Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 10 (2017), pp. 7627-7635

http://www.ripublication.com

On Semi Pre Generalized ωα-Closed Sets in Topological Spaces

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Abstract

In the present paper, we introduce the new class of closed sets called semi pre generalized ωα-closed (briefly spgωα-closed) sets in topological spaces which is properly placed between the class of pre-closed sets and the class of gsp-closed sets and obtained some of their properties Also we define the spgωα-open sets and studied some of their properties.

Keywords - Topological spaces, g-closed sets, ωα- open sets, spgωα--closed sets, spgωα-open sets.

AMS Subject Classifications: 54A05, 54A10

1. INTRODUCTION

Levine [10] introduced and investigated the weaker forms of open sets called semi-open sets in 1963. Andrijevic [2] introduced the notion of semi pre-closed set. The concept of generalized closed (briefly g-closed) sets as a generalization of closed set is defined by Levine [11] in 1970. Later on many researchers like Dontchev [7], Sundaram and Sheik John [20] and others introduced and studied the notion of generalized semi pre-closed sets and ω-closed sets in topological spaces respectively. Recently Benchalli et. al. [4] defined and studied the concept of ωα-closed sets in topological spaces.

The aim of this paper is to introduce the new weaker forms of closed sets called spgωα-closed sets and studied the some of their characterizations and also we define the spgωα-open sets and studied some of their properties.

2. PRELIMINARIES

Throughout this paper, the space (X, τ) (or simply X) always means a topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space (X, τ), then cl(A), int(A) and A^C denote the closure of A, the interior of A and the compliment of A in X respectively.

Definition 2.1: A subset A of a topological space X is called

1) regular open [19] if A =int(cl(A)) and regular closed if A = cl(int(A)).
2) semi-open set [10] if A$\subseteq$ cl(int(A)) and semi-closed set if int(cl(A)) $\subseteq$ A.
3) pre-open set [15] if A $\subseteq$ int(cl(A)) and pre-closed set if cl(int(A)) $\subseteq$ A.
4) α-open set [17] if A $\subseteq$ int(cl(int(A))) and α-closed set if cl(int(cl(A))) $\subseteq$ A.
5) semi-preopen set [2] (= β-open [1]) if A $\subseteq$ cl(int(cl(A))) and semi-pre closed set [2] (= β-closed [1]) if int(cl(int(A))) $\subseteq$ A.

The intersection of all semi-closed (resp. semi-open) subsets of (X, τ) containing A is called the semi-closure (resp. semi-kernel) of A and by scl(A) (resp. sker(A)). Also the intersection of all pre-closed (resp. semi-pre-closed and α-closed) subsets of (X, τ) containing A is called the pre-closure (resp. semi-preclosure and α-closure) of A and is denoted by pcl(A) (resp. spcl(A) and α-cl(A)).

Definition 2.2: A subset A of a topological space X is called a

1. generalized closed (briefly g-closed) set [11] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
2. generalized semi-closed (briefly gs-closed) set[3] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
3. α-generalized closed (briefly αg-closed) set [13] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

4. generalized α-closed (briefly gα-closed) set[12] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in X.
5. generalized pre-closed (briefly gp-closed) set [14] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
6. generalized semi-preclosed (briefly gsp-closed) set[7] if $spcl(A) \subseteq U$ whenever $A\subseteq U$ and U is open in X.
7. weakly generalized closed (briefly wg-closed) set [16] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
8. generalized pre-regular-closed (briefly gpr-closed) set[8] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular-open inX.
9. ω-closed [20] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.
10. g^*-closed set [21] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open set in X.
11. α-generalized regular closed (briefly αgr-closed) set[23] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular-open in X.
12. pre g^*-closed (briefly pg^*-closed) set [9] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is ωα-open in X.
13. g^*-pre closed (briefly g^*p-closed) set[22] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open set in X.
14. ωα-closed [4] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is ω-open in X.
15. generalized ωα-closed (briefly gωα-closed) set[5] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is ωα-open set in X.
16. semi generalized ωα-closed (briefly sgωα-closed) set [18] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is ωα-open set in X.

3. spgωα - CLOSED SETS IN TOPOLOGICAL SPACES

In this section, we introduce semi pre generalized ωα-closed (briefly spgωα-closed) sets in topological spaces and obtained some of their properties.

Definition 3.1: A subset A of a topological space (X, τ) is called semi pre generalized ωα- closed (briefly spgωα-closed) set if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is ωα-open in X.

We denote the set of all spgωα-closed sets in (X, τ) by SPGωα(X, τ).

Theorem 3.2: Every closed set is spgωα-closed.

Proof: Let A be a closed and G be an ωα-open set containing A in X. Since A is closed, we have cl(A) = A. But spcl(A) $\subseteq$ cl(A) is always true. So that spcl(A) $\subseteq$ cl(A) $\subseteq$ G. Therefore spcl(A) $\subseteq$ G. Hence A is spgωα-closed set.

The converse of the above theorem need not be true as seen from the following example.

Example 3.3: Let X= {a, b, c} and τ = {X, ϕ, {a}, {b, c}}. Then the set {a, b} is spgωα-closed but not a closed set in X.

Theorem 3.4: Every pre-closed set is spgωα-closed but not conversely.

Proof: Let A be a pre-closed and G be an ωα-open set in X such that A $\subseteq$ G. Since A is pre-closed, we have pcl(A) = A. But spcl(A) $\subseteq$ pcl(A) is always true. So that spcl(A) $\subseteq$ pcl(A) $\subseteq$ G. Therefore spcl(A) $\subseteq$ G. Hence A is spgωα-closed set.

Example 3.5: Let X= {a, b, c} and τ = {X, ϕ, {a}, {b}, {a, b}}. Then the set A = {b} is spgωα-closed but not pre-closed set in X.

Theorem 3.6: Every α-closed set is spgωα-closed but not conversely.

Proof: Since every α-closed set is pre-closed and theorem 3.4, the proof follows.

Example 3.7: In Example 3.3, the subset A = {a, b} is spgωα-closed but not α-closed set in (X, τ).

Theorem 3.8: Every semi-closed set is spgωα-closed but not conversely.

Proof: Let A be semi-closed set and G be an ωα-open set in X such that A $\subseteq$ G. Since A is semi-closed, we have scl(A) = A $\subseteq$ G. But spcl(A) $\subseteq$ scl(A) is always true. Therefore spcl(A) $\subseteq$ G. Hence A is spgωα-closed set.

Example 3.9 : Let X= {a, b, c} and τ = {X, ϕ, {a},{b, c}}. Then the subset A = {a, b} is spgωα-closed but not semi-closed set in X.

Theorem 3.10: Every sgωα-closed set is spgωα-closed but not conversely.

Proof: Let A be a sgωα-closed and G be an ωα-open set in X such that A $\subseteq$ G. As A is sgωα-closed, we have scl(A) $\subseteq$ G. But spcl(A) $\subseteq$ scl(A) is always true. So that spcl(A) $\subseteq$ scl(A) $\subseteq$ G. Therefore spcl(A) $\subseteq$ G. Hence A is spgωα-closed set.

Example 3.11: Let X= {a, b, c, d} and τ = {X, ϕ, {b, c}, {b, c, d}, {a, b, c}}. Then the subset A = {b, d} is spgωα-closed but not sgωα set in X.

Theorem 3.12: Every gωα-closed set is spgωα-closed but not conversely.

Proof: Since every gωα-closed set is sgωα-closed and the Theorem 3.10, the poof follows.

Example 3.13: In Example 3.5, the subset A = {b} is spgωα-closed but not gωα-closed set in (X, τ).

Theorem 3.14: Every spgωα-closed set is gsp-closed but not conversely.

Proof: Let A be a spgωα-closed and G be an open set in X such that A ⊆ G. Since every open set is ωα-open and A is spgωα-closed, we have spcl(A) ⊆ G. Hence A is gsp-closed.

Example 3.15: Let X= {a, b, c} and τ = {X, ϕ, {a}}. Then the subset A = {a, b} is gsp-closed but not spgωα-closed set in X.

Theorem 3.16: Every spgωα-closed set is gs-closed (resp. wg-closed) but not conversely.

Proof: The proof follows from the definitions.

Example 3.17: In Example 3.15, the subset A = {a, c} is gs-closed (resp. wg-closed) but not spgωα-closed set in (X, τ).

Remark 3.18: The concept of spgωα-closed set is independent of the concept of sets namely g-closed, gp-closed, αg-closed, gpr-closed, αgr-closed, g*-closed, g*p-closed, ωα-closed sets as seen from the following examples.

Example 3.19: In Example 3.15, the subset A = {a, b} is g-closed, gp-closed, αg-closed, αgr-closed, ωα-closed but not spgωα-closed set in (X, τ).

Example 3.20: Let X= {a, b, c} and τ = {X, ϕ, {a}, {a, c}}. Then the subset A = {a, b} is g*-closed, g*p-closed, gpr-closed but not spgωα-closed set in X.

Example 3.21: In Example 3.5, the subset A = {b} is spgωα-closed set but not αgr-closed, g*-closed,αg-closed, g*p-closed, gp-closed, gpr-closed, ωα-closed in (X, τ).

Remark 3.22: Union of two spgωα-closed sets need not be a spgωα-closed set as seen from the following example.

Example 3.23: In Example 3.5, the subsets {a} and {b} are spgωα-closed sets but their union {a} ∪ {b} = {a, b} is not a spgωα-closed set in (X, τ).

Theorem 3.24: If a subset A of X is spgωα-closed, then spcl(A) – A does not contain any non-empty ωα-closed set in (X, τ).

Proof: Suppose that A is spgωα-closed set and F be a non-empty ωα-closed subset of spcl(A) – A. Then F ⊆ spcl(A) ∩ (X –F). Since (X – F) is ωα-open and A is spgωα-closed, spcl(A) ⊆ (X – F). Since (X – F) is ωα-open and A is spgωα-closed, Then spcl(A) ⊆ (X – F). Therefore F ⊆ (X – spcl(A)). Then F ⊆ spcl(A) ∩ (X – spcl(A)) = ϕ. That is F = ϕ. Thus spcl(A) – A does not contain any non-empty ωα-closed set in (X, τ).

Theorem 3.25: For an element x ∈ X, the set X – {x} is spgωα-closed or ωα-open.

Proof: Suppose X – {x} is not ωα-open set. Then X is only ωα-open set containing X – {x} and also (X – {x}) ⊆ X. Hence X – {x} is spgωα-closed set in X.

Theorem 3.26: If a subset of a topological space X is spgωα-closed such that A ⊆ B ⊆ spcl(A), then B is also spgωα-closed.

Proof: Let G be an ωα-open set in X such that B ⊆ G, then A ⊆ G. Since A is spgωα-closed, spcl(A) ⊆ G. By hypothesis, spcl(B) ⊆ spcl(spcl(A)) = spcl(A) ⊆ G. Consequently, spcl(B) ⊆ G. Therefore B is also spgωα-closed set in (X, τ).

The converse of the above theorem need not be true as seen from the following example.

Example 3.27: In Example 3.9, the set A = {a} and B = {a, b} such that A and B are spgωα-closed sets but A ⊆ B ⊄ scl(A).

Theorem 3.28: If A is open and gsp-closed set, then A is spgωα-closed set in X.

Proof: Let A be an open and gsp-closed set in X, Let A ⊆ U and U be a ωα-open in X. Now A ⊆ A. By hypothesis, spcl(A) ⊆ A. That is spcl(A) ⊆ U. Hence A is spgωα-closed in X.

Theorem 3.29: If A is ωα-open and spgωα-closed then A is semi-preclosed in X.

Proof: Let A ⊆ A, where A is ωα-open, Then spcl(A) ⊆ A as A is spgωα-closed in X, But A ⊆ spcl(A) is always true. Therefore A = spcl(A) . Hence A is semi-preclosed in X.

Theorem 3.30: If A is a spgωα-closed set in X and $A \subseteq Y \subseteq X$, then A is a spgωα-closed set relative to Y.

Proof: Let $A \subseteq Y \cap G$, where G is an ωα-open set in X. Then $A \subseteq Y$ and $A \subseteq G$. Since A is spgωα-closed set in X, so spcl(A) $\subseteq$ G which implies that Y $\cap$ splc(A) $\subseteq$ Y $\cap$ G. Hence A is spgωα-closed relative to Y.

Theorem 3.31: If A is both open and g-closed in X, then it is spgωα-closed in X.

Proof: Let A be an open and g-closed set in X. Let $A \subseteq U$ and U be a ωα-open set in X. Now $A \subseteq A$, By hypothesis, cl(A) $\subseteq$ A, so that spcl(A) $\subseteq$ cl(A) $\subseteq$ A, that is spcl(A) $\subseteq$ A. Thus spcl(A) $\subseteq$ U, Hence A is spgωα-closed in X.

Remark 3.32: If A is both open and spgωα-closed in X, then A need not be g-closed in general as seen from the following example.

Example 3.33: In Example 3.5, the subset {b} is open and spgωα-closed, but not g-closed.

Definition 3.34 [4]: The intersection of all ωα-open subsets of (X, τ) containing A is called the ωα-kernel of A and is denoted by ωα-ker(A).

Theorem 3.35: A subset A of X is spgωα-closed if and only if spcl(A) ⊆ ωα-ker(A).

Proof: Suppose that A is spgωα-closed, spcl(A) $\subseteq$ U whenever $A \subseteq U$ and U is ωα-open. Let x ∈ spcl(A) and suppose x ∉ ωα-ker(A), then there is a ωα-open set U containing A such that x is not in U. Since A is spgωα-closed, spcl(A) $\subseteq$ U. We have x is not in spcl(A), which is contradiction. Hence x ∈ ωα-ker(A) and so spcl(A) $\subseteq$ ωα-ker(A).

Conversely, let spcl(A) $\subseteq$ ωα-ker(A). If U is any ωα-open set containing A, then ωα-ker(A) $\subseteq$ U. That is spcl(A) $\subseteq$ ωα-ker(A) $\subset$ U. Therefore A is spgωα-closed in X.

Now we introduce the following.

Definition 3.36: A subset A of a topological space (X, τ) is called semi pre generalized ωα-open (briefly spgωα-open) set in X if A^c is spgωα-closed in (X, τ).

Theorem 3.37: Every singleton point set in a space is either spgωα-open or ωα-open in X.

Proof: Let X be a topological space. Let x ∈ X. We prove {x} is either spgωα-open or ωα-open, i.e. X\{x} is either spgωα-closed or ωα-open. From Theorem 3.25, we have X\{x} is spgωα-closed or ωα-open. Thus {x} is either spgωα-open or ωα-open in X.

Theorem 3.38: A subset A of a topological space X is spgωα-open, then F $\subseteq$ spint(A) whenever F $\subseteq$ A and F is ωα-closed in (X, τ).

Proof: Assume that A is spgωα-open. Then A^c is spgωα-closed. Let F be a ωα-closed set in X contained in A. Then F^c is ωα-open set containing A^c in (X, τ). Since A^c is spgωα-closed, this implies that spcl(A) $\subseteq$ F^c. Taking complements on both sides, we have F $\subseteq$ spint(A).

Theorem 3.39: If spgωα spint(A) $\subseteq$ B $\subseteq$ A and if A is a spgωα-open, then B is a αg*s-open in (X, τ).

Proof: We have spint(A) $\subseteq$ B $\subseteq$ A. Then $A^c \subseteq B^c \subseteq$ spcl(A^c) and since A^c is spgωα-closed set. By the Theorem 3.26, B^c is spgωα-closed. Hence B is a spgωα-open.

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