{11.4}$$

$$\sigma_{zz}=2G\left\{\frac{(2-\nu)}{a^2}\sum\sum\frac{\xi_m^2 J_1^1(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n^1(z)}{\lambda_n}\Omega\right.$$

$$+\frac{2(2-\nu)}{a^2 r}\sum_m\sum_n\frac{\xi_m J_1(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n^1(z)}{\lambda_n}\Omega$$

$$+\frac{2(1-\nu)}{a^2}\sum\sum\frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n^{111}(z)}{\lambda_n}\Omega$$

$$-A\sum\sum\frac{\xi_m^2 J_1^1(r\xi_m)}{[J_1(a\xi_m)]^2}\left[\frac{P_n^1(z)}{\lambda_n}\Omega+B(t)\right]$$

$$-\frac{A}{r}\sum\sum\frac{\xi_m J_1(r\xi_m)}{[J_1(a\xi_m)]^2}\left[\frac{P_n^1(z)}{\lambda_n}\Omega+B(t)\right]\Bigg\}\tag{11.5}$$

$$\sigma_{rz} = 2G\left\{\frac{2(1-\nu)}{a^2}\sum\sum\frac{\xi_m^3 J_1^{11}(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P^1(z)}{\lambda_n}\Omega\right.$$

$$+\frac{2(-\nu)}{a^2}\sum\sum\frac{\xi_m J_1(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n^{11}(z)}{\lambda_n}\Omega$$

$$+\frac{2(1-\nu)}{a^2}\sum\sum\frac{\xi_m}{[J_1(a\xi_m)]^2}\left\{\frac{\xi_m r J_1^1(r\xi_m)}{r_2}\right\}\frac{P_n(z)}{\lambda_n}\Omega$$

$$\left.+A\sum\sum\frac{\xi_m J_1(r\xi_m)}{[J_1(a\xi_m)]^2}\left[\frac{P_n^4(z)}{\lambda_n}\Omega+B(t)\right]\right\} \tag{11.6}$$

where

$$A=\left(\frac{1+\nu}{1-\nu)}\right)\frac{2\alpha_t}{a^2},$$

$$\Omega=e^{-kp^2t}\left[\int_0^t \prod e^{-kp^2t^1}dt^1+\overline{F}^*(m,n)\right],$$

$$B(t)=\int\Omega\ dt$$

12. SPECIAL CASE

Set $F(r,z)=z^2(1-r^2)$ (12.1)

Applying Marchi-Fasulo transform, to (12.1) we obtain

$$\overline{F}(r,n)=(1-r^2)\int_{-h}^{h} z^2 P_n(z)dz$$

$$\overline{F}(r,n)=(1-r^2)\Phi_n\left|\frac{2h^2\sin(a_n h)}{a_n}+\frac{4h\cos(a_n h)}{a_n^{\ 2}}-\frac{4\sin(a_n h)}{a_n^{\ 3}}\right|$$

Where

$$P_n(z)=Q_n\cos(a_n z)-W_n\sin(a_n z)\ ,$$

$$Q_n=a_n(\alpha_1+\alpha_2)\cos(a_n h)+(\beta_1-\beta_2)\sin(a_n h)$$

$$W_n=(\beta_1-\beta_2)\cos(a_n h)+a_n(\alpha_1-\alpha_2)\sin(a_n h)$$

Again on applying Hankel transform, we obtain

$$\overline{F}^{*}(m,n)=\Pi_n\left[\frac{a}{\xi_m}J_1(a\xi_m)-\frac{a(a^2\xi_m^2-4)}{\xi_m^3}J_1(a\xi_m)-\frac{2a^2}{\xi_m^2}J_0(a\xi_m)\right] \tag{12.2}$$

Where

$$\Pi_n=\Phi_n\left[\frac{2h^2\sin(a_nh)}{a_n}+\frac{4h\cos(a_nh)}{a_n{}^2}-\frac{4\sin(a_nh)}{a_n{}^3}\right]$$

And

$$\Phi_n=a_n(\alpha_1+\alpha_2)\cos(a_nh)+(\beta_1-\beta_2)\sin(a_nh).$$

Using equation (7.6.2) in equation (7.3.5) – (7.5.6) one obtains

$$T(r,z,t)=\frac{2}{a^2}\sum_m\sum_n\frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n(z)}{\lambda_n}e^{-kp^2t}\times\left[\int_0^t\prod e^{kp^2t}\,dt^1+\Pi_n\right]$$
$$\times\left(\frac{a}{\xi_m}J_1(a\xi_m)-\frac{a(a^2\xi_m^2-4)}{\xi_m{}^3}J_1(a\xi_m)-\frac{2a^2}{\xi_m^2}J_0(a\xi_m)\right) \tag{12.3}$$

13. NUMERICAL RESULTS

Set $a=2,k=15.9\times10^6,t=1$ second in equation (12.3) we get

$$T(r,z,t)=2\sum_m\sum_n\frac{J_0(r\xi_m)}{[J_1(2\xi_m)]^2}\frac{P_n(z)}{\lambda_n}e^{-(15.9\times10^6)P^2}$$
$$\times\left[\int_0^1\Psi e^{(15.9\times10^6)P^2t^1}\,dt^1+\Pi_n\left(\frac{2}{\xi_m}J_1(2\xi_m)-\frac{8(\xi_m^2-1)}{\xi_m{}^3}J_1(2\xi_m)-\frac{8}{\xi_m^2}J_0(2\xi_m)\right)\right] \tag{13.1}$$

14. CONCLUSION

In both the problems, the temperature distribution, displacement and thermal stresses of a thick circular plate are investigated with known boundary conditions. Finite integral transform techniques are used to obtain numerical results. The results are obtained in terms of Bessel's function in the form of infinite series. Numerical estimated are calculated and depicted graphically.

Any particular cases of special interest can be assigned to the parameters and functions in expressions. The results that are obtained can be useful to the design of structure or machines in engineering applications.

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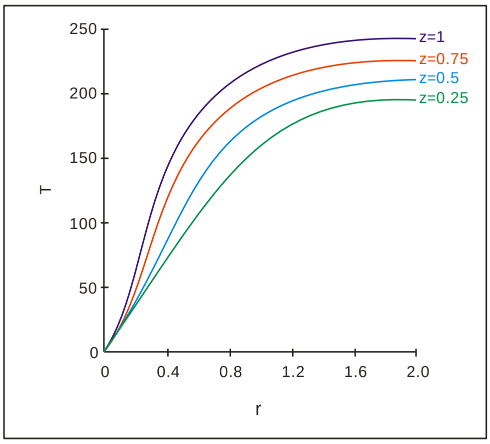
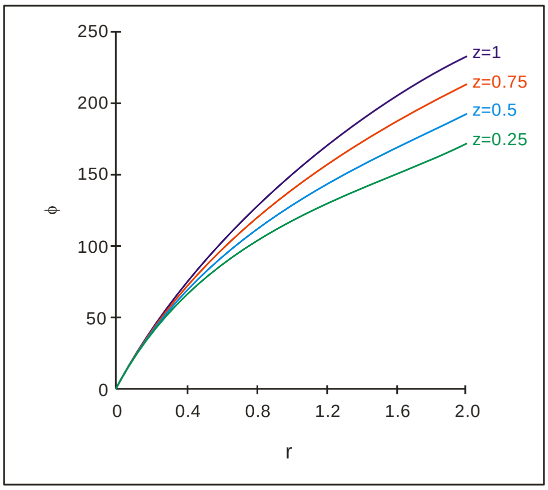
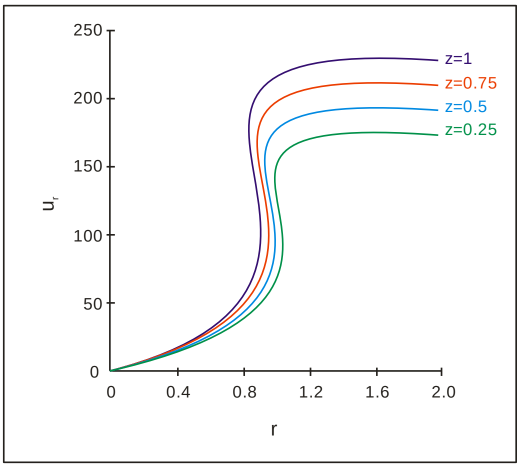


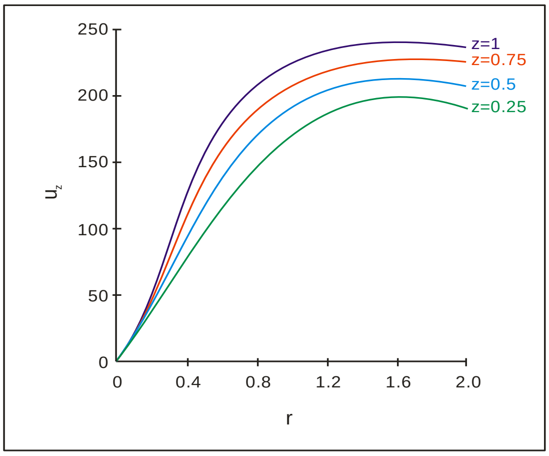
Graph 1: Temperature distribution versus r



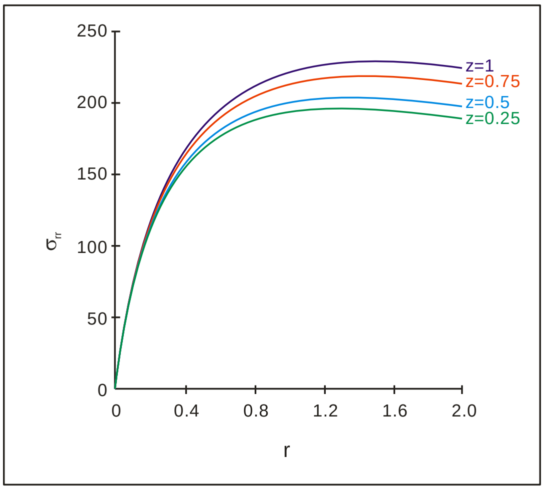
Graph 2: The displacement potential function versus r



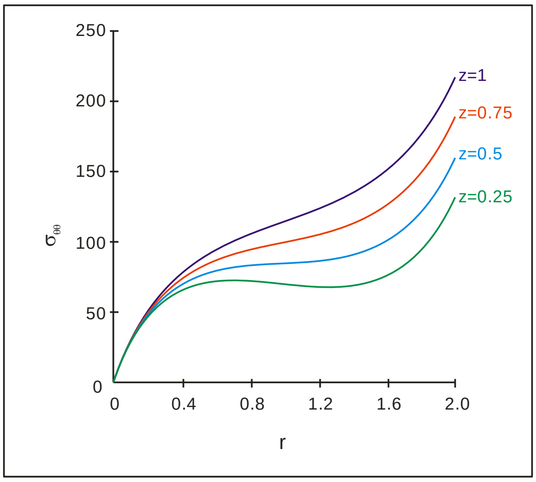
Graph 3: The displacement function versus r



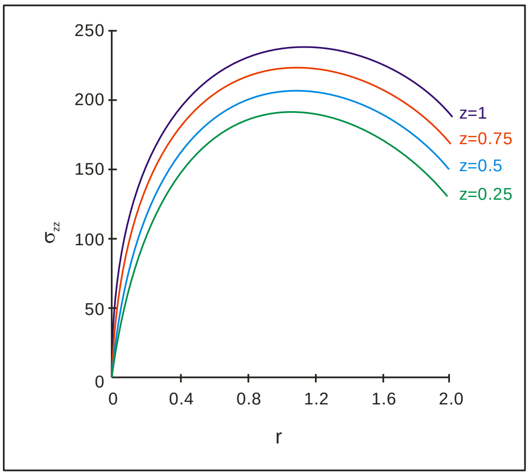
Graph 4: The displacement function versus r



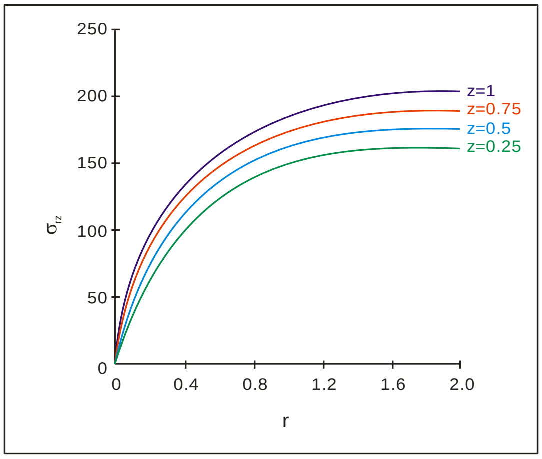
Graph 5: The component of stresses versus r



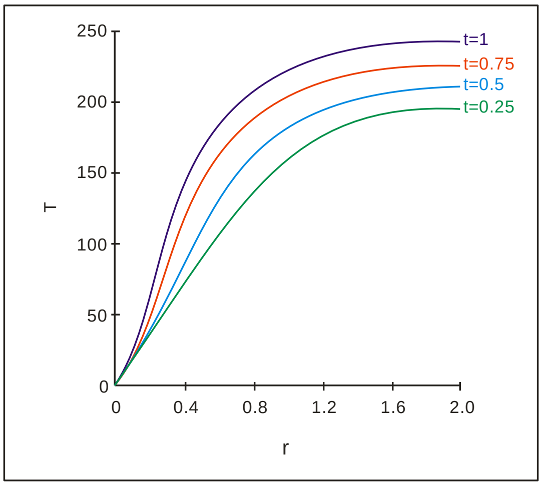
Graph 6: The component of stresses versus r



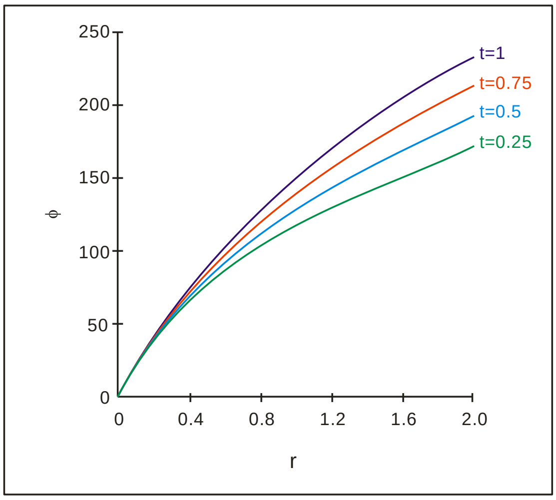
Graph 7: The component of stresses versus r



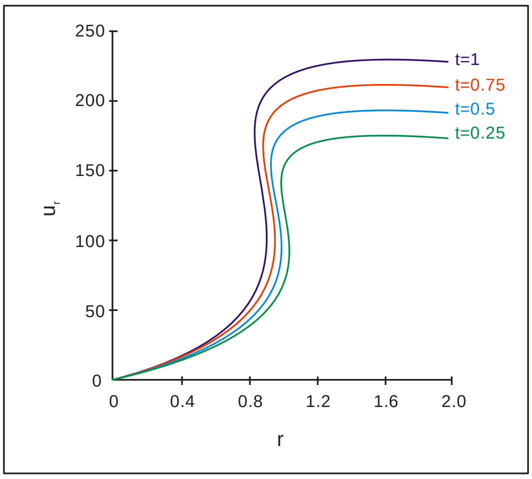
Graph 8: The component of stresses versus r



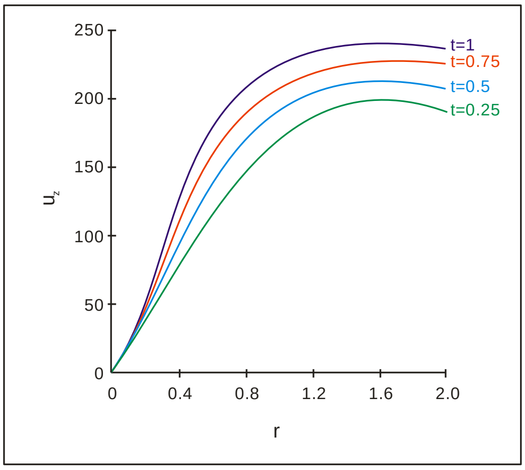
Graph 9: Temperature distribution versus r



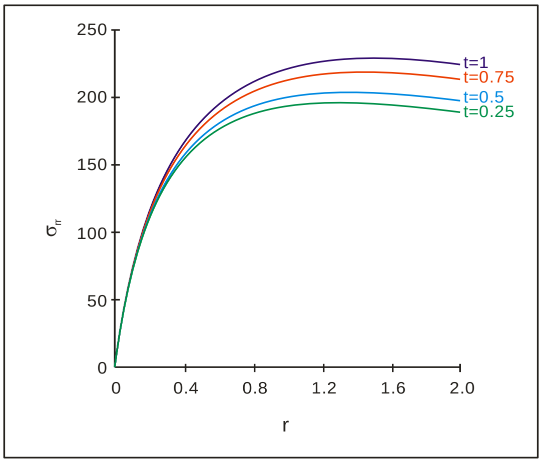
Graph 10: The displacement potential function versus r



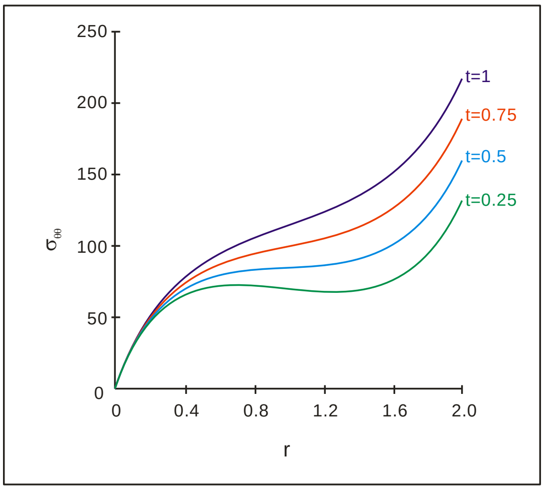
Graph 11: The displacement function versus r



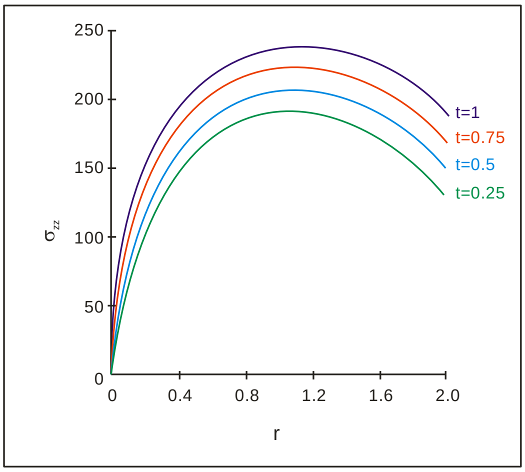
Graph 12: The displacement function versus r



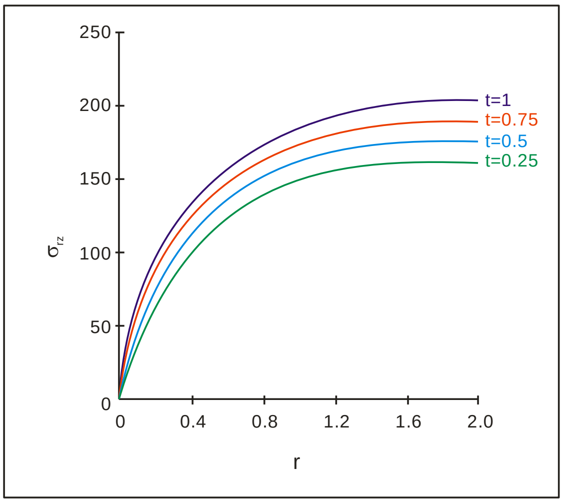
Graph 13: The component of stresses versus r



Graph 14: The component of stresses versus r



Graph 15: The component of stresses versus r



Graph 16: The component of stresses versus r