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Intuitionistic Fuzzy Linear Fractional Programming Problem

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Abstract

Linear Fractional Programming is an optimization technique of dividing a work among the existing persons satisfying the given constraints. Here we consider the Linear Fractional Programming Problem with Symmetric Trapezoidal Intuitionistic Fuzzy Numbers (STIFN) for the decision variables and right side resources of the constraints. We solve the Intuitionistic Fuzzy Linear Fractional Programming Problem (IFLFPP) using Intuitionistic Fuzzy Simplex Algorithm. An illustrated example is given to show the efficiency of the proposed algorithm.

Keywords: Linear Fractional Programming Problem, Intuitionistic Fuzzy Set, Symmetric Trapezoidal Intuitionistic Fuzzy Number, Simplex Method.

2010Mathematics Subject Classification: 90C32, 03E72.

1. INTRODUCTION

The information available in real life is insufficient and uncertain and so the parameters of the problem are considered as fuzzy numbers. Since fuzziness in decision making problems are very limited in scope, the Intuitionistic Fuzzy Set (IFS) theory seems to be applicable to address this issue of uncertainty. Intuitionistic fuzzy sets have been found to be highly useful to deal with vagueness and it is applied in many fields such as decision making, medical diagnosis. Linear programming is an optimization technique which is most frequently applied in real-world problems. Linear Fractional Programming is a technique which is used to solve the problem of maximizing the fraction of two linear functions subject to a set of linear constraints and the non-negativity restrictions.

Intuitionistic fuzzy set was introduced by Attanassov [1] by adding an additional non-membership degree and hesitancy degree. S.K. Bharati and S.R. Singh [2] presented a sign distance method to find the optimal solution of a fully intuitionistic fuzzy linear programming problem. Charnes and Cooper [3] solved linear fractional programming problem by resolving it into two linear programming problems. Das and Hassan Mishmast Nehi [4] proposed a method of ranking intuitionistic fuzzy numbers based on the characteristic values of membership and non-membership functions of an intuitionistic fuzzy number. Mojgan Esmailzadeh and Mojdeh Esmailzadeh [5] presented a metric space method to calculate the distance between Intuitionistic fuzzy sets on the basis of α-cuts.

AL. Nachammai and P. Thangaraj [6] solved intuitionistic fuzzy linear programming problem by using Metric Distance Ranking. Nuran Guzel [7] solved the multi objective linear fractional programming problem by converting MOLFPP into LPP based on the theorem dealing with nonlinear fractional programming with single objective function. Pandy and Punnen [8] developed a Simplex method to solve the piecewise linear fractional programming problem.

R. Parvathi and C. Malathi [9] proposed a new type of arithmetic operations for symmetric trapezoidal intuitionistic fuzzy numbers based on (α, β) cuts. These operations find applications in solving linear programming problems in intuitionistic fuzzy environment. Ratnesh Rajan Saxena and Rashmi Gupta [10] defined an enumeration technique and a lexicographic approach for solving linear fractional fuzzy set covering problem.

Ranking of Intuitionistic Fuzzy Numbers plays a very important role in multi criteria decision making, optimization and in many different fields. Satyajit Das and

Debashree Guha [11] proposed a method for ranking of Intuitionistic Fuzzy Numbers by determining centroid point of Intuitionistic Fuzzy Numbers. S.F. Tantawy [12] proposed an iterative method based on the conjugate gradient projection method to solve linear fractional programming problem.

We consider a linear fractional programming problem in which the decision variables and resources of the constraints are symmetric trapezoidal intuitionistic fuzzy numbers. This paper is outlined as follows. In section 2, some basic definitions of intutitionistic fuzzy number are given. In section 3, fuzzy linear fractional programming problem is given and its solutions are discussed. In section 4, intutionistic fuzzy simplex algorithm is given. In section 5, a numerical example is presented and section 6 concludes the paper.

2. PRELIMINARIES:

Definition 2.1: An Intuitionistic Fuzzy set (IFS) A in X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define the degree of membership and the degree of nonmembership of the element $x \in X$ respectively and for every $x \in X$ in A, $0 \le \mu_A + \nu_A \le 1$ holds.

Definition 2.2: For every common fuzzy subset A in X, Intuitionistic fuzzy index of x in A is defined as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$. It is also known as degree of hesitancy or degree of uncertainty of the element x in A. Obviously, for every $x \in X$, $0 \le \pi_A(x) \le 1$

Definition 2.3: An Intuitionistic Fuzzy number (IFN) $\tilde{A}^I$ is

(i) an intuitionistic fuzzy subset of the real line

(ii) normal, that is there is some $x_0 \in R$ such that $\mu_{\tilde{A}^I}(x_0) = 1, \nu_{\tilde{A}^I}(x_0) = 0$

(iii) convex for the membership function $\mu_{\tilde{A}^I}(x)$, that is

$$\mu_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \ge \min(\mu_{\tilde{A}^I}(x_1), \mu_{\tilde{A}^I}(x_2)) \text{ for every } x_1, x_2 \in R, \lambda \in [0,1]$$

(iv) concave for the non membership function $\nu_{\tilde{A}^I}(x)$, that is

$$\nu_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \le \max(\nu_{\tilde{A}^I}(x_1), \nu_{\tilde{A}^I}(x_2)) \text{ for every } x_1, x_2 \in R, \lambda \in [0,1]$$

Definition 2.4: A Trapezoidal Intuitionistic Fuzzy number (TIFN) $\tilde{A}^I$ is an IFS in R with membership function and nonmembership function as follows.

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \dfrac{x-(a_1-\alpha)}{\alpha}, & a_1-\alpha \le x \le a_1 \\ 1, & a_1 \le x \le a_2 \\ \dfrac{a_2+(\beta-x)}{\beta}, & a_2 \le x \le a_2+\beta \\ 0, & otherwise \end{cases}$$

$$\nu_{\tilde{A}^I}(x) = \begin{cases} \dfrac{a_1-x}{\alpha}, & a_1-\alpha \le x \le a_1 \\ 0, & a_1 \le x \le a_2 \\ \dfrac{x-a_2}{\beta}, & a_2 \le x \le a_2+\beta \\ 1, & otherwise \end{cases}$$

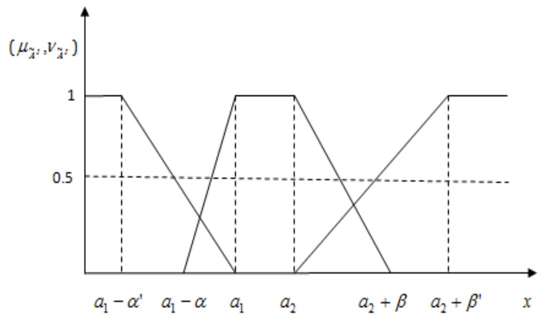

Figure 1: Trapezoidal Intuitionistic Fuzzy Number

where $a_1 \le a_2, \alpha, \beta \ge 0 \;\; \alpha \le \alpha', \beta \le \beta'$

Definition 2.5: Symmetric Trapezoidal Intuitionistic Fuzzy Number (STIFN)

An IFS $\tilde{A}^I$ in R is said to be a Symmetric Trapezoidal Intuitionistic Fuzzy Number (STIFN) if there exist real numbers a_1, a_2, h, h' where $a_1 \le a_2, h \le h', h, h' > 0$ such that the membership and nonmembership functions are as follows.

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \dfrac{x-(a_1-h)}{h}, & a_1-h \le x \le a_1 \\ 1, & a_1 \le x \le a_2 \\ \dfrac{a_2+(h-x)}{h}, & a_2 \le x \le a_2+h \\ 0, & otherwise \end{cases}$$

$$\nu_{\tilde{A}^I}(x) = \begin{cases} \dfrac{a_1-x}{h'}, & a_1-h' \le x \le a_1 \\ 0, & a_1 \le x \le a_2 \\ \dfrac{x-a_2}{h'}, & a_2 \le x \le a_2+h' \\ 1, & otherwise \end{cases}$$

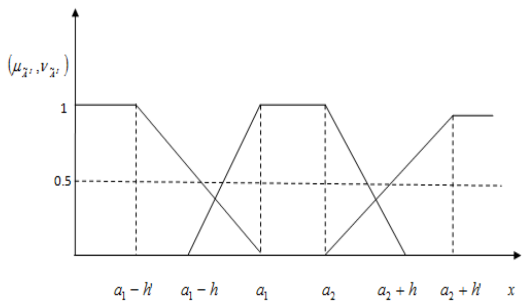

Figure 2: Symmetric Trapezoidal Intuitionistic Fuzzy Number

A STIFN is denoted by $\tilde{A}^I = [a_1-h, a_1, a_2, a_2+h; a_1-h', a_1, a_2, a_2+h']$

If $h = h'$ then the STIFN becomes STFN.

Arithmetic Operations on STIFN

1. Addition:

Let $\tilde{A}^I = [a_1, a_2, h, h; a_1, a_2, h', h']$ and $\tilde{B}^I = [b_1, b_2, k, k; b_1, b_2, k', k']$

then $\tilde{C}^I = \tilde{A}^I + \tilde{B}^I = [a_1 + b_1, a_2 + b_2, h + k, h + k; a_1 + b_1, a_2 + b_2, h'+k', h'+k']$

2. Subtraction:

$$\tilde{C}^I = \tilde{A}^I - \tilde{B}^I = [a_1 - b_2, a_2 - b_1, h + k, h + k; a_1 - b_2, a_2 - b_1, h'+k', h'+k']$$

3. Scalar Multiplication

Let $k \in R.$ Then

$$k\tilde{A}^I = \begin{cases} [ka_1, ka_2, kh, kh; ka_1, ka_2, kh', kh'] & if\ k > 0 \\ [ka_2, ka_1, -kh, -kh; ka_2, ka_1, -kh', -kh'] & if\ k < 0 \end{cases}$$

Ranking Function

A convenient approach for solving IFLFPP is based on the concept of comparison of IF numbers by use of ranking functions. A ranking function is used to order IFN and it is a mapping from IFS to the set of real numbers R. A ranking function $\Re : F(R) \to R$ maps each STIFN into the real line. The following method is suggested for ranking classical intuitionistic fuzzy numbers.

$$\Re\left(\tilde{A}^I\right) = a_1 + a_2 + \frac{1}{2}(h'-h)$$

3. MATHEMATICAL MODEL

The fuzzy linear fractional programming problem (FLFPP) is given by

$$\text{Maximize } \tilde{z}^I = \frac{c\tilde{x}^I + \tilde{\alpha}^I}{d\tilde{x}^I + \tilde{\beta}^I}$$

$$\text{Subject to } \quad A\tilde{x}^I \le \tilde{b}^I$$

$$\tilde{x}^I \ge 0$$

where $c^T \in F(R^n),\ \tilde{x}^I \in F(R^n),\ A \in F(R^{m \times n}),\ \tilde{b}^I \in F(R^m)$

Definition 3.1: Feasible Solution

Any $\tilde{x}^I \in F(R^n)$ which satisfies the constraints and non-negative restrictions of FLFPP is said to be a feasible solution.

Definition 3.2: Initial Basic Feasible Solution

Let $\tilde{x}_B{}^I$ be the initial basic feasible solution such that $\tilde{x}_B{}^I = B^{-1}\tilde{b}^I$

Let $\tilde{z}_{(1)}{}^I = c_B\tilde{x}_B{}^I + \tilde{\alpha}^I$ and $\tilde{z}_{(2)}{}^I = d_B\tilde{x}_B{}^I + \tilde{\beta}^I$

where c_B and d_B are the vectors having their components as the coefficients associated with the basic variables in the numerator and the denominator of the objective function respectively.

4. INTUTIONISTIC FUZZY SIMPLEX ALGORITHM

Step 1: Check whether the objective function is to be maximized or minimized. If it is to be minimized, then convert $Min\,\tilde{z}^I = -Max\left(-\tilde{z}^I\right)$

Step 2: Check whether all $\tilde{b}_i^I$'s are non-negative. If anyone $\tilde{b}_i^I$ is negative, then multiply the inequality by -1. Convert all the inequalities into equalities by introducing slack and/or surplus variables.

Step 3: Compute $\tilde{z}_{(1)}{}^I = c_B\tilde{x}_B{}^I + \tilde{\alpha}^I$ and $\tilde{z}_{(2)}{}^I = d_B\tilde{x}_B{}^I + \tilde{\beta}^I$ and $\tilde{z}^I = \dfrac{\tilde{z}_{(1)}{}^I}{\tilde{z}_{(2)}{}^I}$

Step 4: Obtain an initial fuzzy basic feasible solution to the given FLFPP

Step 5: Compute $\tilde{\Delta}_j{}^I = \tilde{z}_{(2)}{}^I\left(z_{1j} - c_j\right) - \tilde{z}_{(1)}{}^I\left(z_{2j} - d_j\right)$ Where $\begin{array}{l} z_{1j} - c_j = c_B y_j - c_j \\ z_{2j} - d_j = d_B y_j - d_j \end{array}$

Step 6: Examine the sign of Δ_j

(i) If all $\tilde{\Delta}_j{}^I \geq 0$, then the initial basic feasible solution $\tilde{x}_B{}^I$ is an optimum solution.

(ii) Otherwise go to next step.

Step 7: If there are more than one negative Δ_j, then choose the most negative of them. Let it be $\tilde{\Delta}_r{}^I$.

(i) If all $y_{ir} \leq 0$, then there is an unbounded solution to the given problem.

(ii) If atleast one $y_{ir} > 0$, then the corresponding vector $\tilde{y}_r^{\,I}$ enters the basis $\tilde{y}_B^{\,I}$.

Step 8: Compute the ratios $\left\{ \frac{\tilde{x}_{Bi}^{\,I}}{y_{ir}},\ y_{ir} > 0, i = 1,2,...,m \right\}$ and choose the minimum of them. Let the minimum of these ratios be $\frac{\tilde{x}_{Bk}^{\,I}}{y_{kr}}$. Then the vector $\tilde{y}_k^{\,I}$ will leave the basis $\tilde{y}_B^{\,I}$. The element y_{kr} is known as the leading element.

Step 9: Convert the leading element to unity by dividing its row by the leading element itself, and all other elements in its column to zeros by using the following relations. $(new\, y_{ij}) = y_{ij} - \frac{y_{kj}}{y_{kr}} y_{ir}$ and $(new\, y_{kj}) = \frac{y_{kj}}{y_{kr}}$

Step 10: Go to step 6 and repeat the procedure until either an optimum solution is obtained or there is an indication of an unbounded solution.

5. NUMERICAL EXAMPLE

$$Max\ \tilde{z}^I = \frac{6\tilde{x}_1^{\,I} + 5\tilde{x}_2^{\,I}}{2\tilde{x}_1^{\,I} + \tilde{7}^I}$$

Subject to

$$\tilde{x}_1^{\,I} + 2\tilde{x}_2^{\,I} \leq (2,4,1,1;2,4,2,2)$$
$$3\tilde{x}_1^{\,I} + 2\tilde{x}_2^{\,I} \leq (5,7,2,2;5,7,4,4)$$
$$\tilde{x}_1^{\,I}, \tilde{x}_2^{\,I} \geq 0$$

Table 1: Initial Iteration

			c_j	6	5	0	0
			d_j	2	0	0	0
$c_{\mathbf{B}}$	$d_{\mathbf{B}}$	$\tilde{y}_{\mathbf{B}}{}^I$	$\tilde{x}_{\mathbf{B}}{}^I$	$\tilde{y}_1{}^I$	$\tilde{y}_2{}^I$	$\tilde{y}_3{}^I$	$\tilde{y}_4{}^I$
0	0	$\tilde{y}_3{}^I$	$(2,4,1,1;2,4,2,2)$	1	2	1	0
0	0	$\tilde{y}_4{}^I$ $\leftarrow$	$(5,7,2,2;5,7,4,4)$	3*	2	0	1
$\tilde{z}_{(1)}{}^I = c_B \tilde{x}_B{}^I + \tilde{\alpha}^I = \tilde{0}^I$			$z_{1j} - c_j$	-6	-5	0	0
$\tilde{z}_{(2)}{}^I = d_B \tilde{x}_B{}^I + \tilde{\beta}^I = (6,8,1,1;6,8,3,3)$			$z_{2j} - d_j$	-2	0	0	0
			$\tilde{\Delta}_j{}^I$	$\begin{pmatrix} -48,-36,6,6; \\ -48,-36,18,18 \end{pmatrix}$ $\uparrow$	$\begin{pmatrix} -40,-30,5,5; \\ -40,-30,15,15 \end{pmatrix}$	$\tilde{0}^I$	$\tilde{0}^I$

Table 2: First Iteration

0	0	$\tilde{y}_3^{\,I}$ $\leftarrow$	$\left(-\frac{1}{3},\frac{7}{3},\frac{5}{3},\frac{5}{3};-\frac{1}{3},\frac{7}{3},\frac{10}{3},\frac{10}{3}\right)$	0	$\frac{4}{3}*$	1	$-\frac{1}{3}$
6	2	$\tilde{y}_1^{\,I}$	$\left(\frac{5}{3},\frac{7}{3},\frac{2}{3},\frac{2}{3};\frac{5}{3},\frac{7}{3},\frac{4}{3},\frac{4}{3}\right)$	1	$\frac{2}{3}$	0	$\frac{1}{3}$
$\tilde{z}_{(1)}^{\,I}=c_B\tilde{x}_B^{\,I}+\tilde{\alpha}^I$ $=(10,14,4,4;10,14,8,8)$			$z_{1j}-c_j$	0	-1	0	2
$\tilde{z}_{(2)}^{\,I}=\tilde{d}_B^{\,I}\tilde{x}_B^{\,I}+\tilde{\beta}^I=$ $\left(\frac{28}{3},\frac{38}{3},\frac{7}{3},\frac{7}{3};\frac{28}{3},\frac{38}{3},\frac{17}{3},\frac{17}{3}\right)$			$z_{2j}-d_j$	0	$\frac{4}{3}$	0	$\frac{2}{3}$
			$\tilde{\Delta}_j^{\,I}$	$\tilde{0}^I$	$\left(-\frac{94}{3},-\frac{68}{3},\frac{23}{3},\frac{23}{3};-\frac{94}{3},-\frac{68}{3},\frac{49}{3},\frac{49}{3}\right)$ $\uparrow$	$\tilde{0}^I$	$\left(\frac{28}{3},\frac{56}{3},\frac{22}{3},\frac{22}{3};\frac{28}{3},\frac{56}{3},\frac{50}{3},\frac{50}{3}\right)$

Table 3: Final Iteration

5	0	$\tilde{y}_2^{\,I}$	$\left(-\frac{1}{4},\frac{7}{4},\frac{5}{4},\frac{5}{4};-\frac{1}{4},\frac{7}{4},\frac{10}{4},\frac{10}{4}\right)$	0	1	$\frac{3}{4}$	$-\frac{1}{4}$
6	2	$\tilde{y}_1^{\,I}$	$\left(\frac{1}{2},\frac{5}{2},\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{5}{2},3,3\right)$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$
$\tilde{z}_{(1)}^{\,I}=c_B\tilde{x}_B^{\,I}+\tilde{\alpha}^I=\left(\frac{7}{4},\frac{95}{4},\frac{61}{4},\frac{61}{4};\frac{7}{4},\frac{95}{4},\frac{61}{2},\frac{61}{2}\right)$			$z_{1j}-c_j$ $z_{2j}-d_j$	0 0	0 0	$\frac{3}{4}$	$\frac{7}{4}$
$\tilde{z}_{(2)}^{\,I}=d_B\tilde{x}_B^{\,I}+\tilde{\beta}^I=(7,13,4,4;7,13,9,9)$						-1	1
$\tilde{z}^I=\dfrac{\tilde{z}_{(1)}^{\,I}}{\tilde{z}_{(2)}^{\,I}}$			$\tilde{\Delta}_j^{\,I}$	$\tilde{0}^I$	$\tilde{0}^I$	$\left(\frac{28}{4},\frac{134}{4},\frac{73}{4},\frac{73}{4};\frac{28}{4},\frac{134}{4},\frac{149}{4},\frac{149}{4}\right)$	$\left(-\frac{46}{4},\frac{84}{4},\frac{89}{4},\frac{89}{4};-\frac{46}{4},\frac{84}{4},\frac{185}{4},\frac{185}{4}\right)$

From the table, the optimal solution is given by

$$\tilde{x}_1^{\,I}=\left(\frac{1}{2},\frac{5}{2},\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{5}{2},3,3\right),\ \Re\left(\tilde{x}_1^{\,I}\right)=3.75$$

$$\tilde{x}_2^{\,I}=\left(\frac{-1}{4},\frac{7}{4},\frac{5}{4},\frac{5}{4};\frac{-1}{4},\frac{7}{4},\frac{10}{4},\frac{10}{4}\right),\ \Re\left(\tilde{x}_2^{\,I}\right)=2.125$$

$$\text{Max}\,\Re\left(\tilde{z}^I\right)=\frac{\Re\left(\tilde{z}_{(1)}^{\,I}\right)}{\Re\left(\tilde{z}_{(2)}^{\,I}\right)}=\frac{33.125}{22.5}=1.472$$

CONCLUSION

We consider the linear fractional programming problem with Symmetric Trapezoidal Intuitionistic Fuzzy Numbers (STIFN) for the decision variables and right side resources of the constraints. We solve the Intuitionistic Fuzzy Linear Fractional Programming Problem (IFLFPP) by using intuitionistic fuzzy simplex algorithm. An example is illustrated to show the efficiency of the proposed algorithm.

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