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THE UNIQUE PROOF OF BEAL'S CONJECTURE BY BABLU REGAR

If $A^x + B^y = C^z$

Where A,B,C,x,y, and z are positive integers with x,y,z>2, then A,B,C have common prime factor.

Equivalently,

There are no solutions to the above equation in positive integers A,B,C,x,y,z with A,B,C being pairwise co-prime and all of x,y,z being greater than 2.

Bablu Regar

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C/o Narottam Kanwaria

1(B.E; M. Tech in Computer science)

[1, 2] Near Raaj Ka Kuaa, Shahbad Ward Baran, Rajasthan-325205, India.

Abstract

The conjecture was formulated in 1993 by Andrew Beal, a banker and amateur mathematician, while investigating generalization of Fermat's last theorem. Since 1997,Beal has offered a monetary prize for a peer-reviewed proof of this conjecture or a counter example. The value of the prize has increased several times and is currently $1 million.

In some venues, this conjecture has occasionally been referred to as a generalized Fermat equation,the Mauldin conjecture, and the Tijdeman-Zagier conjecture.

Keywords: Beal Conjecture , Co-prime Numbers, Two co-prime number Ratio, Common Prime Factor, Divisibility of Numbers.

THE UNIQUE PROOF OF BEAL'S CONJECTURE BY BABLU REGAR [B.E, M.TECH, in Computer Science]:

The Beal Conjecture: If $A^X + B^Y = C^Z$,where A,B C and X,Y,Z are positive integers with x, y, z > 2, then A, B ,C have a common prime factor.

Equivalently,

There are no solutions to the above equation in positive integers A , B , C, X ,Y, Z, with A , B , and C being pairwise co-prime and all of x, y ,z being greater than 2.

PROOF:

Condition 1 : Powers (x, y ,z) such that X < Y < Z where x,y,z> 2

We have BEAL'S CONJECTURE $A^X + B^Y = C^Z$

Putt: A=ut , B=vt , C= wt

⇨ $(ut)^x + (vt)^{x+1} = (wt)^{x+2}$

Putt w=1 , so, $(w)^{x+2}=(1)^{x+2}=1$ where x>2

⇨ $t^x[(u)^x + (v)^{x+1} t] = t^x [t^2]$ (t^x is common both side)

⇨ $t^2- t(v)^{x+1} - (u)^x = 0$ on transposing

we get quadratic equation in 't' , using Sri Dhara Acharaya formula to get values of 't'.

we know that , if quadratic equation $ax^2 + bx + c = 0$ where a≠0 than

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ or $x = 2c/[-b\pm\sqrt{b^2-4ac}]$

⇨ $t^2 - t(v)^{x+1} - (u)^x = 0$ ……………………………….(i)

Putt v=p such that $-v^{x+1} = -p^x (q^x -1)$ ………(ii)

Where $p =(q^x -1)$ …………(iii)

⇨ $-u^x = (-p^x q^x)(p^x)$

⇨ $u^x = (p^{2x})(q^x)= (p^2 q)^x$

⇨ $u = p^2 q$ ……………………………..(iv)

⇨ $t^2 -v^{x+1} t - u^x =0$ using Sri dhara acharya formula to get value of 't'

⇨ $t= [-(-v^{x+1})\pm\sqrt{(-v^{x+1})^2-4(1)(-u^x)}]/2(1)$

putt the value of v and u from equation (ii) &(iv)

we get $t=[p^{x+1}\pm\sqrt{(-p^{x+1})^2+4(p^2q)^x}]/2$

putt value of $q^x = p+1$ from equation (iii)

⇨ $t=[p^x(p)\pm\sqrt{p^{2x}(p^2)+4p^{2x}(p+1)}]/2$

⇨ $t=p^x[p\pm\sqrt{p^2+4(p+1)}]/2$

⇨ $t=p^x[p\pm\sqrt{(p)^2+2(2p)+2^2}]/2$

using identity $(a+b)^2=a^2+b^2+2(a)(b)$

⇨ $t=p^x[p\pm\sqrt{(p+2)^2}]/2$

⇨ $t=p^x[p\pm(p+2)]/2$

⇨ using positive sign we get $t=p^x[p+(p+2)]/2$

⇨ $t=p^x[2p+2]/2$

⇨ $t=(p^x)[(p+1)(2)]/2$

⇨ $t=p^x(p+1)$

putt value of $p+1=q^x$ from equation (iii)

⇨ $t=p^x q^x$ ……………………………………………………….(v)

we get positive value of 't', $t= p^x q^x$

using negative sign we get the value of 't'

⇨ $t=p^x[p-(p+2)]/2$

⇨ $t=p^x[p-p-2]1/2$

⇨ $t=p^x(-2)/2$

⇨ $t=-p^x$ value of 't' is negative , which is not permitted because we have already A, B, C positive value.

⇨ $t=-p^x$ absurd

we have original equation $A^X + B^Y = C^Z$ where x,y ,z>2

⇨ $(ut)^x + (vt)^{x+1} = (wt)^{x+2}$ …………………………….(vi)

Putt all the values of in the form of p,q in in equation (vi).

⇨ $t=p^x q^x$; $v=p$; $u=p^2 q$; $w=1$ from above equations

we get Beal's conjecture generalization equation in form of p,q

⇨ $(ut)^x + (vt)^{x+1} = (wt)^{x+2}$

⇨ $[(p^2q)(p^x q^x)]^x + [(p)(p^x q^x)]^{x+1} = [(1)(p^x q^x)]^{x+2}$

⇨ $[P^{x+2}q^{x+1}]^{x}$ $+[p^{x+1}q^{x}]^{x+1}$ $=[p^{x}\ q^{x}]^{x+2}$

Putt value of $p= q^{x}-1$ from equation (iii)

⇨ $\mathbf{[(q^{x}-1)^{x+2}(q)^{x+1}]^{x} + [(q^{x}-1)^{x+1}(q)^{x}]^{x+1} = [(q^{x}-1)^{x}(q)^{x}]^{x+2}}$ ……(vii)

Where $q\neq 1$; $x>2$

We have equation (vii) Generalization form of Beal's conjecture.

Equation (vii) has infinitely many solutions where the bases shere a positive integer common factor

We have common factor in equation : $[(q^{x}-1)^{x}(q)^{x}]$

We have common prime factor : $(q^{x}-1)$ and (q)

⇨ A, B, C positive value are

⇨ $A = (q)^{x+1}(q^{x}-1)^{x+2}$ …………………(viii)

⇨ $B = (q)^{x}(q^{x}-1)^{x+1}$ …………………………………(ix)

⇨ $C = (q)^{x}(q^{x}-1)^{x}$ …………………………………..(x)

Example: Putt $q=2$ and $x=3$ $x>2$

In equation no. (vii)

We get

⇨ $\mathbf{[(q^{x}-1)^{x+2}(q)^{x+1}]^{x} + [(q^{x}-1)^{x+1}(q)^{x}]^{x+1} = [(q^{x}-1)^{x}(q)^{x}]^{x+2}}$

⇨ Here $q^{x}-1=2^{3}-1=7$ and $q^{x}=2^{3}=8$

⇨ $[(7)^{3+2}(2)^{3+1}]^{3} + [(7)^{3+1}(2)^{3}]^{3+1} = [(7)^{3}(2)^{3}]^{3+2}$

⇨ $[(7)^{5}(2)^{4}]^{3} + [(7)^{4}(2)^{3}]^{4} = [(7)^{3}(2)^{3}]^{5}$

⇨ A, B, C positive value are

⇨ $A= (7)^{5}(2)^{4}$, $B=(7)^{4}(2)^{3}$, $C=(7)^{3}(2)^{3}$

We have common factor = $(7)^{3}(2)^{3}$

We have common prime factor = 7 and 2

We know that

Common factor $\neq$ common prime factor.

We proved that

Beal's conjecture have A, B , C positive integer common prime factor.

Beal's conjecture have not A, B ,C positive integer co-prime values.

Where x , y , z >2

Condition 2 :

Powers (x, y ,z) such that X > Y > Z where x,y,z> 2

We have BEAL'S CONJECTURE $A^X + B^Y = C^Z$

Putt: A=ut , B=vt , C= wt

⇨ $(ut)^{x+2} + (vt)^{x+1} = (wt)^{x}$

Putt u=1 , so, $(u)^{x+2}=(1)^{x+2}=1$ where x>2

⇨ $t^x[t^2 (u)^{x+2} + (v)^{x+1} t] = t^x [w^x]$ (t^x is common both side)

⇨ $t^2 + t(v)^{x+1} - (w)^x = 0$ on transposing

we get quadratic equation in 't'.

putt $(v)^{x+1}=p^x(q^x-1)$
where $p=(q^x-1)$ …………(i)

so, we get $v=p$ …………………………..(ii)

$-(w)^x= - (p^2q)^x$

$W=p^2q$ ………………..(iii)

⇨ $t^2+t(v)^{x+1} -w^x =0$ ……………………..(iv)

we get quadratic equation in 't' , using factorization method to get values of 't'.

we get two values of 't' :

⇨ $t^2+t(v)^{x+1} -w^x =0$

putt v, w values from equation (ii) and (iii) in equation (iv)

we get

⇨ $t^2+t(p)^{x+1}-(p^2q)^x=0$ putt $p^{x+1}=p^x(q^x-1)$

⇨ $t^2+t[p^x(q^x-1)]-p^{2x}q^x=0$

⇨ $t^2+t(p^xq^x)-tp^x-p^{2x}q^x=0$

⇨ $t(t+p^xq^x)-p^x(t+p^xq^x)=0$

we have common $(t+p^xq^x)$

⇨ $(t+p^xq^x)(t-p^x)=0$

If $(t+p^xq^x)=0$

⇨ $t=-p^xq^x$ negative value which is not permitted.

And if $(t-p^x)=0$

⇨ $t= p^x$ positive value is permitted.

Putt all the values : $t=p^x$;$u=1$; $w=p^2q$; $v=p$ in equation

⇨ $(ut)^{x+2} + (vt)^{x+1} = (wt)^{x}$

We get generalization equation

⇨ $[p^x]^{x+2} +[p^{x+1}]^{x+1} =[p^{x+2}q]^x$ where $p=q^x-1$

Form in 'q'

⇨ $\mathbf{[(q^x-1)^x]^{x+2} +[(q^x-1)^{x+1}]^{x+1}=[(q^x-1)^{x+2}\, q]^x}$ …..(v)

where $q\neq1$

$X>2$

Example: putt $q=2$; $x=3$ than $q^x-1=2^3-1=7$

⇨ $[7^3]^5 +[(7)^4]^4 =[7^5(2)]^3$

We have common prime factor = 7

We have common factor = 7^3

Also we have $A= 7^3$; $B=7^4$; $C=2 (7)^5$ positive integers values.

Power $X=5$; $Y=4$;$Z=3$ $X,Y,Z>2$

We know that

Common factor ≠ common prime factor.

We proved that

Beal's conjecture have A, B , C positive integer common prime factor.

Beal's conjecture have not A, B ,C positive integer co-prime values.

Condition 3:

Powers (x, y ,z) such that X < Y > Z where x,y,z> 2

We have BEAL'S CONJECTURE $A^X + B^Y = C^Z$

Putt: A=ut , B=vt , C= wt

⇨ $(ut)^x + (vt)^{x+2} = (wt)^{x+1}$

Putt v=1 , so, $(v)^{x+2}=(1)^{x+2}=1$ where x>2

⇨ $t^x[t^2 (v)^{x+2} - (w)^{x+1} t + u^x] = 0$ (t^x is common both side)

⇨ $t^2 - t (w)^{x+1} + u^x = 0$ on transposing

we get quadratic equation in ' t' .

putt $-(w)^{x+1} = -p^x(q^x+1)$

where $p=(q^x+1)$ …………(i)

so, we get w=p …………………………..(ii)

$(u)^x= (p^2q)^x$

$u=p^2q$ ………………..(iii)

⇨ $t^2-t(w)^{x+1} +u^x =0$ ………………………..(iv)

we get quadratic equation in ' t' , using factorization method to get values of 't'.

we get two values of 't' :

⇨ $t^2-t(w)^{x+1} +u^x =0$

putt u, w values from equation (ii) and (iii) in equation (iv)

we get

⇨ $t^2-t (w)^{x+1}+(p^2q)^x=0$ putt $-w^{x+1}=-p^x(q^x+1)$

⇨ $t^2-t[p^x(q^x+1)]+p^{2x}q^x=0$

⇨ $t^2-t(p^xq^x)-tp^x+p^{2x}q^x=0$

⇨ $t(t-p^xq^x)-p^x(t-p^xq^x)=0$

⇨ we have common $(t-p^xq^x)$

⇨ $(t-p^xq^x)(t-p^x)=0$

If $(t-p^xq^x)=0$

⇨ $t=p^xq^x$ positive value is permitted.

And if $(t-p^x)=0$

⇨ $t=p^x$ positive value is permitted.

Putt all the values : $t=p^x$;$v=1$; $u=p^2q$; $w=p$ in equation

⇨ $(ut)^x + (vt)^{x+2} = (wt)^{x+1}$

We get generalization equation

⇨ $[P^2qp^x]^x + [p^x]^{x+2} = [pp^x]^{x+1}$ where $p=q^x+1$

Form in 'q'

⇨ $\mathbf{[q(q^x+1)^{x+2}]^x + [(q^x+1)^x]^{x+2} = [(q^x+1)^{x+1}]^{x+1}}$ **…..(v)**

where $X>2$

Example: putt $q=2$; $x=3$ than $q^x+1=2^3+1=8+1=9=3^2$

⇨ $[2(3^2)^5]^3 + [(3^2)^3]^5 = [(3^2)^4]^4$

We have common prime factor = 3

We have common factor = 3^6

We supply $t=p^xq^x$

Putt all the values : $t=p^xq^x$;$v=1$; $u=p^2q$; $w=p$ in equation

⇨ $(ut)^x + (vt)^{x+2} = (wt)^{x+1}$

We get generalization equation

⇨ $[P^2qp^xq^x]^x + [p^xq^x]^{x+2} = [pp^xq^x]^{x+1}$ where $p=q^x+1$

Form in 'q'

⇨ $\mathbf{[(q)^{x+1}(q^x+1)^{x+2}]^x + [(q)^x(q^x+1)^x]^{x+2} = [(q)^x(q^x+1)^{x+1}]^{x+1}}$……..(vi)

where $X>2$

Example: putt $q=2$; $x=3$ than $q^x+1=2^3+1=8+1=9=3^2$

⇨ $[2^4(3^2)^5]^3 + [2^3(3^2)^3]^5 = [2^3(3^2)^4]^4$

We have common prime factor = 2 and 3

We have common factor = $(3^6)(2^3)$

Also we have $A= 2^4(3^2)^5$; $B= 2^3(3^2)^3$; $C= 2^3(3^2)^4$ positive integers values.

Power $X=3$; $Y=5$; $Z=4$ $X,Y,Z>2$

We know that

Common factor ≠ common prime factor.

We proved that

Beal's conjecture have A, B , C positive integer common prime factor.

Beal's conjecture have not A, B ,C positive integer co-prime values.

Aliter Method:

The Unique proof of Beal's conjecture is the following conjecture in number theory:

If $A^x + B^y = C^z$

There are no solutionsto the above equation in positive integers A,B,C,x,y,z with A,B,C being pairwise co-prime and all of x,y,z being greater than 2.

Power putt $x=4$; $z=4$ $y=n$ where $y>2$; $y\neq4$

$$A^x + B^y = C^z$$

On transposing

$B^n = C^4 - A^4=(C^2-A^2)(C^2+A^2)=(C+A)(C-A)(C^2+A^2)$

Assume that $C+A=p^n$……………….(i)

$C-A=q^n$…………………(ii)

$C^2+A^2=r^n$………………..(iii)

Where p ,q ,r are co-prime numbers.

Now we have $B^n=(p^n)(q^n)(r^n)$……………..(iv)

We get from equation (i) and (ii)

$C=[p^n+q^n]/2$ and $A=[p^n-q^n]/2$; $CA=[p^{2n}-q^{2n}]/4$

Using factorization formula $[C^2+A^2]=[(C+A)^2-2CA]$

We have $C^2+A^2=r^n$, $(C+A)^2-2CA=r^n$……………(v)

We get $r^n=p^{2n}-[2(p^{2n}-q^{2n})]/4=[p^{2n}+q^{2n}]/2$………(vi)

Putt in equation no.(iv)

$B^n=(p^n)(q^n)(r^n)$

$B^n=p^n\ q^n\ [p^{2n}+q^{2n}]/2=p^nq^nq^{2n}[1+(p/q)^{2n})]/2$

$B^n=p^nq^{3n}[1+(p/q)^{2n})]/2$………………(vii)

Where p, q are prime numbers.

We know that

Two prime number ratio (p/q) always gives non-integer value.

In the equation no. (vii) we have factor $[1+(p/q)^{2n})]/2$ always gives non- integer value that is not permitted in the co-prime factor of 'B'. So we have not co-prime bases in Beal's equation.

If the factor $[1+(p/q)^{2n})]/2$ gives value integer

In only one case: putt p=q in above factor

Putt (p/q)=1

We get $[1+(p/p)^{2n})]/2=[1+(1)^{2n}]/2=[1+1]/2=1$.

Because we have $(1)^{2n}=1$. Then equation will be

$$A^x + B^y = C^z$$

Putt value A, B, C, and x=4 ; y=n ; z=4

We get

⇨ $[(p^n-q^n)/2]^4 + [pq^3]^n = [(p^n+q^n)/2]^4$

Where p=q

Putt p=q

⇨ $[(p^n-p^n)/2]^4 + [pp^3]^n = [(p^n+p^n)/2]^4$

⇨ $[(0)/2]^4 + [p^4]^n = [(2p^n)/2]^4$

⇨ $[(0]^4 + [p^4]^n = [(p)^n]^4$ ……………(viii)

Putt 0=(0)(p) in equation (viii)

⇨ $\mathbf{[(0)(p)]^4 + [p^4]^n = [(p)^n]^4}$…………(ix)

Equation no. (ix) shows that

⇨ A, B, C have common factor 'p'

⇨ A, B, C have not co-prime factor.

If we have: $p \neq q$

$$A^x + B^y = C^z$$

Putt A= $[(p^n-q^n)/2]$; B= $p\,q\,[(p^{2n}+q^{2n})/2]^{1/n}$; C= $[(p^n+q^n)/2]$

We get

⇨ $[(p^n-q^n)/2]^4 + [pq\,\{(p^{2n}+q^{2n})/2\}^{1/n}]^n = [(p^n+q^n)/2]^4$

⇨ Where $n>2$, $n \neq 4$; $p>q$; p ,q prime number

Example:

Putt prime number p=5 ;q=3; and power n=3

⇨ $[(5^3-3^3)/2]^4 \quad +[(5)\ (3)\{(5^6+3^6)/2\}^{1/3}]^3 \quad =[(5^3+3^3)/2]^4$

⇨ $[49]^4 \quad +[(5)(3)\ (37)^{1/3}(221)^{1/3}]^3 \ =[76]^4$

⇨ $[7^2]^4 \quad +[(5)(3)\ (37)^{1/3}(221)^{1/3}]^3 \ =[19\ (2)^2]^4$

⇨ 5764801+27597375 =33362176

⇨ 33362176=33362176 Hence proved.

In above equation have $A=7^2$; $B=(5)(3)\ (37)^{1/3}(221)^{1/3}$; $C=19\ (2)^2$

Bases 'B'have not integer value.

We proved that

There are no solutionsto the above equation in positive integers A,B,C,x,y,z with A,B,C being pairwise co-prime and all of x,y,z being greater than 2.