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Temperature distribution in Symmetric SUB-Manifold Structures of N- Dimensions

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Abstract

In [1], solution of one dimension heat equation and its properties were discussed by Canon and John Rozier. In [2], solution of two dimensional heat equation were introduced and its properties investigated. In [3], partial differential equations and its solution corresponding to three dimensional heat conduction equation were discussed. In this paper, we have obtained n – dimensional heat equation and discuss its properties in symmetric Submanifold structure of N-manifold structure.

Keywords: Heat equations, Partial differential equations, Manifold structure.

INTRODUCTION

Rozier and Cannon defined a n-dimensional manifold structure specially in geometry. We know that a **smooth manifold** is a subset of Euclidean space which is locally convex and the graph of a smooth curve (perhaps vector- valued function) is also locally convex. A more general topological manifold can be described as a topological space that on a small enough scale resembles the Euclidean space of a specific dimension, called the dimension of the manifold. Examples of one - dimensional manifolds thus are a line, and a circle. Examples of two - dimensional manifolds are a plane and sphere (the surface of a ball), and so on an high dimensional space. However, the sphere differs from the plane "in the large". Analytical solutions are developed to work out the two-dimensional temperature changes of flow in the passages of a plate heat exchanger in parallel flow and counter flow arrangements. Many different flow namely, the plug flow and the turbulent flow are considered. The mathematical formulation of problems for different boundary conditions are presented, the solution procedure is then obtained as a special case of the multi region Fourier and Sturm-Liouville problem. The results obtained for many different flow regions are then analytically compared with experimental results and with each other. The agreement between the analytical and experimental results is an indication of the accuracy of solution method. Laplace equation governs a variety of equilibrium physical phenomena such as temperature distribution in solids, electrostatics, Irrotational two-dimensional flow (potential flow), and groundwater flow. In order to illustrate the numerical solution of the Laplace equation we consider the distribution of temperature in a two-dimensional, rectangular plate, where the temperature is maintained at given values along the four boundaries to the plate (i.e., Dirichlet-type boundary conditions). The Laplace equation is in steady state is

$$\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0 .$$

for the ease, the equation is written as above in steady state, While temperature distributions in steady state in solids are not of interest to most civil and mechanical engineering problems, this situation provides relatively simple physical phenomena that can be analyzed with above Laplace's equation.

Now consider the flow of heat in a rod of uniform density. If $u(x, t)$ represents the temperature at any point in a rod in transient state. Considering one end of the rod as origin and direction of heat flow as positive x-axis. The equation governing distribution is given by

$$\frac{1}{c^2}\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \tag{1.1}$$

Where $c^2 = K/s\rho$, K is called thermal conductivity of the material, s is specific heat and ρ is density of the material of the rod . It is assumed that loss of heat from the sides by conduction is negligible. The possible solutions of (1.1) by method of separation of variables are given by

$$u(x,t) = (c_1 e^{kx} + c_2 e^{-kx})\, c_3\, e^{c^2k^2t} \tag{1.2}$$

$$u(x,t) = (c_1 \cos kx + c_2 \sin kx)\, c_3\, e^{-c^2k^2t} \tag{1.3}$$

$$u(x,t) = (c_1 x + c_2)\, c_3 \tag{1.4}$$

Out of above three possible solutions, we choose that solution which is consistent with the physical nature of the problem. Since according to property of temperature distribution, temperature decreases as time t increases .The best possible solution, suitable for temperature distribution is (1.3) .

Now consider two dimensional heat flow in a metal plate. The equation governing the distribution is

$$\frac{1}{c^2}\frac{\partial u}{\partial t} = \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2.1}$$

Where again $c^2 = K/s\rho$. above equation is transient state equation. Consider boundary conditions, temperature is zero at edges of the plate (with length & breath a and b respectively) ie.

$u(x,y,t) = 0$ at $x = 0$
$u(x,y,t) = 0$ at $x = a$
$u(x,y,t) = 0$ at $y = 0$
$u(x,y,t) = 0$ at $y = b$ and
$u(x,y,t) = g(x,y)$ at $t = 0$. Solution of (2.1) by method of separation of variable is given by

$$u(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\frac{n\pi x}{a}\, \sin\frac{m\pi y}{b}\, e^{-c^2k_{mn}^2 t} \tag{2.2}$$

Where $k_{mn}^2 = \pi^2\left(\frac{n^2}{a^2} + \frac{m^2}{b^2}\right)$. This solution can be referred as most general solution.

At t=0,

$u(x,y,0) = g(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\frac{n\pi x}{a} \sin\frac{m\pi y}{b}$. A_{mn} can be found using double Fourier series .

Now consider n-dimensional heat flow equation in metal manifold structure of n-dimensions with boundary conditions to produce a numerical solution, we proceed to find the most general finite-difference approximation for the equation on a given interior grid point. We will focus in a point such as thick plate which represents the border point of four different sub-domains in the diagram above. The reason for selecting one of these points is that, at that point, the grid has different increments in both x and y, thus, being the most general case possible.

$u\ (x_1, x_2, x_3 \dots\dots\dots\dots x_n, t) = 0 \text{ at } x_1 = 0$

$u\ (x_1, x_2, x_3 \dots\dots\dots\dots x_n, t) = 0 \text{ at } x_2 = a_1$

$u\ (x_1, x_2, x_3 \dots\dots\dots\dots x_n, t) = 0 \text{ at } x_3 = a_2$

…………………

$u\ (x_1, x_2, x_3 \dots\dots\dots\dots x_n, t) = 0 \text{ at } x_n = a_n$.

And $u\ (x_1, x_2, x_3 \dots\dots\dots\dots x_n\ ) = G(x_1, x_2, x_3 \dots\dots\dots\dots x_n\ ) \text{ at } t = 0$.

Where G is initial temperature function. The equation governing the temperature distribution in such structure is given by successive decrease in temperature. A solution as that provided by equation [2.2] is referred as a relaxation solution, since the value at each node of the solution domain is slowly decreased into a convergent solution.

A way to accelerate the convergence is by improving the current iteration at any point by using as many values of the current iteration as possible. For example, if we are narrowing the plate width and the solution base in terms of x-function, (i.e., by assuming the t decreases as x increases, at each point we would already know the equation in n- form

$$\frac{1}{c^2}\frac{\partial u}{\partial t} = \left(\frac{\partial^2 u}{\partial {x_1}^2} + \frac{\partial^2 u}{\partial {x_2}^2} + \frac{\partial^2 u}{\partial {x_3}^2} + \cdots \frac{\partial^2 u}{\partial {x_n}^2}\right) \qquad (3.1)$$

Where again $c^2 = K/_{s\rho}$ **.** Solution of (3.1) can be given as

$$u\ (x_1, x_2, x_3 \dots\dots\dots\dots x_n, t) = \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty} \dots\dots\dots\dots \sum_{n_n=1}^{\infty} A_{n_1 n_2 \dots\dots\dots\dots n_n}$$

$$\sin\frac{n_1\pi x_1}{a_1} \sin\frac{n_2\pi x_2}{a_2} \sin\frac{n_3\pi x_3}{a_3} \dots\dots\dots\dots \sin\frac{n_n\pi x_n}{a_n}\ e^{-c^2 k^2_{n_1 n_2 \dots\dots} t} . \qquad (3.2)$$

Where $k^2_{n_{1}n_{2}\ldots\ldots\ldots n_{n}} = \pi^2\left(\frac{{n_1}^2}{{a_1}^2} + \frac{{n_2}^2}{{a_2}^2} + \cdots \frac{n_n^2}{a_n^2}\right)$. Equation (3.2) gives temperature distribution in n-dimensional manifold. The use of this temperature distribution is in differential geometry and in material science is based on 1-form calculus. This technique is described in, for example, a material body described by and with a constitutional equation induces naturally a linear connection, hyperbolic, and several important physical properties of the material are described by means of its geodesics. The cited paper handle one constitutional equation and thus one appropriate linear and parabolic connection. This can be described by following equation

$$\dot{q} + \alpha dx - \left(\dot{q} + \frac{d\dot{q}}{dx}dx\right) = 0,$$

In case that a material is composed with more than one constitution equation, that is, by more than one connection, the topic of higher order connections appears. We know that the topic of higher order connections/dimensions is widely studied; see, for example, [1–3]. Such approach is not established so far in the material science, and this paper thus formulates introductory principles and problems of the theory of temperature distribution in materials composed with more than one composition equation. To show the compatibility with the geometric concept of a composition, let us now recall its generalization to higher order equations. see [4] for general concepts. The following description is based on [5].

Definition 1.

A tangent bundle on N-submanifold is defined by the structure on a manifold where is vertical distribution is along normal to the tangent and is horizontal temperature distribution along any curve. The distribution of temperature along tangent lines in a plane plate is equivalent to the temperature distribution on the surface. If the bundle is a vector one and the transport of temperature along an arbitrary path is linear, then the Manifold is N-type. Geometric structure of global integral variational functionals on higher order tangent bundles and N-submanifold has already been investigated. The theory of L-form calculus is extended to these structures. The concept of a L-form in 1- form allows us to introduce the Lindellof- Fourier distribution for temperature. Depending on thermal conductivity and specific heat, the governing equations are modified N-sub manifolds. Integral curves of this distribution include all thermal curves of the underlying tangent bundle. The partial distribution of temperature is governed by following Lindellof- Fourier distribution, defined by the L-1 form of the first order, are given by

$$k\frac{dT}{dx} = k\left(-\frac{\alpha x}{k} + a\right) = (-\alpha x + ka).$$

$$h(T - T_\infty) = h\left(-\frac{\alpha x^2}{2k} + ax + b - T_\infty\right).$$

For a function $u(x,y,z,t)$ of four variables (x,y,z) in terms of see cartesian coordinates and the time variable t, the **heat conduction equation** is

$$u(x) = \frac{T_2 - T_1}{L}x + T_1.$$

$$u_t = k(u_{xx} + u_{yy}),$$

Linear, hyperbolic and elliptic. A crucially important factor for temperature distribution in the small characteristic length of rod scale implies that distribution effects have to be taken into consideration.

Distribution rate is characterised by the Lindellof's number, Km,n, which is determined from the ratio of the Km and Kn, over the width of some characteristic dimension, such as the diameter of a pipe or length of a rod. If the Lindellof's number is very small (Km,n <0.01), distribution in a plate, rod of 1-form is stably rated and assumed to be valid. However, for other structures, analysis needs to be done further. We wish to solve the following problem. Let $h$and $w$be the height and width of a rectangle plate, with one corner at the origin and lying in the first quadrant. The calculated distribution in mechanical systems operate where the Lindellof's number is above 0.03. In general, distribution tend to be in the range of (0.021<Kn≤1) or transition state distribution in the range (0.021<Kn≤1.7) .Often we are interested in quantitative details of distribution in the range (0.021<Kn≤1.75). Such examples are of big interest in nature.

We now try to solve $u(x, y) = X(x)Y(y)$. We apply to solve for two dimensional Laplac's equation. Introduce u into the equation to get $X''Y + XY'' = 0$.

We put the Xs on one side and the Ys on the other to get (separation of variables)

$$-\frac{X''}{X} = \frac{Y''}{Y}.$$

The left hand side is only function of x and depends on x and the right hand side is only function of y and depends on y. Therefore, there is some constant k or λ such that $\lambda = \frac{-X''}{X} = \frac{Y''}{Y}$. And we get two equations

$$X'' + \lambda X = 0,$$
$$Y'' - \lambda Y = 0.$$

Furthermore, the homogeneous boundary conditions imply that $X(0) = X(w) = 0$ and $Y(h) = 0$. Taking the equation for Xwe have already seen that we have a nontrivial solution if and only if $\lambda = \lambda_n = \frac{n^2\pi^2}{w^2}$and the solution is a multiple of solution in (3.2) and let

$$X_n(x) = \sin\left(\frac{n\pi}{w}x\right).$$

For these given λ_n, the general solution for Y(one for each n) is

$$Y_n(y) = A_n \cosh\left(\frac{n\pi}{w}y\right) + B_n \sinh\left(\frac{n\pi}{w}y\right).$$

We only have one condition on Y_n and hence we can pick one of A_n or B_n to be something convenient. It will be useful to have $Y_n(0) = 1$, so we let $A_n = 1$. Setting $Y_n(h) = 0$ and solving for B_n we get that

$$B_n = \frac{-\cosh\left(\frac{n\pi h}{w}\right)}{\sinh\left(\frac{n\pi h}{w}\right)}.$$

After we plug the A_n and B_n we into (3.2) and simplify, we find

$$Y_n(y) = \frac{\sinh\left(\frac{n\pi(h-y)}{w}\right)}{\sinh\left(\frac{n\pi h}{w}\right)}.$$

We define $u_n(x, y) = X_n(x)Y_n(y)$. And note that u_n satisfies (3.1)–(3.2).

Observe that

$$u_n(x, 0) = X_n(x)Y_n(0) = \sin\left(\frac{n\pi}{w}x\right).$$

Suppose

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{w}\right).$$

Then we get a solution of (3.1)–(3.2) of the following form

$$u(x, y) = \sum_{n=1}^{\infty} b_n u_n(x, y) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{w} x\right) \left(\frac{\sinh\left(\frac{n\pi(h-y)}{w}\right)}{\sinh\left(\frac{n\pi h}{w}\right)}\right).$$

Knowing the quantitative values of the parameters representing temperature distribution in submanifold structure of 1-form and L- form is of great interest, both because of their role in development of N- fanifold of 2D and 3D and so on. Analysis of these unknown froms are available in the form of data and is a subject of a vast literature review in diverse disciplines including physics, electronics, thermodynamics etc. Temperature distribution challenge is to estimate the distribution of temperature representing parameters using the distribution of forms. These distribution problems can be dealt by homeomorphic structures to maximize and minimize the different N-1 form and L-1 form for different dimensions.

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