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Cost-Benefit Analysis of Two Non-Identical Units Cold Standby System Subject to Heavy Rain with Partially Operative after Repair

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Abstract

The aim of this paper is to improve the reliability through regenerative points technique. In this paper two non-identical units in the cold standby system are considered. Each unit of the system has three modes via; operative, partially operative and failed. The failure and repair times follow exponential and general time distributions. We transform the basic equations of the proposed model into an integro-differential equations and solve it by semi-markov processes. Various reliability parameters such as mean time to system failure, availability, busy period, number of visit periods by repairman and profit have been computed and analyzed by graphical illustrations.

Keywords: dissimilar cold stand by system, operative, partially operative and failed.

INTRODUCTION

To improve the reliability of the systems many researchers have been worked on the reliability modeling.Osaki [1] and Taneja etl. [2] investigated reliability models of such systems with different failure rate and different repair facilities.Singh and Taneja [3] and Malhotra and Taneja [6] studied comparative study of the systems. Environmental conditions cannot be control which may fluctuate due to changing climate. Therefore, Goel and Sharma[4], Gupta and Goel[5] have obtained reliability measures of cold standby repairable systems operating under different weather conditions. Unprecedented heavy rain that hits Chennai city and other part of Tamilnadu during December 2015. All flight operations at Chennai airport had been partially suspended until further review and a number of flights have been damaged as waters inundated the runway and the tarmac. SpiceJet, Air India and Indigo said the airfield had been closed due to flooding. While considering above facts and practical situations in mind, here reliability measures of a system of non-identical units operating under heavy rain obtained using semi-Markov process and regenerative point technique. In such situation we can improve the reliability of the system to first repair the system become partially operative after the repair of partially operative unit the system become operative.

This paper is organized as follows:

Briefly mentioned all sections and subsections.

Model and transition probabilities and mean sojourn times have been developed and they are given below:

- Mean Times to system failure
- availability analysis
- busy period analysis of the repairman
- busy period analysis of the repairman
- profit analysis
- particular cases

The assumptions for the model are given below:

- both units are non-identical
- Starting the unit is not partially fail but after repair the unit becomes partially operative
- Only one repairman with the system

NOTATIONS

λ_1	: Failure rate from operative unit first
λ_2	: Failure rate from operative unit second
⬭	:up state
□	: failed state
$G_1(t)$, $G_2(t)$, $g_1(t)$ $g_2(t)$	: C.d.f. and P.d.f. of the repair time of unit first and second respectively
Op	: operative unit
cs	: cold standby
fur	: first unit is failure under repair
Fur	: second unit is failure under repair

PouR : partial operative repair is continuing on the unit

FuR : repair is continuing on the unit

Fwr : waiting for repair

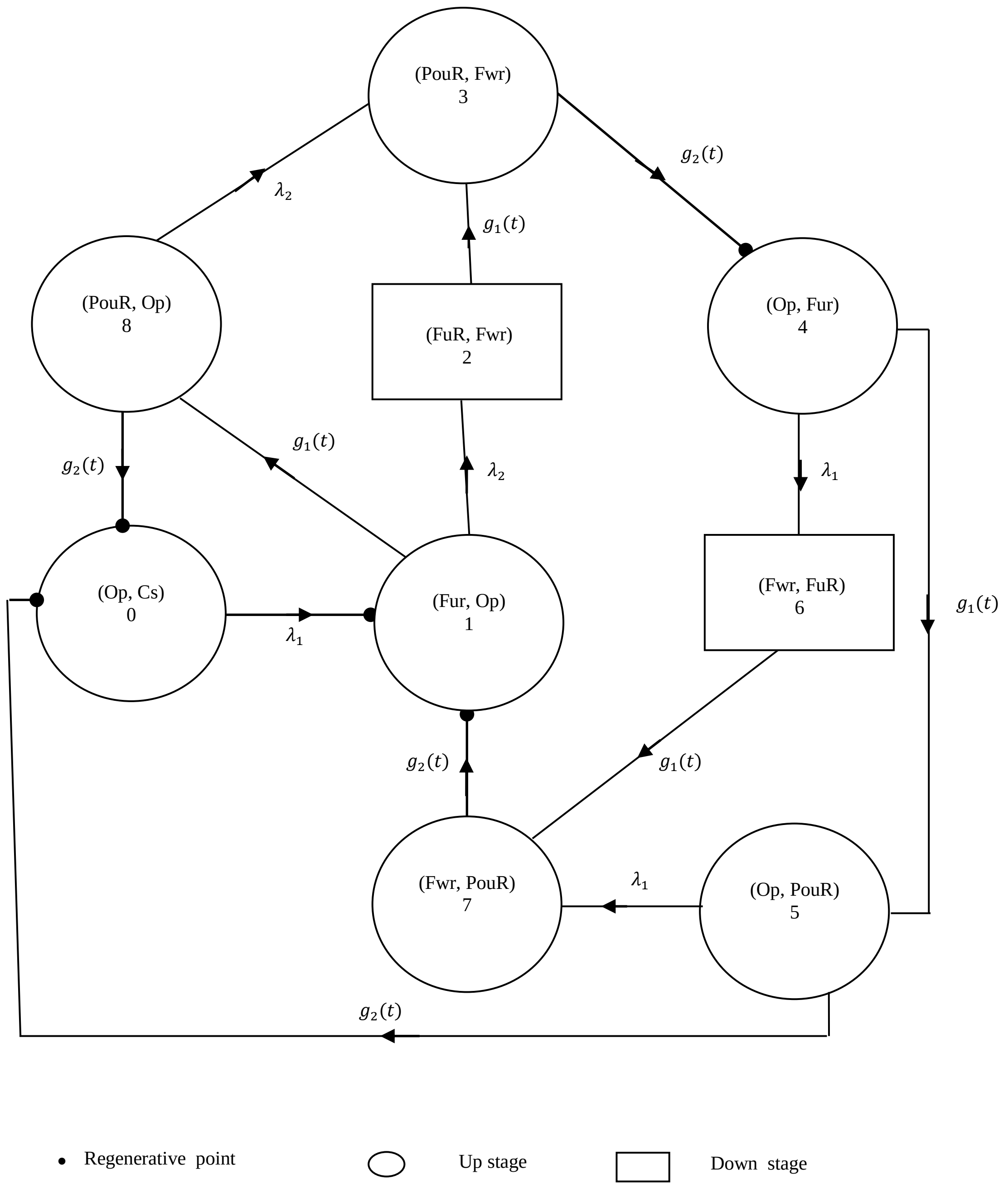

Figure 1: State Transition Diagram

TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

The epochs of entry into states 0, 1 and 4 regenerative points and thus these are regenerative states. States 2, 3, 5, 6, 7, and 8 are non regenerative states.

The non zero elements p_{ij} of the transition probability for the system are found out as

$p_{ij}= \lim_{s\to 0} q_{ij}^{*}(s)$

$p_{01}=1$, $p_{18}=g_1^*(\lambda_2)$, $p_{12}=(1- g_1^*(\lambda_2))$

$p_{23}=g_1^*(0)$, $p_{34}=g_2^*(0)$, $p_{45}=g_1^*(\lambda_1)$, $p_{46}=(1- g_1^*(\lambda_1))$

$p_{50}=g_2^*(\lambda_1)$, $p_{57}=(1- g_2^*(\lambda_1))$, $p_{80}=g_2^*(\lambda_2)$, $p_{83}=(1- g_2^*(\lambda_2))$

$p_{67}=g_1^*(0)$, $p_{71}=g_2^*(0)$

$p_{10}^{8}=g_1^*(\lambda_2)\, g_2^*(\lambda_2)$, $p_{14}^{83}=g_1^*(\lambda_2)\,(1-g_2^*(\lambda_2))$, $p_{14}^{23}=(1-g_1^*(\lambda_2))$

$p_{40}^{5}=g_1^*(\lambda_1)\, g_2^*(\lambda_1)$, $p_{41}^{57}=g_1^*(\lambda_1)(1- g_2^*(\lambda_1))$, $p_{41}^{67}=(1-g_1^*(\lambda_2))$

For the transition probabilities, it can be verified that

$p_{01}=p_{10}^{8}+p_{14}^{83}+p_{14}^{23}=p_{40}^{5}+p_{41}^{67}+p_{41}^{57}=p_{12}+p_{18}=p_{45}+p_{46}=p_{50}+p_{57}=p_{80}+p_{83}=1$ and

$p_{23}=p_{34}=p_{67}=p_{71}=1$

The mean sojourn times μ_i in state 'i' are given by

$\mu_0=\frac{1}{\lambda_1}$, $\mu_1=\frac{1}{\lambda_2}(1-g_1^*(\lambda_2))$, $\mu_2=-g_1^{*'}(0)=\mu_6$, $\mu_3=-g_1^{*'}(0)=\mu_7$,

$\mu_4=\frac{1}{\lambda_1}(1-g_1^*(\lambda_1))$, $\mu_5=\frac{1}{\lambda_1}(1-g_1^*(\lambda_1))$,

$\mu_8=\frac{1}{\lambda_2}(1-g_1^*(\lambda_2))$

The unconditional mean time taken by the system to transit from any state 'i' when time is counted from epoch at entrance into state 'i' is stated as:

$m_{ij}=\int_0^\infty t\, q_{ij}(t)dt=-q_{ij}^{*\prime}(0)$

Thus,

$m_{01}=\mu_0$, $m_{12}+m_{18}=\mu_1$, $m_{45}+m_{46}=\mu_4$, $m_{50}+m_{57}=\mu_5$

$m_{80}+m_{83}=\mu_8$, $m_{23}=m_{67}=\mu_2$, $m_{34}=m_{71}=\mu_3$

$m^{8}{}_{10}+m_{14}^{83}+m_{14}^{23}=k_1=\mu_1+p_{18}(\mu_8+p_{83}\mu_8)+p_{12}(\mu_2+\mu_3)$

$m^{5}{}_{40}+m_{41}^{67}+m_{41}^{57}=k_2=\mu_4+p_{45}(\mu_5+p_{57}\mu_7)+p_{46}(\mu_2+\mu_3)$

$m_{12}+m_{10}^{8}+m_{14}^{83}=k_3=\mu_1+p_{18}\mu_8+p_{18}p_{83}\mu_3$

$m_{46}+m_{41}^{57}+m_{40}^{5}=k_4=\mu_4+p_{45}\mu_5+p_{45}p_{57}\mu_7$

ANALYSIS OF MEAN TIME TO SYSTEM FAILURE
Regarding the failed states as absorbing states and applying the arguments used for regenerative processes, the following recursive relation for $\phi_i(t)$ are obtained

$\phi_0(t) = Q_{01}(t)Ⓢ\phi_1(t)$

$\phi_1(t) = Q_{12}(t) + Q_{10}^{(8)}(t)Ⓢ\phi_0(t) + Q_{14}^{(83)}(t)Ⓢ\phi_4(t)$

$\phi_4(t) = Q_{46}(t) + Q_{40}^{(5)}(t)Ⓢ\phi_0(t) + Q_{41}^{(57)}(t)Ⓢ\phi_1(t)$

Taking Laplace- Stieltjes Transforms (L.S.T) of these relations and solving them by Cramer's rule for $\phi_0^{**}(s)$.

Now, the mean time to system failure (MTSF) when the system starts from the state '0' is

$$T_0 = \lim_{s\to 0} \frac{1-Q_0^{**}(s)}{s} = \lim_{s\to 0} \frac{1-\frac{N(S)}{D(S)}}{s} = \lim_{s\to 0} \frac{D(s) - N(s)}{sD(s)} = \frac{D'(0)-N'(0)}{D(0)} = \frac{N}{D}$$

Where N $= k_3 + \mu_0(p_{10}^{8} + p_{12} + p_{40}^{5}p_{14}^{53} + p_{46}p_{14}^{83}) + k_4p_{14}^{83}$

And

D$=1 - p_{41}^{57}p_{14}^{83} - p_{40}^{5}p_{14}^{83} - p_{10}^{8}$

AVAILABILITY ANALYSIS
The availability of a system is defined as the probability that the system is operating and provides service when requested.

Using the probabilistic argument and $A_i(t)$ as the probability of unit entering into up state at time t, given that the unit entered is regenerative state i at t=0, the following recursive relation are obtained.

$A_0(t) = M_0(t) + q_{01}(t) © A_1(t)$

$A_1(t) = M_1(t) + q_{10}^{(8)}(t) © A_0(t) + (q_{14}^{(23)}(t) + q_{14}^{(83)}(t)) © A_4(t)$

$A_4(t) = M_4(t) + q_{40}^{(5)}(t) © A_0(t) + (q_{41}^{(67)}(t) + q_{41}^{(57)}(t)) © A_1(t)$

Where $M_0(t) = e^{-\lambda_1 t}dt$, $M_1(t) = e^{-\lambda_2 t}\overline{G_1}(t)dt$

$M_4(t) = e^{-\lambda_1 t}\overline{G_1}(t)dt$

Taking Laplace transforms(L.T.) of these relations and solving them by crammer rule for A_0^*(s), we obtain

$A_0^*(s)=\frac{N_1(s)}{D_1(s)}$

Where,

$N_1(s)$ = M_0^*(s) - M_0^*(s) $q_{41}^{*(67)}$(s) $q_{41}^{*(23)}$(s) - M_0^*(s)$q_{41}^{*(67)}$(s)$q_{14}^{*(83)}$(s) - M_0^*(s) $q_{41}^{*(57)}$(s) $q_{14}^{*(23)}$(s) - M_0^*(s)$q_{41}^{*(57)}$(s) $q_{14}^{*(83)}$(s)+ M_1^*(s)q_{01}^*(s) + M_4^*(s) q_{01}^*(s)$q_{14}^{*(23)}$(s) +M_4^*(s) M_{01}^*(s) $q_{14}^{*(83)}$(s) D_1(s) = 1-$q_{41}^{*(67)}$(s)$q_{14}^{*(23)}$(s)- q_{14}^*(s)$q_{41}^{*(67)}$(s)

\- $q_{14}^{*(23)}$(s) $q_{41}^{*(57)}$(s)- $q_{14}^{*(83)}$(s) $q_{41}^{*(57)}$(s)-q_{01}^*(s) $q_{10}^{*(8)}$(s)$-q_{01}^*$(s)$q_{40}^{*(5)}$(s) $q_{14}^{*(23)}$(s)

\- q_{01}^*(s)$q_{40}^{*(5)}$(s) $q_{14}^{*(83)}$(s)

The steady state availability of the system is given by

$A_0= \lim_{s\to 0}(sA_0^*(s)) =\lim_{s\to 0}(s\frac{N_1(s)}{D_1(s)}) = \frac{N_1(0)}{D_1'(0)}= \frac{N_1}{D_1}$

$N_1 =\mu_0(p_{10}^{(8)} + p_{40}^{(5)} + p_{40}^{(5)}p_{10}^{(8)})+ \mu_1 + \mu_1(1 - p_{40}^{(5)})$

$D_1=k_1+k_2(p_{21}^{(3)}+p_{21}^{(3)})+ \mu_0(p_{10}^{(8)} + p_{40}^{(5)}p_{14}^{(23)} + p_{40}^{(5)}p_{14}^{(83)})$

Where, k_1 and k_2 is already specified.

BUSY PERIOD ANALYSIS OF THE REPAIR MAN

B_i(t) = Probability that the repair man is busy at instant t, given that the system entered regenerative state i at t=0,

B_0(t) = q_{01}(t) © B_1(t)

B_1(t) = W_1(t)+ $q_{10}^{(8)}$ (t) © B_0(t)+ ($q_{14}^{(23)}$ (t) +$q_{14}^{(83)}$ (t))© B_4(t)

B_4(t) = W_4(t)+ $q_{40}^{(5)}$ (t) © B_0(t)+ ($q_{41}^{(67)}$ (t) +$q_{41}^{(57)}$ (t))© B_1(t)

Where W_1(t)= $e^{-\lambda_2 t}\overline{G_1}$(t)dt+($\lambda_2$ $e^{-\lambda_2 t}$©1) $\overline{G_1}$(t)dt

W_4(t)= $e^{-\lambda_1 t}\overline{G_1}$(t)dt+($\lambda_1$ $e^{-\lambda_1 t}$©1) $\overline{G_1}$(t)dt

Taking Laplace transforms (L.T.) of these relations and solving them by applying crammer's rule for B_0^*(s), we obtain

$$B_0^*(s)=\frac{N_2(s)}{D_1(s)}$$

Where

$$N_2(s)=q_{01}^*(s)\,W_1^*(s)(1+(q_{14}^{*(23)}(s)+q_{14}^{*(83)}(s))$$

And $D_1(s)$ is already specified.

In steady state, the total fraction of time for which the system is under repair is given by $B_0=\lim_{s\to 0}(sB_0^*(s))=\lim_{s\to 0}(s\frac{N_2(s)}{D_1(s)})=\frac{N_2(0)}{D_1'(0)}=\frac{N_2}{D_1}$

Where,

$$N_2=k_5(1+(q_{14}^{(23)}+q_{14}^{(83)})$$

Where$k_5=W_1^*(0)=W_4^*(0)$ and D_1 is already specified.

EXPECTED NUMBER OF VISITS BY THE REPAIR MAN

We define

$V_0(t)$ = expected number of visits by the repair man in (0,t], given that the system started from the regenerative state i at t=0

$$V_0(t)=Q_{01}(t)\,Ⓢ\,(1+V_1(t))$$

$$V_1(t)=Q_{10}^{(8)}(t)\,Ⓢ\,V_0(t)+(Q_{14}^{(23)}(t)+Q_{14}^{(83)}(t))\,Ⓢ\,V_4(t)$$

$$V_4(t)=Q_{40}^{(5)}(t)\,Ⓢ\,V_0(t)+(Q_{41}^{(67)}(t)+Q_{41}^{(57)}(t))\,Ⓢ\,V_1(t)$$

Taking Laplace – Stieltjes Transforms (L.S.T.) of these relations and solving them by applying crammer rule for $V_0^{**}(s)$, we obtain

$$V_0^{**}(s)=\frac{N_3(s)}{D_1(s)}$$

Where

$$N_3(s)=Q_{01}^{**}(s)-Q_{01}^{**}(s)\,Q_{41}^{**(67)}(s)\,Q_{14}^{**(23)}(s)-Q_{01}^{**}(s)\,Q_{41}^{**(67)}(s)\,Q_{14}^{**(83)}(s)-Q_{01}^{**}(s)\,Q_{41}^{**(57)}(s)\,Q_{14}^{**(23)}(s)-Q_{01}^{**}(s)\,Q_{41}^{**(57)}(s)\,Q_{14}^{**(83)}(s)$$

And $D_1(\mathrm{s})$ is already specified.

$V_0 = \lim_{s\to 0}(sV_0^*(s)) = \lim_{s\to 0}(s\frac{N_3(s)}{D_1(s)}) = \frac{N_3(0)}{D_1'(0)} = \frac{N_3}{D_1}$

Where,

$N_3 = p_{10}^{(8)} + p_{40}^{(5)} + p_{40}^{(5)} p_{10}^{(8)}$

And D_1 is already specified.

COST- BENEFIT ANALYSIS

The expected total profit incurred to the system in steady state is given by

$P = C_0A_0 - C_1B_0 - C_2V_0$

Where

C_0= revenue per unit up time of the system

C_1= cost per unit time for which expert repairman is busy in repairing in the field unit

C_2= cost per visit of expert.

GRAPHICAL STUDY

Let us assume that the repair rate are exponentially distributed as under :

$g_1(t) = \alpha_1 e^{-\alpha_1 t}$ and $g_2(t) = \alpha_2 e^{-\alpha_2 t}$

$p_{01} = 1$, $p_{18} = \alpha_1/(\lambda_2 + \alpha_1)$,

$p_{12} = \lambda_2/(\lambda_2 + \alpha_1)$, $p_{23} = 1$, $p_{34}=1$, $p_{45} = \alpha_1/(\lambda_1 + \alpha_1)$,

$p_{46} = \lambda_1/(\lambda_1 + \alpha_1)$, $p_{50} = \alpha_2/(\lambda_1 + \alpha_2)$, $p_{57} = \lambda_1/(\lambda_1 + \alpha_2)$

$p_{67}=1$, $p_{71}=1$, $p_{80} = \alpha_1/(\lambda_1 + \alpha_1)$, $p_{83} = \alpha_1/(\lambda_1 + \alpha_1)$,

$p_{10}^{(8)} = \frac{\alpha_1}{\lambda_2+\alpha_1}.\alpha_2/(\lambda_2 + \alpha_2)$, $p_{14}^{(83)} = \frac{\alpha_1}{\lambda_2+\alpha_1}.\alpha_2/(\lambda_2 + \alpha_2)$, $p_{14}^{(23)} = \frac{\lambda_2}{\lambda_2+\alpha_1}$ $p_{41}^{(67)} = \frac{\lambda_1}{\lambda_1+\alpha_1}$
$p_{41}^{(57)} = \frac{\alpha_1}{\lambda_1+\alpha_1}.\lambda_1/(\lambda_1 + \alpha_2)$, $p_{40}^{(5)} = \frac{\alpha_1}{\lambda_1+\alpha_1}.\alpha_2/(\lambda_1 + \alpha_2)$, $\mu_1 = 1/\alpha_1$, $\mu_1 = 1/(\lambda_2 + \alpha_1)$,
$\mu_2 = 1/\alpha_1$, $\mu_3 = 1/\alpha_2$, $\mu_4 = 1/(\lambda_1 + \alpha_1)$, $\mu_5 = 1/(\lambda_1 + \alpha_2)$, $\mu_6 = 1/\alpha_1$, $\mu_7 = 1/\alpha_2$, $\mu_8 = 1/(\lambda_2 + \alpha_2)$.

GRAPHICAL INTERPRETATION

By giving some numerical values to the parameters involved , various graphs has been plotted using particular case and the following interpretations have been drawn.

Fig. 2 Shows the behaviour operative unit for different values of repair rate (α_1).From the graph, we can see of MTSF with respect to failure rate (λ_1) of the that the MTSF decreases as λ_1 increases, but has higher values for higher values of α_1.

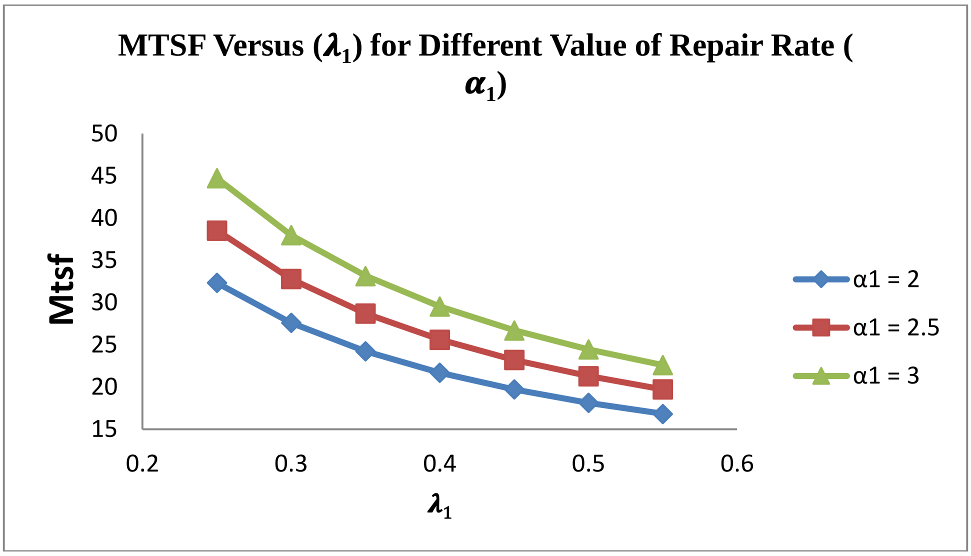

Fig. 2

Fig. 3 Shows the behaviour of availability (A_0) with respect to failure rate (λ_1) of the operative unit for different values of repair rate (α_1).From the graph, we can see that the A_0 decreases as λ_1 increases, but has higher values for higher values of α_1.

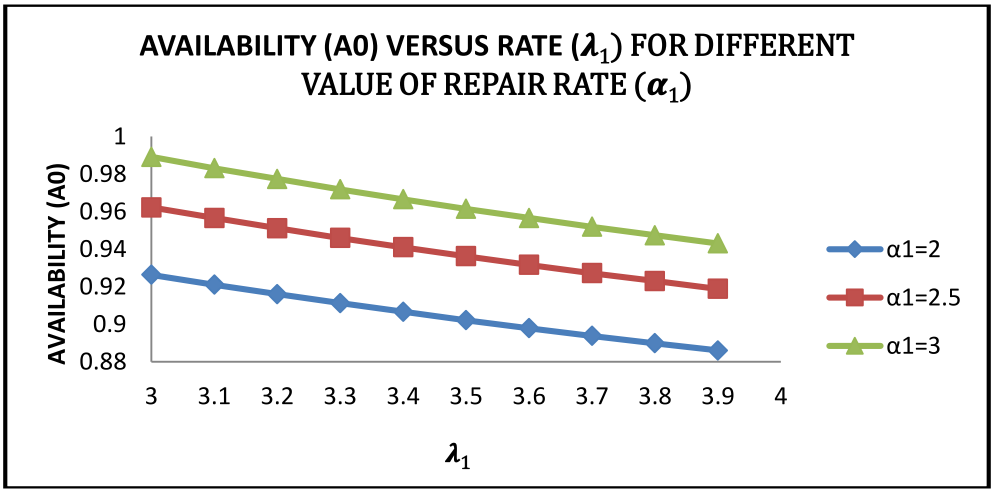

Fig. 3

Fig.4 Shows the behavior of profit (P) with respect to cost per unit visit up revenue (C_0) for different values of cost per visit (C_2) of the repairman.It reaveals that profit (P) increase with increase in the values of C_0,but it gets lowered for higher of C_2.

Following conclusions can also be drawn from the graph.

(i) For C_2=10,the profit (P) is > or= or< 0 according as C_0 is > or= or< 19.5591.Thus, the price of profit should be fixed in such a way so that the revenue is atleast 19.5591.

(ii) For C_2=30,the profit (P_2) is > or= or< 0 according as C_0 is > or= or< 58.675.Thus, the price of profit should be fixed in such a way so that the revenue is atleast 58.675.

(iii) For C_2=50,the profit (P_2) is > or= or< 0 according as C_0 is > or= or< 97.7958.Thus, the price of profit should be fixed in such a way so that the revenue is atleast 97.7958.

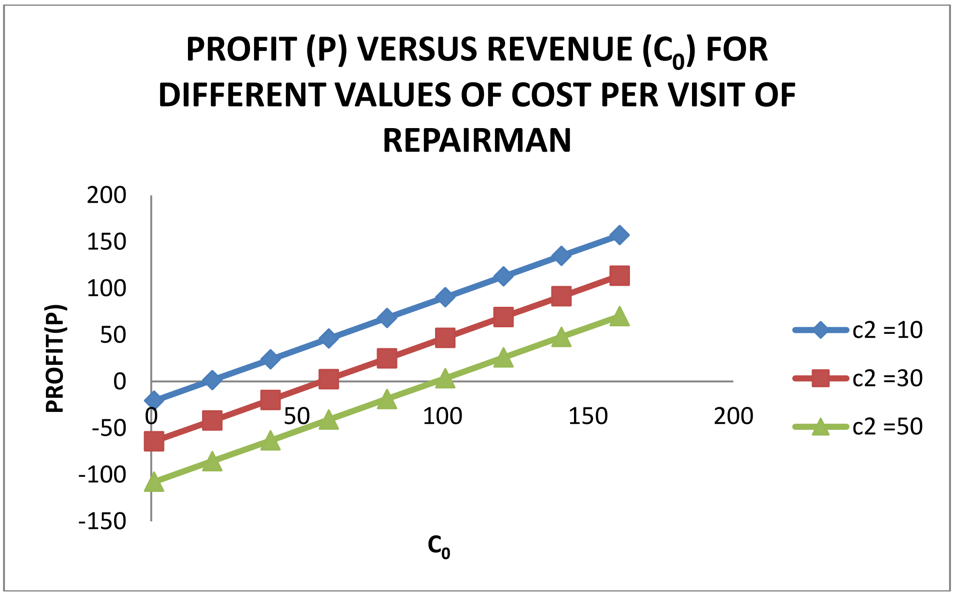

Figure 4

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