$$\int_{-1}^{+1}|R(t)|\,dt=\int_{-1}^{+1}\left|(2m-1)P_{m+2}(t)-(4m+2)P_m(t)+(2m+3)P_{m-2}(t)\right|dt.$$

Using Cauchy-Schwarz's inequality, we get

$$\left[\int_{-1}^{+1}|R(t)|\,dt\right]^2\le\int_{-1}^{+1}dt\int_{-1}^{+1}\left(\left|(2m-1)P_{m+2}(t)-(4m+2)P_m(t)+(2m+3)P_{m-2}(t)\right|\right)^2dt,$$

$$=4\int_0^1\left[(2m-1)^2P_{m+2}^2(t)+(4m+2)^2P_m^2(t)+(2m+3)^2P_{m-2}^2(t)\right]dt,$$

$$=4.3\frac{(2m+3)^2}{2m-3},$$

$$\left[\int_{-1}^{+1}|R(t)|\,dt\right]\le\frac{2\sqrt{3}(2m+3)}{\sqrt{2m-3}}. \tag{27}$$

Using equation (27) in equation (26), we get

$$|g_{nm}|\le\frac{2^{-(5k+1)/2}K}{(2m-1)(2m+3)\sqrt{(2m+1)}}\frac{2\sqrt{3}(2m+3)}{\sqrt{2m-3}},$$

$$=\frac{2^{-5k/2}K\sqrt{6}}{(2m-1)\sqrt{(2m+1)(2m-3)}},$$

$$|g_{nm}|\le\frac{\sqrt{6}K}{2^{5k/2}(2m-1)\sqrt{(2m+1)(2m-3)}}. \tag{28}$$

Hence desire.

Theorem (Mean square error): Let the function $D^\alpha f(x)\in C^2[0,1],$ and $D^\alpha f(x)$ exists bounded second derivative then we have the following accuracy estimation.

$$\varepsilon_n\le\sum_{n=2^k}^{+\infty}\sum_{m=M}^{+\infty}\frac{\sqrt{6}K}{(n)^{5/2}(2m-1)\sqrt{(2m+1)(2m-3)}}.$$

Where

$$\varepsilon_n=\int_0^1\left[\sum_{n=0}^{+\infty}\sum_{m=0}^{+\infty}g_{nm}\psi_{nm}(x)-\sum_{n=0}^{2^k-1}\sum_{m=0}^{M-1}g_{nm}\psi_{nm}(x)\right]dx.$$

Proof: Let us consider the quantity

$$\varepsilon^2=\int_0^1\left[\sum_{n=0}^{+\infty}\sum_{m=0}^{+\infty}g_{nm}\psi_{nm}(x)-\sum_{n=0}^{2^k-1}\sum_{m=0}^{M-1}g_{nm}\psi_{nm}(x)\right]^2dx, \tag{29}$$

$$\varepsilon^2 = \int_0^1 \left[\sum_{n=2^k}^{+\infty} \sum_{m=M}^{+\infty} g_{nm}\psi_{nm}(x) \right]^2 dx = \sum_{n=2^k}^{+\infty} \sum_{m=M}^{+\infty} \int_0^1 g_{nm}^2 \psi_{nm}^2(x)dx, \tag{30}$$

Using the property of orthonormal wavelets

$$\int_0^1 \psi_{nm}(x)\psi_{nm}^T(x)dx = I. \tag{31}$$

$$\varepsilon^2 = \sum_{n=2^k}^{+\infty} \sum_{m=M}^{+\infty} g_{nm}^2,$$

From equation (28), we have

$$\varepsilon_n \le \sum_{n=2^k}^{+\infty} \sum_{m=M}^{+\infty} \frac{\sqrt{6}K}{(n)^{5/2}(2m-1)\sqrt{(2m+1)(2m-3)}}.$$

Hence desire.

ELECTRICAL CIRCUITS

In this section, we apply Legendre wavelet collocation method and find the approximate solution of fractional circuits and comparing their solutions with the corresponding classical solutions.

Solution of LC circuit

Consider the LC circuit, only charged capacitor and inductor are present in the circuit and its differential equation is given as follows.

$$J''(t) + \frac{1}{LC}J(t) = 0. \tag{32}$$

with $J(0) = J_0$ and $J'(0) = 0.$

The classical solution of equation (32) is

$$J(t)_{LC} = J_0 Cos(\omega_0 t), \tag{33}$$

where $\omega_0^2 = 1/LC.$

Now, we analyse equation (32) using fractional calculus, we replace $J''(t)$ by $D^\alpha J(t)$, where $\alpha \in (1,2).$

In the sense of Riemann-Liouville derivative, we get the fractional order LC circuit and its differential equation as

$$D^\alpha J(t) + \omega_0^2 J(t) = 0. \tag{34}$$

with $J(0)=J_0$ and $D^{\alpha}J(0)=0.$ (35)

We use equation (12) to approximate $D^{\alpha}J(t)$ as

$$D^{\alpha}J(t)=\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1}u_{nm}\psi_{nm}(t)=U^{T}\Psi(t), \tag{36}$$

Integrating equation (36) with respect to t over the interval $[0,t]$, we get

$$DJ(t)=DJ(0)+U^{T}P_{\hat{m}\times\hat{m}}^{\alpha-1}\Psi(t),$$

Using condition (35), yield

$$J(t)=J_0+U^{T}P_{\hat{m}\times\hat{m}}^{\alpha}\Psi(t). \tag{37}$$

Substituting equations (35-36) in equation (33), we obtain

$$U^{T}\Psi(t)+\omega_0^2\left(J_0+U^{T}P_{\hat{m}\times\hat{m}}^{\alpha}\Psi(t)\right)=0. \tag{38}$$

From equation (16), we can approximate J_0 as:

$$J_0=\left[J_0,J_0,...,J_0\right]_{1\times\hat{m}}\phi_{\hat{m}\times\hat{m}}^{-1}\psi(t). \tag{39}$$

Substituting equation (39) in equation (38), we have

$$U^{T}\Psi(t)+\omega_0^2U^{T}P_{\hat{m}\times\hat{m}}^{\alpha}\Psi(t)=-\omega_0^2[J_0,J_0,...,J_0]\phi_{\hat{m}\times\hat{m}}^{-1}\Psi(t), \tag{40}$$

$$U^{T}\left(I+\omega_0^2P_{\hat{m}\times\hat{m}}^{\alpha}\right)=-\omega_0^2[J_0,J_0,...,J_0]\phi_{\hat{m}\times\hat{m}}^{-1},$$

$$U^{T}=-\omega_0^2[J_0,J_0,...,J_0]\phi_{\hat{m}\times\hat{m}}^{-1}\left(I+\omega_0^2P_{\hat{m}\times\hat{m}}^{\alpha}\right)^{-1}. \tag{41}$$

Hence required

$$J(t)=[J_0,J_0,...,J_0]\phi_{\hat{m}\times\hat{m}}^{-1}\Psi(t)+-\omega_0^2[J_0,J_0,...,J_0]\phi_{\hat{m}\times\hat{m}}^{-1}\left(I+\omega_0^2P_{\hat{m}\times\hat{m}}^{\alpha}\right)^{-1}P_{\hat{m}\times\hat{m}}^{\alpha}\Psi(t). \tag{42}$$

By solving the above system (41) of linear equations, we can find the value of vector U. substituting value of U in equation (37), hence we required the numerical results of the LC circuit for different value of k, M and α . Here, solution obtained by the proposed LWM approach for $\alpha=1.50, 1.75, 1.999$ and $k=2$ and $M=3$ is graphically show in figure 1. As it can be clearly seen, for $\alpha=1.999$ and $k=2$ and $M=3$, the fractional RC circuit graph behave similar to the classical solution graph for $\alpha=1$. It is show that the proposed LWM approach is more close to the exact solution. Table 1 describes the efficiency of the proposed method by comparing with the classical solution at $\alpha=1$. Table 1 show that very high accuracies are obtained for $k=2$ and $M=3$ by the present method.

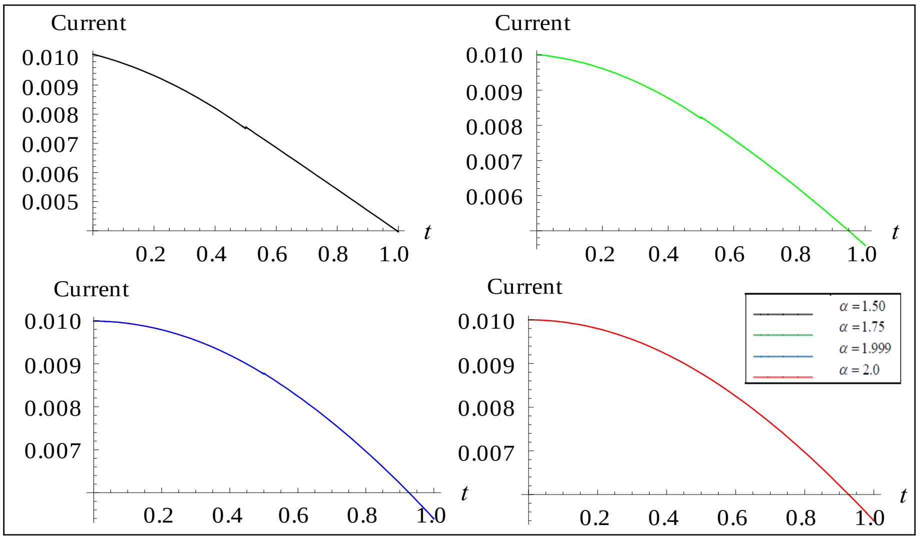

Figure 1. Current versus time graph $(L=1, C=1, J_0=0.01\ and\ \alpha=1.50, 1.75\ and\ 1.999)$.

Table 1. Numerical results of LC circuit for $(L=1, C=1, J_0=0.01\ and\ \alpha=1.5, 1.75\ and\ 1.999)$.

t		$\alpha=1.50$	$\alpha=1.75$	$\alpha=1.999$	$\alpha=2$
		LW	LW	LW	CS
0.1		9.7394×10^{-3}	9.8706×10^{-3}	9.9382×10^{-3}	9.9500×10^{-3}
0.2		9.3239×10^{-3}	9.6153×10^{-3}	9.7888×10^{-3}	9.8006×10^{-3}
0.3		8.8129×10^{-3}	9.2517×10^{-3}	9.5430×10^{-3}	9.5533×10^{-3}
0.4		8.2064×10^{-3}	8.7801×10^{-3}	9.2007×10^{-3}	9.2106×10^{-3}
0.5		7.5620×10^{-3}	8.2393×10^{-3}	8.7757×10^{-3}	8.7758×10^{-3}
0.6		6.8577×10^{-3}	7.5956×10^{-3}	8.2444×10^{-3}	8.2534×10^{-3}
0.7		6.1371×10^{-3}	6.9084×10^{-3}	7.6406×10^{-3}	7.6484×10^{-3}
0.8		5.4179×10^{-3}	6.1776×10^{-3}	6.9640×10^{-3}	6.9671×10^{-3}
0.9		4.6945×10^{-3}	5.4032×10^{-3}	6.2147×10^{-3}	6.2161×10^{-3}

Solution of RC circuit

Consider the RC circuit differential equation given in equation (2), only charged capacitor and resistor are present to the circuit and its differential equation is given as follows

$$CV'(t)+\frac{1}{R}V(t)=0. \tag{43}$$

with condition $V(0)=V_0\ and\ V'(0)=0.$ (44)

The classical solution of equation (43) is

$$V(t)_{RC} = V_0 e^{-\frac{1}{RC}t}. \quad (45)$$

Now, we consider equation (43) using fractional calculus, we replace $V'(t)$ by $D^{\alpha}V(t)$, where $\alpha \in (0,1)$. In the sense of Riemann-Liouville derivative, we get the fractional order RC circuit and its differential equation as

$$D^{\alpha}V(t) + \frac{1}{RC}V(t) = 0 \quad (46)$$

with condition $V(0) = V_0 \; and \; D^{\alpha}V(0) = 0.$ (47)

We use equation (12) to approximate $D^{\alpha}V(t)$ as

$$D^{\alpha}V(t) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} w_{nm}\psi_{nm}(t) = W^T\Psi(t), \quad (48)$$

Integrating equation (48) with respect to t, over $[0,t]$, we get

$$V(t) = V(0) + W^T P^{\alpha}_{\hat{m}\times\hat{m}}\Psi(t), \quad (49)$$

Similarly equation (39), we can approximate V_0 as

$$V(0) = V_0 = \left[V_0, V_0, ..., V_0\right]_{1\times\hat{m}} \phi^{-1}_{\hat{m}\times\hat{m}}\psi(t). \quad (50)$$

So, equation (49) become

$$V(t) = \left[V_0, V_0, ..., V_0\right]_{1\times\hat{m}} \phi^{-1}_{\hat{m}\times\hat{m}}\psi(t) + W^T P^{\alpha}_{\hat{m}\times\hat{m}}\Psi(t), \quad (51)$$

Substituting equations (48) and equation (51) in equation (46), we obtain

$$W^T\Psi(t) + \frac{1}{RC}\left(\left[V_0, V_0, ..., V_0\right]_{1\times\hat{m}} \phi^{-1}_{\hat{m}\times\hat{m}}\psi(t) + W^T P^{\alpha}_{\hat{m}\times\hat{m}}\Psi(t)\right) = 0$$

$$W^T\Psi(t) + \frac{1}{RC}W^T P^{\alpha}_{\hat{m}\times\hat{m}}\Psi(t) = -\frac{1}{RC}[V_0, V_0, ..., V_0]\phi^{-1}_{\hat{m}\times\hat{m}}\Psi(t),$$

$$W^T\left(I + \frac{1}{RC}P^{\alpha}_{\hat{m}\times\hat{m}}\right) = -\frac{1}{RC}[V_0, V_0, ..., V_0]\phi^{-1}_{\hat{m}\times\hat{m}},$$

$$W^T = -\frac{1}{RC}[V_0, V_0, ..., V_0]\phi^{-1}_{\hat{m}\times\hat{m}}.\left(I + \frac{1}{RC}P^{\alpha}_{\hat{m}\times\hat{m}}\right)^{-1}. \quad (52)$$

Hence required

$$V(t) = [V_0, V_0, ..., V_0]\phi^{-1}_{\hat{m}\times\hat{m}}\Psi(t) + -\frac{1}{RC}[V_0, V_0, ..., V_0]\phi^{-1}_{\hat{m}\times\hat{m}}.\left(I + \frac{1}{RC}P^{\alpha}_{\hat{m}\times\hat{m}}\right)^{-1} P^{\alpha}_{\hat{m}\times\hat{m}}\Psi(t). \quad (53)$$

We solving the above system (52)of linear equations and obtain the value of vector w. Substituting the value of vector W in equation (51), hence we require the approximate results of the RC circuit model for different value of k, M and α . Here, we use the proposed LWM approach for $\alpha = 0.25, 0.75, 0.999$ and $k = 2$ and $M = 3$. This has been seen from figure 2 that the obtain solution for $\alpha = 0.999$ and $k = 2$ and $M = 3$, the fractional RC circuit graph behave similar to the classical solution graph for $\alpha = 1$. It is show that the proposed LWCM approach is more close to the exact solution. Table 2 describes the good organization of the proposed method by comparing with the classical solution at $\alpha = 1$. Table 2 also shows that, very high accuracies are obtained for $k = 2$ and $M = 3$ by the present method.

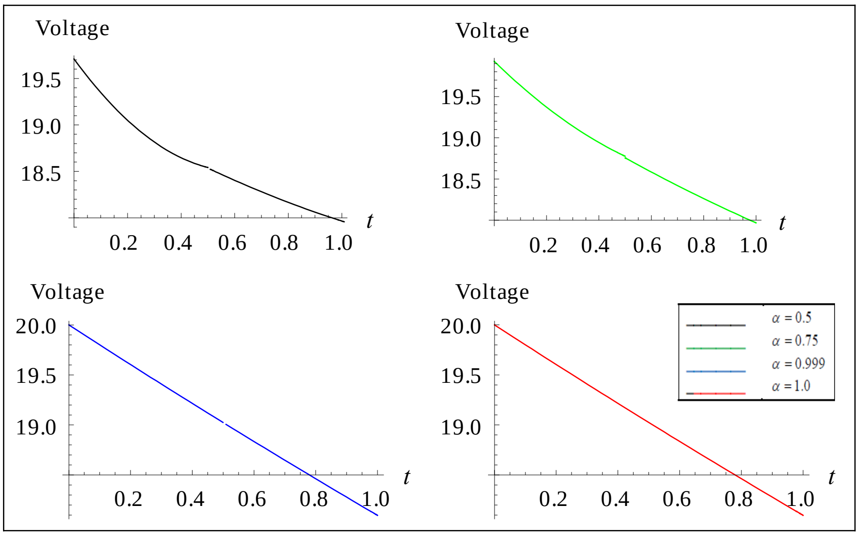

Figure 2. Voltage versus time graph $(R = 10, C = 1, V_0 = 20 \textit{ and } \alpha = 0.5, 0.75 \textit{ and } 0.999)$.

Table 2. Numerical results of RC circuit for $(R = 10, C = 1, J_0 = 20 \textit{ and } \alpha = 0.5, 0.75 \textit{ and } 0.999)$.

t	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 0.999$	$\alpha = 1$
	LW	LW	LW	CS
0.1	19.3481	19.6340	19.8012	19.8010
0.2	19.0500	19.3717	19.6039	19.6040
0.3	18.8163	19.1413	19.4086	19.4089
0.4	18.6471	19.9428	19.2154	19.2158
0.5	18.4976	18.7573	19.0242	19.0246
0.6	18.3669	18.5854	18.8349	18.8353
0.7	18.2450	18.4207	18.6475	18.6479
0.8	18.1319	18.2630	18.4620	18.4623
0.9	18.0275	18.1124	18.2784	18.2786

Solution of RLC circuit

Here, we analyze RLC circuit differential equation given in equation (3). LCR circuit consisting of three kinds of circuit elements: a resistor, an inductor and a capacitor its differential equation is given as follows

$$LQ^{''}(t)+RQ^{'}(t)+\frac{1}{C}Q(t)=0. \tag{54}$$

With the condition $Q(0)=Q_0,\ Q^{'}(0)=0.$ (55)

The classical solution of equation (54) is

$$Q(t)_{RLC}=Q_0 e^{-\frac{R}{2L}t} Cos\left(\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}t\right). \tag{56}$$

Now, we analyse equation (54) using fractional calculus, we replace $Q^{''}(t)$ by $D^{2\alpha}Q(t)$, where $\alpha\in(0,1)$. In the sense of Riemann-Liouville derivative, we get the fractional order RLC circuit and its differential equation as

$$L\left(D^{2\alpha}Q\right)(t)+R\left(D^{\alpha}Q\right)(t)+\frac{1}{C}Q(t)=0. \tag{57}$$

With the condition $Q(0)=Q_0\ and\ D^{\alpha}Q(0)=0.$ (58)

We use equation (12) to approximate $\left(D^{2\alpha}Q\right)(t)$ as

$$\left(D^{2\alpha}Q\right)(t)=\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} y_{nm}\psi_{nm}(t)=Y^T\Psi(t), \tag{59}$$

Integrating equation (59) with respect to t, over $[0,t]$ and using condition (58), we get

$$\left(D^{\alpha}Q\right)(t)=\left(D^{\alpha}Q\right)(0)+Y^T P^{\alpha}_{\hat{m}\times\hat{m}}\Psi(t)=Y^T P^{\alpha}_{\hat{m}\times\hat{m}}\Psi(t), \tag{60}$$

and

$$Q(t)=Q(0)+Y^T P^{2\alpha}_{\hat{m}\times\hat{m}}\Psi(t). \tag{61}$$

Similarly equation (39), we can approximate Q_0 as

$$Q(0)=Q_0=[Q_0,Q_0,...,Q_0]\phi^{-1}_{\hat{m}\times\hat{m}}\Psi(t). \tag{62}$$

From equation (61) and equation (62), we have

$$Q(t)=[Q_0,Q_0,...,Q_0]\phi^{-1}_{\hat{m}\times\hat{m}}\Psi(t)+Y^T P^{2\alpha}_{\hat{m}\times\hat{m}}\Psi(t). \tag{63}$$

Substituting equations (59,60) and equation (63) in equation (57), we obtain

$$Y^T\Psi(t)+\frac{R}{L}Y^T P^{\alpha}_{\hat{m}\times\hat{m}}\Psi(t)+\frac{1}{LC}\left([Q_0,Q_0,...,Q_0]\phi^{-1}_{\hat{m}\times\hat{m}}\Psi(t)+Y^T P^{2\alpha}_{\hat{m}\times\hat{m}}\Psi(t)\right)=0, \tag{64}$$

Let $\frac{1}{LC} = \mu^2$ and $\frac{R}{L} = \sigma^2$, then equation (64) become

$$Y^T \Psi(t) + \sigma^2 Y^T P^{\alpha}_{\hat{m}\times\hat{m}} \Psi(t) + \mu^2 [Q_0, Q_0, ..., Q_0] \phi^{-1}_{\hat{m}\times\hat{m}} \Psi(t) + \mu^2 Y^T P^{2\alpha}_{\hat{m}\times\hat{m}} \Psi(t) = 0$$

$$Y^T + \sigma^2 Y^T P^{\alpha}_{\hat{m}\times\hat{m}} + \mu^2 Y^T P^{2\alpha}_{\hat{m}\times\hat{m}} = -\mu^2 [Q_0, Q_0, ..., Q_0] \phi^{-1}_{\hat{m}\times\hat{m}},$$

$$Y^T \left(I + \sigma^2 P^{\alpha}_{\hat{m}\times\hat{m}} + \mu^2 P^{2\alpha}_{\hat{m}\times\hat{m}} \right) = -\mu^2 [Q_0, Q_0, ..., Q_0] \phi^{-1}_{\hat{m}\times\hat{m}},$$

$$Y^T = -\mu^2 [Q_0, Q_0, ..., Q_0] \phi^{-1}_{\hat{m}\times\hat{m}} . \left(I + \sigma^2 P^{\alpha}_{\hat{m}\times\hat{m}} + \mu^2 P^{2\alpha}_{\hat{m}\times\hat{m}} \right)^{-1}. \quad (65)$$

Hence required

$$Q(t) = [Q_0, Q_0, ..., Q_0] \phi^{-1}_{\hat{m}\times\hat{m}} \Psi(t) - \mu^2 [Q_0, Q_0, ..., Q_0] \phi^{-1}_{\hat{m}\times\hat{m}} \left(I + \sigma^2 P^{\alpha}_{\hat{m}\times\hat{m}} + \mu^2 P^{2\alpha}_{\hat{m}\times\hat{m}} \right)^{-1} P^{2\alpha}_{\hat{m}\times\hat{m}} \Psi(t). \quad (66)$$

We manipulate the above system (65) of linear equations and obtain the unknown vector Y. Having these value of vector Y in equation (63), we get numerical results of RLC circuit for different value of k, M and α. Here, solution obtained by the proposed LWM approach for $\alpha = 0.25, 0.75, 0.999$ and $k = 2$ and $M = 3$ is graphically shown in figure 3. As it can be clearly seen, for $\alpha = 0.999$ and $k = 2$ and $M = 3$, the fractional RLC circuit graph behave similar to the classical solution graph for $\alpha = 2$. It is show that the proposed LWCM approach is more close to the exact solution. Table 3 describes the effectiveness of the proposed method by comparing with the classical solution at $\alpha = 2$. Table 3 shows that very high accuracies are obtained for $k = 2$ and $M = 3$ by the present method.

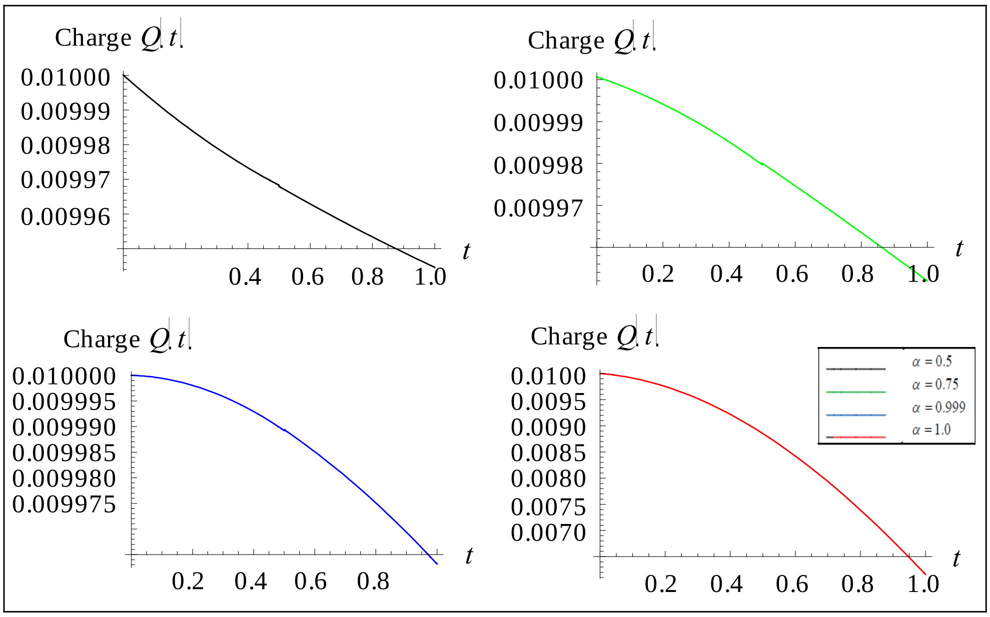

Figure 3. Charge $Q(t)$ versus time graph

$\left(R = 10, L = 10, C = 10, V_0 = 0.01 \text{ and } \alpha = 0.5, 0.75 \text{ and } 0.999\right)$.

Table 3. Numerical results of RC circuit for
$\left(R=10, L=10, C=10, V_0=0.01 \text{ and } \alpha=0.5, 0.75 \text{ and } 0.999\right)$.

t	$\alpha=0.5$	$\alpha=0.75$	$\alpha=0.999$	$\alpha=2$
	LW	LW	LW	CS
0.1	9.9924×10^{-3}	9.9977×10^{-3}	9.9994×10^{-3}	9.9928×10^{-3}
0.2	9.9853×10^{-3}	9.9941×10^{-3}	9.9980×10^{-3}	9.7523×10^{-3}
0.3	9.9790×10^{-3}	9.9899×10^{-3}	9.9958×10^{-3}	9.5205×10^{-3}
0.4	9.9733×10^{-3}	9.9851×10^{-3}	9.9929×10^{-3}	9.2197×10^{-3}
0.5	9.9679×10^{-3}	9.9800×10^{-3}	9.9893×10^{-3}	8.8529×10^{-3}
0.6	9.9629×10^{-3}	9.9746×10^{-3}	9.9850×10^{-3}	8.4235×10^{-3}
0.7	9.9580×10^{-3}	9.9691×10^{-3}	9.9803×10^{-3}	7.9354×10^{-3}
0.8	9.9534×10^{-3}	9.9635×10^{-3}	9.9750×10^{-3}	7.3927×10^{-3}
0.9	9.9489×10^{-3}	9.9578×10^{-3}	9.9693×10^{-3}	6.8001×10^{-3}

Solution of RL circuit

Finally, we consider RL circuit differential equation given in equation (4). RL circuit consists only resistor, inductor and a non-variant voltage source are present in the circuit and its differential equation is given as follows

$$LJ'(t)+RJ(t)=V. \tag{67}$$

with $J(0)=J_0$ and V is the constant voltage source.

The classical solution of equation (67) is

$$J(t)=\left[I_0-\frac{VL}{R}\right]e^{-\frac{R}{L}t}+\frac{VL}{R}. \tag{68}$$

Now, we analyse equation (67) using fractional calculus, we replace $J'(t)$ by $D^\alpha J(t)$, where $\alpha\in(0,1)$. In the sense of Riemann-Liouville derivative, we get the fractional order RL circuit and its differential equation as

$$\left(D^\alpha J\right)(t)+\frac{R}{L}J(t)=\frac{V}{L}. \tag{69}$$

Let $\frac{R}{L}=\sigma^2$ and $\frac{V}{L}=\rho^2$, then equation (69) become

$$\left(D^\alpha J\right)(t)+\sigma^2 J(t)=\rho^2. \tag{70}$$

We use equation (12) to approximate $\left(D^{\alpha}J\right)(t)$ as

$$\left(D^{\alpha}J\right)(t)=\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1}z_{nm}\psi_{nm}(t)=Z^{T}\Psi(t), \tag{71}$$

Integrating equation (71) with respect to t , over $[0,t]$, we get

$$J(t)=J(0)+Z^{T}P^{\alpha}_{\hat{m}\times\hat{m}}\Psi(t), \tag{72}$$

Similarly equation (39), we can approximate J_0 and ρ^2 as

$$J(0)=J_0=[J_0,J_0,...,J_0]_{1\times\hat{m}}\phi^{-1}_{\hat{m}\times\hat{m}}\Psi(t) \text{ and } \rho^2=[\rho^2,\rho^2,...,\rho^2]_{1\times\hat{m}}\phi^{-1}_{\hat{m}\times\hat{m}}\Psi(t). \tag{73}$$

Substituting equations (71-73) in equation (70), we obtain

$$Z^{T}\Psi(t)+\sigma^2\left([J_0,J_0,...,J_0]_{1\times\hat{m}}\phi^{-1}_{\hat{m}\times\hat{m}}\Psi(t)+Z^{T}P^{\alpha}_{\hat{m}\times\hat{m}}\Psi(t)\right)=[\rho^2,\rho^2,...,\rho^2]_{1\times\hat{m}}\phi^{-1}_{\hat{m}\times\hat{m}}\Psi(t),$$

$$Z^{T}\Psi(t)+\sigma^2[J_0,J_0,...,J_0]_{1\times\hat{m}}\phi^{-1}_{\hat{m}\times\hat{m}}\Psi(t)+\sigma^2Z^{T}P^{\alpha}_{\hat{m}\times\hat{m}}\Psi(t)=[\rho^2,\rho^2,...,\rho^2]_{1\times\hat{m}}\phi^{-1}_{\hat{m}\times\hat{m}}\Psi(t)$$

$$Z^{T}+\sigma^2[J_0,J_0,...,J_0]_{1\times\hat{m}}\phi^{-1}_{\hat{m}\times\hat{m}}+\sigma^2Z^{T}P^{\alpha}_{\hat{m}\times\hat{m}}=[\rho^2,\rho^2,...,\rho^2]_{1\times\hat{m}}\phi^{-1}_{\hat{m}\times\hat{m}}$$

$$Z^{T}=\left([\rho^2,\rho^2,...,\rho^2]_{1\times\hat{m}}-\sigma^2[J_0,J_0,...,J_0]_{1\times\hat{m}}\right)\phi^{-1}_{\hat{m}\times\hat{m}}.\left(I+\sigma^2P^{\alpha}_{\hat{m}\times\hat{m}}\right)^{-1}. \tag{74}$$

Hence required

$$J(t)=[J_0,J_0,...,J_0]_{1\times\hat{m}}\phi^{-1}_{\hat{m}\times\hat{m}}+\left([\rho^2,\rho^2,...,\rho^2]_{1\times\hat{m}}-\sigma^2[J_0,J_0,...,J_0]_{1\times\hat{m}}\right)\phi^{-1}_{\hat{m}\times\hat{m}}.\left(I+\sigma^2P^{\alpha}_{\hat{m}\times\hat{m}}\right)^{-1}.P^{\alpha}_{\hat{m}\times\hat{m}}\Psi(t). \tag{75}$$

By solving the above system (74) of linear equations, we can find the value of coefficient vector Z . Inserting the value of vector Z in equation (72), hence we obtain the numerical results for different value of k,M and α . Here, solution obtained by the proposed LWM approach for $\alpha=0.25,0.75,0.999$ and $k=2$ and $M=3$ is graphically show in figure 4. As it can be clearly seen, for $\alpha=0.999$ and $k=2;\ M=3,$ the fractional RL circuit graph behave similar to the classical solution graph for $\alpha=1$. It is show that the proposed LWM approach is more close to the exact solution. Table 4 shows the capability of the presented method and also shows that very high accuracies are obtained for $k=2$ and $M=3$.

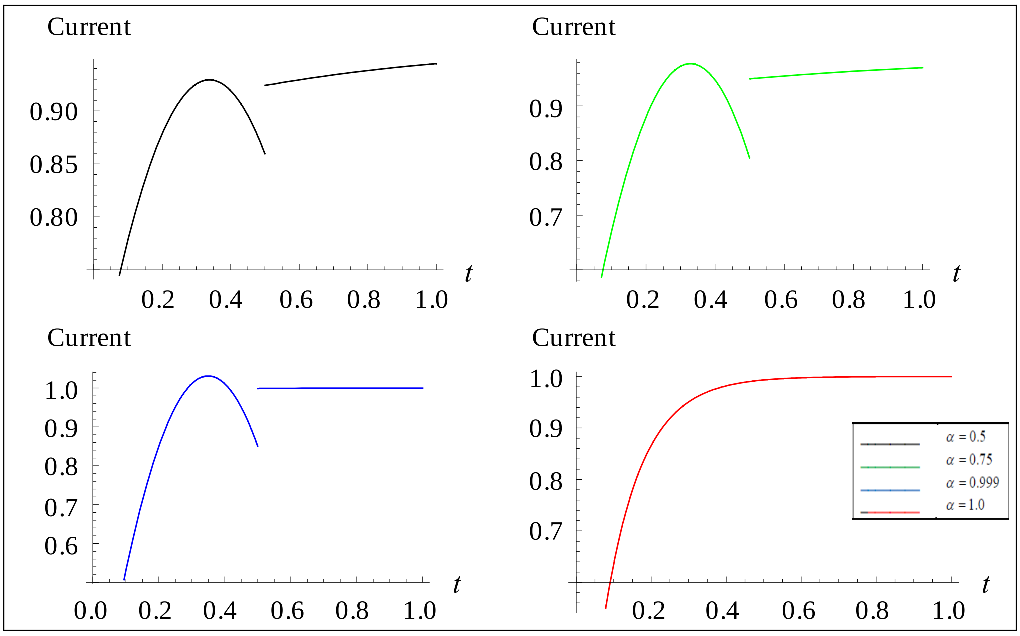

Figure 4. Current versus time graph
$\left(R = 10, L = 1, V = 10, V_0 = 0.01 \; and \; \alpha = 0.5, 0.75 \; and \; 0.999\right)$.

Table 4. Numerical results of RL circuit for
$\left(R = 10, L = 1, V = 10, V_0 = 0.01 \; and \; \alpha = 0.5, 0.75 \; and \; 0.999\right)$.

x	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 0.999$	$\alpha = 1$
	LW	LW	LW	CS
0.1	7.7867×10^{-1}	6.6899×10^{-1}	5.2978×10^{-1}	6.3579×10^{-1}
0.2	8.8787×10^{-1}	8.8797×10^{-1}	8.5076×10^{-1}	8.6602×10^{-1}
0.3	9.2552×10^{-1}	9.7257×10^{-1}	1.0111×10^{-1}	9.5071×10^{-1}
0.4	9.1906×10^{-1}	9.4748×10^{-1}	1.0108×10^{-1}	9.8186×10^{-1}
0.5	9.2404×10^{-1}	9.4989×10^{-1}	9.9895×10^{-1}	9.9332×10^{-1}
0.6	9.2929×10^{-1}	9.5497×10^{-1}	9.9938×10^{-1}	9.9755×10^{-1}
0.7	9.3397×10^{-1}	9.5953×10^{-1}	9.9967×10^{-1}	9.9909×10^{-1}
0.8	9.3808×10^{-1}	9.6359×10^{-1}	9.9983×10^{-1}	9.9966×10^{-1}
0.9	9.4162×10^{-1}	9.6713×10^{-1}	9.9986×10^{-1}	9.9987×10^{-1}

CONCLUSION

In this paper, the Legendre wavelet method (LWM) is applied to obtain approximate analytical solutions of the fractional electrical circuit models. It can be concluded that, LWM is very powerful and efficient technique for finding approximate solutions for many real life problems [14-16]. The main advantage of the method is its fast convergence to the solution. It has been shown in the theorem 6.1, by increasing k and order m of Legendre polynomial, the Legendre wavelet series converges very fast. See, in the figure 1, figure 2 and figure 3 approximate solutions graph behave as similar to the classical solution but $\alpha = 1.999$ the Caputo Fabrizio approach [1] shows damping and behave very differently for LC circuit. As similar proposed method present good approximated results for RC, LCR and RL at $\alpha = 0.999$ but the Caputo fractional derivative [1] graph for RL circuit coincide with the classical solution but diverges to very large positive values as time progresses. The numerical results obtained here, conform to its high degree of accuracy. Such analysis can be further applied to other physical models to develop a better understanding of use of wavelets in real life problems. The implementation of this method is a very easy acceptable and valid. The solutions of the electrical circuit equations are presented graphically and in tabular form.

ACKNOWLEDGMENT

The authors are very thankful to respected Dr. Sag Ram Verma, Department of Mathematics and Statistics, Gurukula Kangri University, Haridwar, for encouragement and support.

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