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# Metric Dimension of Some Path Related Graphs[1]

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**Abstract**

In this paper, we find an upper bound of the metric dimension of power of paths and complement of paths. Also, we determine the metric dimension for $P_n^2$, $P_n^3$, $P_n^4$ where $P_n$ is a path of length n. Finally, we investigate the metric dimension of certain permutation of paths of odd order.

**AMS subject classification:**
**Keywords:**

## 1. Introduction

Throughout this paper we will consider finite simple connected graphs. For a graph $G = (V, E)$, the distance between two vertices $u, v \in V$ is the length of a shortest $u - v$ path in $G$. Let $W = \{w_1, w_2, \ldots, w_k\}$ be an ordered subset of V and let $v \in V$. We can associate with $v$ an ordered k-tuple that gives the distance from $v$ to each of the vertices in $W$, denoted by $d(v, W) = (d(v, w_1), \ldots, d(v, w_k))$. The set $W$ is called a resolving set of $G$ if for every two distinct vertices $u, v \in V$, we have $d(u, W) \neq d(v, W)$. A basis of $G$ is a resolving set of $G$ with minimum cardinality, and the metric dimension of $G$ refers to it's cardinality and is denoted by $dim(G)$.

The metric dimension was first introduced in 1975 by Slater [7] and studied independently by Harary and Melter [3] in 1976. Khuller et al. [5] studied the metric dimension

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motivated by the robot navigation in a graph space. A resolving set for a graph corresponds to the presence of distinctively labelled (landmark) nodes in the graph. It is assumed that a robot can detect the distance to each node of the landmarks, and hence uniquely determine its location in the graph.

Gerey and Johnson [4] showed that determining the metric dimension of an arbitrary graph is an NP-complete problem. Melter and Tomescu [6] studied the metric dimension problem for grid graph. Caceres et al. [1] studied the metric dimension of graphs which obtained by the cartesian product of two or more graphs. Chartrand et al. [2] find all graphs of order n having metric dimension 1, n-2 or n-1. From their work, we mention the following result that we will use later.

**Theorem 1.1.** **[2]** Let $G$ be a connected graph of order $n \geq 2$. Then $dim(G) = 1$ if and only if $G = P_n$.

In this paper, we study the metric dimension of power of paths and complement of paths. Also, we determine the metric dimension of certain permutation of paths of odd order.

## 2. Metric dimension of power of paths

In this section we find an upper bound of the metric dimension of power of paths. Then we determine the metric dimension for some powers of paths. We start with the following lemma that gives an upper bound for the metric dimension of power of paths.

**Lemma 2.1.** Let $G = P_n^k$ where $n \geq k+1$. Then $dim(G) \leq k$.

*Proof.* Suppose that $v(P_n) = \{v_0, v_1, \dots, v_{n-1}\}$ and take $W = \{v_0, v_1, \dots, , v_{k-1}\}$. We want to show that $W$ is a resolving set by showing that $d(v_i, W) \neq d(v_j, W)$ for all $i \neq j$.

Suppose that $v_i$ and $v_j$ are two distinct vertices. Using the division algorithm there exists integers $q_i, q_j, r_i, r_j$ with $i = q_i k + r_i$, $j = q_j k + r_j$, and $0 \leq r_i, r_j \leq k-1$.
Observe that
if $r_i = 0$, then $d(v_i, W) = (q_i, q_i, \dots, q_i)$,
if $1 \leq r_i \leq k-1$, then $d(v_i, W) = (q_i+1, q_i+1, \dots, q_i+1, q_i, q_i, \dots, q_i)$, where the last $q_i + 1$ in the $r_i^{th}$ position.
If $r_j = 0$, then $d(v_j, W) = (q_j, q_j, \dots, q_j)$.
If $1 \leq r_j \leq k-1$, then $d(v_j, W) = (q_j+1, q_j+1, \dots, q_j+1, q_j, q_j, \dots, q_j)$, where the last $q_j + 1$ in the $r_j^{th}$ position.
Since $v_i \neq v_j$, then the $q_i$ and $q_j$ are distinct or $r_i$ and $r_j$ are distinct. Hence, $d(v_i, W) \neq d(v_j, W)$, for all $i \neq j$. Thus, $dim(G) \leq k$. ■

We give the following example to explain Lemma 2.1.

**Example 2.2.** Consider $G = P_{14}^3$. The vertices of $P_{14}^3$ are $\{v_0, v_1, \dots, v_{13}\}$. Take $W = \{v_0, v_1, v_2\}$.

Observe that
$d(v_0, W) = (0, 1, 1)$,
$d(v_1, W) = (1, 0, 1)$,
$d(v_2, W) = (1, 1, 0)$,
$d(v_3, W) = (1, 1, 1)$,
$d(v_4, W) = (2, 1, 1)$,
$d(v_5, W) = (2, 2, 1)$,
$d(v_6, W) = (2, 2, 2)$,
$d(v_7, W) = (3, 2, 2)$,
$d(v_8, W) = (3, 3, 2)$,
$d(v_9, W) = (3, 3, 3)$,
$d(v_{10}, W) = (4, 3, 3)$,
$d(v_{11}, W) = (4, 4, 3)$,
$d(v_{12}, W) = (4, 4, 4)$ and
$d(v_{13}, W) = (5, 4, 4)$.

Now, we find the metric dimension of $P_n^2$, $P_n^3$, $P_n^4$.

**Lemma 2.3.** Let $G = P_n^2, n > 3$. Then $dim(G) = 2$.

*Proof.* Let $P_n$ be the graph with $v(P_n) = \{v_0, v_1, \ldots, v_{n-1}\}$. Take $W$ =$\{v_0, v_1\}$. According to Lemma 2.1, W is a resolving set and $dim(G) \leq 2$. Since $G$ is not a path, then according to Theorem 1.1 we get $dim(G) = 2$. ■

**Lemma 2.4.** Let $G = P_n^3, n > 4$. Then $dim(G) = 3$.

*Proof.* By Lemma 2.1 $dim(G) \leq 3$. We want to show $dim(G) > 2$. Assume that $W_1 = \{v_i, v_j\}$ is a resolving set for some $i, j$ with $i < j \leq n - 1$. Using the division algorithm, there exists $q, r$ integers with $j - i = 3q + r$, $0 \leq r \leq 2$.

We have three cases

- Case 1: If r=0, then $j - i = 3q$. We have $d(v_{i+1}, W_1) = d(v_{i+2}, W_1) = (1, q)$. Hence $W_1$ is not a resolving set.
- Case 2: If r=1, then $j - i = 3q + 1$. We have $d(v_{i+1}, W_1) = d(v_{i+2}, W_1) = (1, q)$. Hence $W_1$ is not a resolving set.
- Case 3: If r=2, then $j - i = 3q + 2$. We have $d(v_{i+2}, W_1) = d(v_{i+3}, W_1) = (1, q)$. Hence $W_1$ is not a resolving set.

So, $dim(G) > 2$. Using Lemma 2.1, we get $W = \{v_0, v_1, v_2\}$ is a resolving set.
Hence $dim(G) = 3$. ■

**Lemma 2.5.** Let $G = P_n^4, n > 5$. Then $dim(G) = 4$.

*Proof.* By Lemma 2.1, $dim(G) \leq 4$. Assume that $W_1 = \{v_i, v_j, v_k\}$ is a resolving set for some $i, j, k$ with $0 \leq i < j < k \leq n - 1$.

Using the division algorithm, there exists integers $q_1, q_2, r_1$ and $r_2$ such that
$j - i = 4q_1 + r_1,\ 0 \le r_1 \le 3.$
$k - j = 4q_2 + r_2,\ 0 \le r_2 \le 3.$
We have four cases for $r_1$

- Case 1: $r_1 = 0$ and hence $j - i = 4q_1$. We have

  1. $k - j = 4q_2$. In this subcase we get
     $d(v_{j-1}, W_1) = d(v_{j-2}, W_1) = (q_1, 1, q_2 + 1)$.
     Hence $W_1$ is not a resolving set.
  2. $k - j = 4q_2 + 1$. In this subcase we get
     $d(v_{j-1}, W_1) = d(v_{j-2}, W_1) = (q_1, 1, q_2 + 1)$.
     Hence $W_1$ is not a resolving set.
  3. $k - j = 4q_2 + 2$. In this subcase we get
     $d(v_{j-1}, W_1) = d(v_{j-2}, W_1) = (q_1, 1, q_2 + 1)$.
     Hence $W_1$ is not a resolving set.
  4. $k - j = 4q_2 + 3$. In this subcase we get
     $d(v_{j-2}, W_1) = d(v_{j-3}, W_1) = (q_1, 1, q_2 + 2)$.
     Hence $W_1$ is not a resolving set.

- Case 2: $r_1 = 1$ and hence $j - i = 4q_1 + 1$. We have

  1. $k - j = 4q_2$. In this subcase we get
     $d(v_j + 1, W_1) = d(v_j + 2, W_1) = (q_1 + 1, 1, q_2)$.
     Hence $W_1$ is not a resolving set.
  2. $k - j = 4q_2 + 1$. In this subcase we get
     $d(v_{j+1}, W_1) = d(v_{j-2}, W_1) = (q_1 + 1, 1, q_2)$.
     Hence $W_1$ is not a resolving set.
  3. $k - j = 4q_2 + 2$. In this subcase we get
     $d(v_{j-1}, W_1) = d(v_{j-2}, W_1) = (q_1, 1, q_2 + 1)$.
     Hence $W_1$ is not a resolving set.
  4. $k - j = 4q_2 + 3$. In this subcase we get
     $d(v_{j+1}, W_1) = d(v_{j+2}, W_1) = (q_1 + 1, 1, q_2 + 2)$.
     Hence $W_1$ is not a resolving set.

- Case 3: $r_1 = 2$ and hence $j - i = 4q_1 + 2$ .We have

  1. $k - j = 4q_2$. In this subcase we get
     $d(v_{j+1}, W_1) = d(v_{j+2}, W_1) = (q_1 + 1, 1, q_2)$.
     Hence $W_1$ is not a resolving set.
  2. $k - j = 4q_2 + 1$. In this subcase we get
     $d(v_{j-2}, W_1) = d(v_{j-3}, W_1) = (q_1, 1, q_2 + 1)$.
     Hence $W_1$ is not a resolving set.

3. $k - j = 4q_2 + 2$. In this subcase we get
   $d(v_{j+1}, W_1) = d(v_{j+2}, W_1) = (q_1 + 1, 1, q_2 + 1)$.
   Hence $W_1$ is not a resolving set.
4. $k - j = 4q_2 + 3$. In this subcase we get
   $d(v_{j-1}, W_1) = d(v_{j+1}, W_1) = (q_1 + 1, 1, q_2 + 1)$.
   Hence $W_1$ is not a resolving set.

- Case 4: $r_1 = 3$ and hence $j - i = 4q_1 + 3$. We have

1. $k - j = 4q_2$. In this subcase we get
   $d(v_{j-1}, W_1) = d(v_{j-2}, W_1) = (q_1 + 1, 1, q_2 + 1)$.
   Hence $W_1$ is not a resolving set.
2. $k - j = 4q_2 + 1$. In this subcase we get
   $d(v_{j-1}, W_1) = d(v_{j-2}, W_1) = (q_1 + 1, 1, q_2 + 1)$.
   Hence $W_1$ is not a resolving set.
3. $k - j = 4q_2 + 2$. In this subcase we get
   $d(v_{j-1}, W_1) = d(v_{j-2}, W_1) = (q_1 + 1, 1, q_2 + 1)$.
   Hence $W_1$ is not a resolving set.
4. $k - j = 4q_2 + 3$. In this subcase we get
   $d(v_j - 1, W_1) = d(v_j + 1, W_1) = (q_1 + 1, 1, q_2 + 1)$.
   Hence $W_1$ is not a resolving set.

Thus $dim(G) > 3$. Using Lemma 2.1 $W = \{v_0, v_1, v_2, v_3\}$ is a resolving set with four elements. So, $dim(G) = 4$. ■

## 3. Metric dimension of permutation of paths

We investigate the metric dimension of certain permutation of paths of odd order.

**Theorem 3.1.** Let $G = P_{2n-1}(\alpha)$ where $\alpha$ is the permutation $\alpha = (v_1, v_{2n+1})$ $(v_2, v_{2n+2}) \dots (v_{2n-2}, v_{4n-2})(v_{2n-1}, v_{2n})$.
This graph is given in Figure 1. Then $dim(G) = 2$.

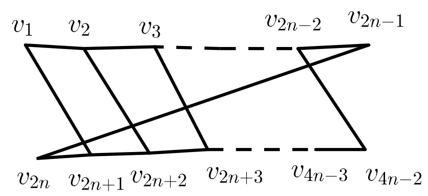


Figure 1: The graph $G = P_{2n-1}(\alpha)$.

*Proof.* Let $G = P_{2n-1}(\alpha)$ be the graph with vertices

$$\{v_1, v_2, \dots, v_{2n-1}, v_{2n}, \dots, v_{4n-2}\},$$

where

$$\alpha = (v_1, v_{2n+1})(v_2, v_{2n+2}) \dots (v_{2n-2}, v_{4n-2})(v_{2n-1}, v_{2n}).$$

The graph is shown in Figure 1. Let $W = \{v_1, v_{n+1}\}$. We want to prove that $W$ is a resolving set by showing that $d(v_i, W) \neq d(v_j, W)$ for all $i \neq j$. Obsreve that
$d(v_1, W) = (0, n)$,
$d(v_2, W) = (1, n-1)$,
$d(v_3, W) = (2, n-2)$,
$d(v_4, W) = (3, n-3)$,
$\vdots$
$d(v_n, W) = (n-1, 1)$,
$d(v_{n+1}, W) = (n, 0)$,
$d(v_{n+2}, W) = (n, 1)$,
$d(v_{n+3}, W) = (n-1, 2)$,
$d(v_{n+4}, W) = (n-2, 3)$,
$\vdots$
$d(v_{2n-1}, W) = (3, n-2)$,
$d(v_{2n}, W) = (2, n-1)$,
$d(v_{2n+1}, W) = (1, n)$,
$d(v_{2n+2}, W) = (2, n)$,
$d(v_{2n+3},\text{W}) = (3, n-1)$,
$d(v_{2n+4}, W) = (4, n-2)$,
$d(v_{2n+5}, W) = (5, n-3)$,
$\vdots$
$d(v_{3n}, W) = (n, 2)$,
$d(v_{3n+1}, W) = (n+1, 1)$,
$d(v_{3n+2}, W) = (n+1, 2)$,
$d(v_{3n+3}, W) = (n, 3)$,
$\vdots$
$d(v_{4n-3}, W) = (6, n-3)$,
$d(v_{4n-2}, W) = (5, n-2)$.
Since $G$ is not a path, then $dim(G) > 1$. Thus $dim(P_{2n-1}(\alpha)) = 2$. ■

We give the following example to explain Theorem 3.1.

**Example 3.2.** Consider $G = P_{13}(\alpha)$. The vertices of $P_{13}(\alpha)$ are $\{v_1, v_2, \dots, v_{26}\}$. Take $W = \{v_1, v_8\}$. Observe that
$d(v_1, W) = (0, 7)$, $d(v_2, W) = (1, 6)$, $d(v_3, W) = (2, 5)$,
$d(v_4, W) = (3, 4)$, $d(v_5, W) = (4, 3)$, $d(v_6, W) = (5, 2)$,
$d(v_7, W) = (6, 1)$, $d(v_8, W) = (7, 0)$, $d(v_9, W) = (7, 1)$,

$d(v_{10}, W) = (6, 2)$, $d(v_{11}, W) = (5, 3)$, $d(v_{12}, W) = (4, 4)$,
$d(v_{13}, W) = (3, 5)$, $d(v_{14}, W) = (2, 6)$, $d(v_{15}, W) = (1, 7)$,
$d(v_{16}, W) = (2, 7)$, $d(v_{17}, W) = (3, 6)$, $d(v_{18}, W) = (4, 5)$,
$d(v_{19}, W) = (5, 4)$, $d(v_{20}, W) = (6, 3)$, $d(v_{21}, W) = (7, 2)$,
$d(v_{22}, W) = (8, 1)$, $d(v_{23}, W) = (8, 2)$, $d(v_{24}, W) = (7, 3)$,
$d(v_{25}, W) = (6, 4)$, and $d(v_{26}, W) = (5, 5)$.

## 4. Metric dimension of complement of paths

In this section, our goal is to find an upper bound of the metric dimension of the complement of paths. Let $P_n$ be the path with vertices $\{v_1, v_2, \ldots, v_n\}$. In the following theorem we show that the metric dimension of the complement of this path $\leq \left\lfloor \frac{2n+2}{5} \right\rfloor$.

**Theorem 4.1.** Let $G = \bar{P}_n$. Then $dim(\bar{P}_n) \leq \left\lfloor \frac{2n+2}{5} \right\rfloor$.

*Proof.* Using the division algorithm there exist $k$ and $r$ with $n = 5k + r,\ 0 \leq r \leq 4$. So, we have five cases to consider.

- Case(1) n=5k, k≥2. In this case $\left\lfloor \frac{2n+2}{5} \right\rfloor = 2k$.

  Let $W = \{v_1, v_5\} \bigcup \{v_{5i+2}, v_{5i+4} : 1 \leq i \leq k-1\}$. We want to prove that $W$ is a resolving set by showing that $d(v_j, W) \neq d(v_l, W)$ for all $j \neq l$. Obsreve that
  $d(v_1, W) = (0, 1, 1, \ldots, 1)$, $d(v_2, W) = (2, 1, 1, \ldots, 1)$,
  $d(v_3, W) = (1, 1, 1, \ldots, 1)$, $d(v_4, W) = (1, 2, 1, \ldots, 1)$,
  $d(v_5, W) = (1, 0, 1, \ldots, 1)$, $d(v_6, W) = (1, 2, 2, 1, \ldots, 1)$.
  If $i > 1$, then $d(v_{5i}, W) = (1, 1, 1, \ldots, 1, 2, 1, \ldots, 1)$, where the 2 in the $(2i)^{th}$ position.
  If $i > 1$, then $d(v_{5i+1}, W) = (1, 1, 1, \ldots, 1, 2, 1, \ldots, 1)$, where the 2 in the $(1+2i)^{th}$ position.
  If $i > 0$, then $d(v_{5i+2}, W) = (1, 1, 1, \ldots, 1, 0, 1, \ldots, 1)$, where the 0 in the $(1+2i)^{th}$ position.
  If $i > 0$, then $d(v_{5i+3}, W) = (1, 1, 1, \ldots, 1, 2, 2, 1, \ldots, 1)$, where the 2's in the $(1+2i)^{th}$ and $(2+2i)^{th}$ positions.
  If $i > 0$, then $d(v_{5i+4}, W) = (1, 1, 1, \ldots, 1, 0, 1, \ldots, 1)$, where the 0 in the $(2+2i)^{th}$ position. So, $\dim(\bar{P}_n) \leq \left\lfloor \frac{2n+2}{5} \right\rfloor$.

- Case(2) n=5k+1, k≥3. In this case $\left\lfloor \frac{2n+2}{5} \right\rfloor = 2k$.

  Let $W = \{v_3, v_{5k}\} \bigcup \{v_{5i}, v_{5i+3} : 1 \leq i \leq k-1\}$. We want to prove that $W$ is a resolving set by showing that $d(v_j, W) \neq d(v_l, W)$ for all $j \neq l$. Obsreve that
  $d(v_1, W) = (1, 1, 1, \ldots, 1)$, $d(v_2, W) = (2, 1, 1, \ldots, 1)$,

$d(v_3, W) = (0, 1, 1, \ldots, 1)$, $d(v_4, W) = (2, 2, 1, \ldots, 1)$.
If $i > 0$, then $d(v_{5i}, W) = (1, 1, 1, \ldots, 1, 0, 1, \ldots, 1)$, where the 0 in the $(2i)^{th}$ position.
If $i > 0$, then $d(v_{5i+1}, W) = (1, 1, 1, \ldots, 1, 2, 1, \ldots, 1)$, where the 2 in the $(2i)^{th}$ position.
If $i > 0$, then $d(v_{5i+2}, W) = (1, 1, 1, \ldots, 1, 2, 1, \ldots, 1)$, where the 2 in the $(1 + 2i)^{th}$ position.
If $i > 0$, then $d(v_{5i+3}, W) = (1, 1, 1, \ldots, 1, 0, 1, \ldots, 1)$, where the 0 in the $(1 + 2i)^{th}$ position.
If $i > 0$, then $d(v_{5i+4}, W) = (1, 1, 1, \ldots, 1, 2, 2, 1, \ldots, 1)$, where the 2's in the $(1 + 2i)^{th}$ and $(2 + 2i)^{th}$ positions. So, $\dim(\bar{P}_n) \leq \left\lfloor \frac{2n+2}{5} \right\rfloor$.

- Case(3) n=5k+2, k≥1 and $\left\lfloor \frac{2n+2}{5} \right\rfloor = 2k + 1$.

  Let $W = \{v_3\} \bigcup \{v_{5i}, v_{5i+2} : 1 \leq i \leq k\}$. We want to prove that $W$ is a resolving set by showing that $d(v_j, W) \neq d(v_l, W)$ for all $j \neq l$. Obsreve that
  $d(v_1, W) = (1, 1, 1, \ldots, 1)$, $d(v_2, W) = (2, 1, 1, \ldots, 1)$,
  $d(v_3, W) = (0, 1, 1, \ldots, 1)$, $d(v_4, W) = (2, 2, 1, \ldots, 1)$.
  If $i > 0$, then $d(v_{5i}, W) = (1, 1, 1, \ldots, 1, 0, 1, \ldots, 1)$, where the 0 in the $(2i)^{th}$ position.
  If $i > 0$, then $d(v_{5i+1}, W) = (1, 1, 1, \ldots, 1, 2, 2, 1, \ldots, 1)$, where the 2's in the $(2i)^{th}$ and $(1 + 2i)^{th}$ positions.
  If $i > 0$, then $d(v_{5i+2}, W) = (1, 1, 1, \ldots, 1, 0, 1, \ldots, 1)$, where the 0 in the $(1 + 2i)^{th}$ position.
  If $i > 0$, then $d(v_{5i+3}, W) = (1, 1, 1, \ldots, 1, 2, 1, \ldots, 1)$, where the 2 in the $(1 + 2i)^{th}$ position.
  If $i > 0$, then $d(v_{5i+4}, W) = (1, 1, 1, \ldots, 1, 2, 1, \ldots, 1)$, where the 2 in the $(2 + 2i)^{th}$ position. So, $\dim(\bar{P}_n) \leq \left\lfloor \frac{2n+2}{5} \right\rfloor$.

- Case(4) n=5k+3, k≥1 and $\left\lfloor \frac{2n+2}{5} \right\rfloor = 2k + 1$. This case is similar to case (3).

- Case(5) n=5k+4, k≥1 and $\left\lfloor \frac{2n+2}{5} \right\rfloor = 2k + 2$.

  Let $W = \{v_1, v_{5k+4}\} \bigcup \{v_{5i-1}, v_{5i+1} : 1 \leq i \leq k\}$. We want to prove that $W$ is a resolving set by showing that $d(v_j, W) \neq d(v_l, W)$ for all $j \neq l$. Obsreve that
  $d(v_1, W) = (0, 1, 1, \ldots, 1)$, $d(v_2, W) = (2, 1, 1, \ldots, 1)$,
  $d(v_3, W) = (1, 2, 1, \ldots, 1)$, $d(v_4, W) = (1, 0, 1, \ldots, 1)$.
  If $i > 0$, then $d(v_{5i}, W) = (1, 1, 1, \ldots, 1, 2, 2, 1, \ldots, 1)$, where the 2's in the $(2i)^{th}$ and $(1 + 2i)^{th}$ positions.
  If $i > 0$, then $d(v_{5i+1}, W) = (1, 1, 1, \ldots, 1, 0, 1, \ldots, 1)$, where the 0 in the

$(1+2i)^{th}$ position.
If $i > 0$, then $d(v_{5i+2}, W) = (1, 1, 1, \ldots, 1, 2, 1, \ldots, 1)$, where the 2 in the $(1+2i)^{th}$ position.
If $i > 0$, then $d(v_{5i+3}, W) = (1, 1, 1, \ldots, 1, 2, 1, \ldots, 1)$, where the 2 in the $(2+2i)^{th}$ position.
If $i > 0$, then $d(v_{5i+4}, W) = (1, 1, 1, \ldots, 1, 0, 1, \ldots, 1)$, where the 0 in the $(2+2i)^{th}$ position. So, $\dim(\bar{P}_n) \leq \left\lfloor \frac{2n+2}{5} \right\rfloor$

■

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