Proof. case (i) : If $\mathcal{G}_{e_k}(P^*_{n\text{-}1}, i-1) \neq \phi$ and $\mathcal{G}_{e_k}(P^*_n - \{2n\}, i-1) = \phi$ then by theorem 3.3 (i), we have $\mathcal{G}_{e_k}(P^*_n, i) = \mathcal{G}_{e_k}(P^*_n, n) = \{2, 4, 6, ..., 2n\}$. Therefore, $g_{e_k}(P^*_n, i) = 1$. Also, $\mathcal{G}_{e_k}(P^*_{n-1}, i-1) = \mathcal{G}_{e_k}(P^*_{n-1}, n-1) = \{2, 4, 6, ..., 2n-2\}$. Therefore, $g_{e_k}(P^*_{n-1}, i-1) = 1$. $g_{e_k}(P^*_n - \{2n\}, i-1) = 0$. Therefore, the theorem holds.

case(ii). If $\mathcal{G}_{e_k}(P^*_{n\text{-}1}, i-1) = \phi$ and $\mathcal{G}_{e_k}(P^*_n - \{2n\}, i-1) \neq \phi$ then by theorem 3.3 (ii), we have $\mathcal{G}_{e_k}(P^*_n, i) = \mathcal{G}_{e_k}(P^*_n, 2n-2) = \{2, 4, 5, 6, ..., 2n\}$. Therefore, $g_{e_k}(P^*_n, i) = 1$.
Also, $g_{e_k}(P^*_n - \{2n\}, i-1) = g_{e_k}(P^*_n - \{2n\}, 2n-3) = \{2, 4, 5, 6, ..., 2n-1\}$. Therefore, $g_{e_k}(P^*_n - \{2n\}, i-1) = 1$. Also, $\mathcal{G}_{e_k}(P^*_{n\text{-}1}, i-1) = 0$. Therefore, the theorem holds. .

case(iii) : If $\mathcal{G}_{e_k}(P^*_{n\text{-}1}, i-1) \neq \phi$ and $\mathcal{G}_{e_k}(P^*_n - \{2n\}, i-1) \neq \phi$ then
by theorem 3.3 (iii), we have, $\mathcal{G}_{e_k}(P^*_n, i) = A_1 \cup A_2$
where $A_1 = \left\{X \cup \{2n\} / X \in \mathcal{G}_{e_k}(P^*_{n\text{-}1}, i-1)\right\}$ and
$A_2 = \left\{X \cup \{2n\} / X \in \mathcal{G}_{e_k}(P^*_n - \{2n\}, i-1)\right\}$.
Therefore $|A_1| = \left|g_{e_k}(P^*_{n\text{-}1}, i-1)\right|$ and $|A_2| = \left|g_{e_k}(P^*_n - \{2n\}, i-1)\right|$.
Since for every $X_1 \in A_1$ and $X_2 \in A_2$, we have $2n-1 \in X_2$ but $2n-1 \notin X_1$. Therefore, $A_1 \cap A_2 = \phi$. Hence, $g_{e_k}(P^*_n, i) = g_{e_k}(P^*_{n\text{-}1}, i-1) + g_{e_k}(P^*_n - \{2n\}, i-1)$

Table 1. $g_{e_k}(P^*_n, i)$ *and* $g_{e_k}(P^*_n - \{2n\}, i)$

I	2	3	4	5	6	7	8	9	10	11	12	13	14
P^*_2	1												
$P^*_3 - \{6\}$	0	1											
P^*_3	0	1	1										
$P^*_4 - \{8\}$	0	0	1	1									
P^*_4	0	0	1	2	1								
$P^*_5 - \{10\}$	0	0	0	1	2	1							
P^*_5	0	0	0	1	3	3	1						
$P^*_6 - \{12\}$	0	0	0	0	1	3	3	1					

P_6^*	0	0	0	0	1	4	6	4	1				
$P_7^* - \{14\}$	0	0	0	0	0	1	4	6	4	1			
P_7^*	0	0	0	0	0	1	5	10	10	5	1		
$P_8^* - \{16\}$	0	0	0	0	0	0	1	5	10	10	5	1	
P_8^*	0	0	0	0	0	0	1	6	15	20	15	6	1

Remark:
By similar way we can construct edge fixed geodominating sets for other similar non-pendant edges.

Theorem 3.6. Let e_k be a non-pendant edge which is common to both $3̧^*$ and $^*_{Q} - \{2ℬ\}$. Then;

(i) For every $n \geq 3$, $G_{e_k}(P_n^*, x) = x[G_{e_k}(P_{n-1}^*, x) + G_{e_k}(P_n^* - \{2n\}, x)]$ with the initial values $G_{e_k}(P_2^*, x) = x^2$ and $G_{e_k}(P_3^* - \{6\}, x) = x^3$.

(ii) For every $n \geq 4$, $G_{e_k}(P_n^* - \{2n\}, x) = x[G_{e_k}(P_{n-2}^*, x) + G_{e_k}(P_{n-1}^* - \{2n-2\}, x)]$ with the initial values $G_{e_k}(P_2^*, x) = x^2$ and $G_{e_k}(P_3^* - \{6\}, x) = x^3$.

(iii) For every $n \geq 4$, $G_{e_k}(P_n^*, x) = x^n(1+x)^{n-2}$, $G_{e_k}(P_n^* - \{2n\}, x) = x^n(1+x)^{n-3}$.

Proof of (i). By theorem 3.5,

$$\sum_{i=n}^{2n-2} g_{e_k}(P_n^*, i)\, x^i = \sum_{i=n}^{2n-2} g_{e_k}(P_{n-1}^*, i-1)\, x^i + \sum_{i=n}^{2n-2} g_{e_k}(P_n^* - \{2n\}, i-1)\, x^i$$

$$G_{e_k}(P_n^*, x) = x\left[\sum_{i=n}^{2n-2} g_{e_k}(P_{n-1}^*, i-1)\, x^{i-1} + \sum_{i=n}^{2n-2} g_{e_k}(P_n^* - \{2n\}, i-1)\, x^{i-1}\right]$$

$$= x[G_{e_k}(P_{n-1}^*, x) + G_{e_k}(P_n^* - \{2n\}, x)]$$

Proof of (ii). By theorem 2.5,

$$g_{e_k}(P_n^* - \{2n\}, i) = g_{e_k}(P_{n-2}^*, i-2) + g_{e_k}(P_{n-1}^* - \{2n-2\}, i-2)$$

$$\sum_{i=n}^{2n-3} g_{e_k}(P_n^* - \{2n\}, i)\, x^i = \sum_{i=n}^{2n-3} g_{e_k}(P_{n-2}^*, i-2)\, x^i + \sum_{i=n}^{2n-3} g_{e_k}(P_{n-1}^* - \{2n-2\}, i-2)\, x^i$$

$$G_{e_k}(P_n^* - \{2n\}, x) = x^2\left[\sum_{i=n}^{2n-3} g_{e_k}(P_{n-2}^*, i-2)x^{i-2} + \sum_{i=n}^{2n-3} g_{e_k}(P_{n-1}^* - \{2n-2\}, i-2)x^{i-2}\right]$$

$$= x^2[G_{e_k}(P^*_{n-2}, x) + G_{e_k}(P^*_{n-1} - \{2n-2\}, x)] \ .$$

Proof of (iii) We shall prove both equalities together by induction on *n*. The result is true for *n* = 4, because $G_{e_k}(P^*_4, x) = x^4(1+x)^{4-2} = x^4 + 2x^5 + x^6$.
$G_{e_k}(P^*_4 - \{8\}, x) = x^4(1+x)^{4-3} = x^4 + x^5$. Assume that the result is true for all natural numbers less than *n*. We prove the result for *n*. We have $G_{e_k}(P^*_{n-1}, x) = x^{n-1}(1+x)^{n-3}$ and $G_{e_k}(P^*_{n-1} - \{2n-2\}, x) = x^{n-1}(1+x)^{n-4}$.

Now by part (i) and induction hypothesis,

$$\begin{aligned} G_{e_k}(P^*_n, x) &= x[G_{e_k}(P^*_{n-1}, x) + G_{e_k}(P^*_n - \{2n\}, x)] \\ &= x[x^{n-1}(1+x)^{n-3} + x^n(1+x)^{n-3}] \\ &= x^n(1+x)^{n-3}(1+x) \\ &= x^n(1+x)^{n-2} \end{aligned}$$

Now by part (ii) and induction hypothesis,

$$\begin{aligned} G_{e_k}(P^*_n - \{2n\}, x) &= x^2[G_{e_k}(P^*_{n-2}, x) + G_{e_k}(P^*_{n-1} - \{2n-2\}, x)] \\ &= x^2[x^{n-2}(1+x)^{n-4} + x^{n-1}(1+x)^{n-4} \\ &= x^n(1+x)^{n-3} \end{aligned}$$

Theorem 3.7. Let $\mathrm{H}_{\mathrm{N}} = \{1,3\}$ be a non- pendant edge of the centipede $P^*_n, n \geq 2$. Suppose that $n \geq 3$. Then for every $n \leq i \leq 2n-2$, $g_{e_k}(P^*_n, i) = \binom{n-2}{i-n}$ and for every $n \leq i \leq 2n-3$, $g_{e_k}(P^*_n - \{2n\}, i) = \binom{n-3}{i-n}$.

Proof. We shall prove both equalities together by induction on *n*. We have the result for $n = 3$ from Table 2.1. Now suppose the results are true for all natural numbers less than *n*, and prove them for *n*. By theorem 2.5 and induction hypothesis we have,

$$\begin{aligned} g_{e_k}(P^*_n - \{2n\}, i) &= g_{e_k}(P^*_{n-2}, i-2) + g_{e_k}(P^*_{n-1} - \{2n-2\}, i-2) \\ &= \binom{n-4}{i-n} + \binom{n-4}{i-n-1} \\ &= \binom{n-3}{i-n} \end{aligned}$$

Now by theorem 3.5,

we have $g_{e_k}(P^*_n, i) = g_{e_k}(P^*_{n-1}, i-1) + g_{e_k}(P^*_n - \{2n\}, i-1)$

$$= \binom{n-3}{i-n} + \binom{n-3}{i-n-1}$$

$$= \binom{n-2}{i-n}$$

Corollary 3.8. Let $e_k = \{1,3\}$ be a non- pendant edge of the centipede $P_n^*, n \geq 2$. Then the following properties hold for coefficients of $G_{e_k}(P_n^*, x)$ and $G_{e_k}(P_n^* - \{2n\}, x)$ for every $n \geq 3$;

(i) $g_{e_k}(P_n^*, 2n-2) = 1, g_{e_k}(P_n^*, n) = 1$

(ii) $g_{e_k}(P_n^*, 2n-3) = n-2,\ g_{e_k}(P_n^*, n+1) = n-2$

(iii) $g_{e_k}(P_n^*, 2n-4) = \dfrac{(n-2)(n-3)}{2}, g_{e_k}(P_n^*, n+2) = \dfrac{(n-2)(n-3)}{2}$

(iv) $g_{e_k}(P_n^*, 2n-5) = \dfrac{(n-2)(n-3)(n-4)}{6}, g_{e_k}(P_n^*, n+3) = \dfrac{(n-2)(n-3)(n-4)}{6}$

(v) $g_{e_k}(P_n^* - \{2n\}, 2n-3) = 1, g_{e_k}(P_n^* - \{2n\}, n) = 1,$

(vi) $g_{e_k}(P_n^* - \{2n\}, 2n-4) = n-3,\ g_{e_k}(P_n^* - \{2n\}, n+1) = n-3,$

(vii) $g_{e_k}(P_n^* - \{2n\}, 2n-5) = \dfrac{(n-3)(n-4)}{2}, g_{e_k}(P_n^* - \{2n\}, n+2) = \dfrac{(n-3)(n-4)}{2}$

(viii) If $S_n = \sum_{i=n}^{2n-2} g_{e_k}(P_n^*, i)$ then for every $n \geq 3,\ S_n = 2(S_{n-1})$ with the initial value $S_2 = 2$.

(ix) If $S_n = \sum_{i=n}^{2n-3} g_{e_k}(P_n^* - \{2n\}, i)$ then for every $n \geq 4,\ S_n = 2(S_{n-1})$

Proof. The properties (i) to (vii) hold, by theorem 3.7.

Proof of (viii). $S_n = \sum_{i=n}^{2n-2} g_{e_k}(P_n^*, i)$

$$= \sum_{i=n}^{2n-2} g_{e_k}(P_{n-1}^*, i-1) + \sum_{i=n}^{2n-2} g_{e_k}(P_n^* - \{2n\}, i-1)$$

$$= \sum_{i=n}^{2n-3} \binom{n-3}{i-n} + \sum_{i=n+1}^{2n-3} \binom{n-3}{i-n-1}$$

$$= 2\sum_{i=n}^{2n-3} \binom{n-3}{i-n}$$

$$= 2\sum_{i=n}^{2n-3} g_{e_k}(P_{n-1}^*, i-1)$$

$$= 2S_{n-1}$$

Proof of (ix). $S_n = \sum_{i=n}^{2n\text{-}3} g_{e_k}(P_n^* - \{2n\}, i)$

$$= \sum_{i=n}^{2n\text{-}3} g_{e_k}(P_{n-2}^*, i-2) + \sum_{i=n}^{2n\text{-}3} g_{e_k}(P_{n-1}^* - \{2n-2\}, i-2)$$

$$= \sum_{i=n\text{-}1}^{2n\text{-}4} \binom{n-4}{i-n} + \sum_{i=n+1}^{2n\text{-}3} \binom{n-4}{i-n-1}$$

$$=2 \sum_{i=n+1}^{2n\text{-}3} \binom{n-4}{i-n-1}$$

$$= 2 \sum_{i=n}^{2n\text{-}3} g_{e_k}(P_{n-1}^* - \{2n-2\}, i-2)$$

$$= 2\, S_{n-1}$$

4. EDGE FIXED GEODOMINATING SETS OF $\mathrm{P}_\mathrm{n}^* - \{2n\}$ AND CENTIPEDES FOR PENDANT EDGES

For the construction of edge fixed geodominating sets of centipede P_n^*, we need to investigate the edge fixed geodominating sets of $\mathrm{P}_\mathrm{n}^* - \{2n\}$. In this section, we investigate edge fixed geodominating sets of centipedes. We construct $\mathrm{g}_{\mathrm{e_k}}(\mathrm{P}_\mathrm{n}^*, i)$ from $\mathrm{g}_{\mathrm{e_k}}(\mathrm{P}_\mathrm{n\text{-}1}^*, i-1)$ and $\mathrm{g}_{\mathrm{e_k}}(\mathrm{P}_\mathrm{n}^* - \{2n\}, i-1)$. The families of these edge fixed geodominating sets can be empty or otherwise. Thus we have four combinations, whether these two families are empty or not.

Lemma 4.1. Let $e_k = \{1, 2\}$ be a pendant edge of the centipede $^*_{\mathrm{Q}}$3 ≥ 2 .QFor every $n \in N$:

(i) $g_{e_k}(P_n^*) = n-1$

(ii) $g_{e_k}(P_n^* - \{2n\}) = n-1$

(iii) $\mathcal{G}_{e_k}(P_n^*, i) = \phi$ if and only if $i < n-1 \; or \; i > 2n-2$

(iv) $\mathcal{G}_{e_k}(P_n^* - \{2n\}, i) = \phi$ if and only if $i < n-1 \; or \; i > 2n-3$

Proof. Proof is similar to 2.1.

Lemma 4.2. Let $e_k = \{1, 2\}$ be a pendant edge of the centipede $^*_{\mathrm{Q}}$3 ≥ 2 .QIf $\mathcal{G}_{e_k}(P_{n\text{-}2}^*, i-2) = \phi$ and $\mathcal{G}_{e_k}(P_{n\text{-}1}^* - \{2n-2\}, i-2) = \phi$ then $\mathcal{G}_{e_k}(\mathrm{P}_\mathrm{n}^* - \{2n\}, i) = \phi$

Proof. Proof is similar to 2.2.

Lemma 4.3. Let $e_k = \{1,2\}$ be a pendant edge of the centipede ${}^{*}_{Q}$3 ≥ 2 Q Suppose $\mathcal{G}_{e_k}(P_{\text{n}}^* - \{2n\}, i) \neq \phi$ then

(i) $\mathcal{G}_{e_k}(P_{n\text{-}2}^*, i-2) \neq \phi$ and $\mathcal{G}_{e_k}(P_{n\text{-}1}^* - \{2n-2\}, i-2) = \phi$ iff $i = n-1$

(ii) $\mathcal{G}_{e_k}(P_{n\text{-}2}^*, i-2) = \phi$ $\mathcal{G}_{e_k}(P_{n\text{-}1}^* - \{2n-2\}, i-2) \neq \phi$ iff $i = 2n-3$

(iii) $\mathcal{G}_{e_k}(P_{n\text{-}2}^*, i-2) \neq \phi$ and $\mathcal{G}_{e_k}(P_{n\text{-}1}^* - \{2n-2\}, i-2) \neq \phi$ iff $n \leq i \leq 2n-4$

Proof. Proof is similar to 2.3

Theorem 4.4. Let $e_k = \{1,2\}$ be a pendant edge of the centipede ${}^{*}_{Q}$3 ≥ 2 Q Suppose $\mathcal{G}_{e_k}(P_n^* - \{2n\}, i) \neq \phi$,

(i) If $\mathcal{G}_{e_k}(P_{n\text{-}2}^*, i-2) \neq \phi$ and $\mathcal{G}_{e_k}(P_{n\text{-}1}^* - \{2n-2\}, i-2) = \phi$, then

$$\mathcal{G}_{e_k}(P_n^* - \{2n\}, i) = \left\{\{2n-1, 2n-2\} \cup X \,/\, X \in \mathcal{G}_{e_k}(P_{n\text{-}2}^*, i-2)\right\}$$

(ii) If $\mathcal{G}_{e_k}(P_{n\text{-}2}^*, i-2) = \phi$ and $\mathcal{G}_{e_k}(P_{n\text{-}1}^* - \{2n-2\}, i-2) \neq \phi$, then

$$\mathcal{G}_{e_k}(P_n^* - \{2n\}, i) = \left\{\{2n-1, 2n-2\} \cup X \,/\, X \in \mathcal{G}_{e_k}(P_{n\text{-}1}^* - \{2n-2\}, i-2)\right\}$$

(iii) If $\mathcal{G}_{e_k}(P_{n\text{-}2}^*, i-2) \neq \phi$ and $\mathcal{G}_{e_k}(P_{n\text{-}1}^* - \{2n-2\}, i-2) \neq \phi$, then

$$\mathcal{G}_{e_k}(P_n^* - \{2n\}, i) = \left\{\{2n-1, 2n-2\} \cup X \,/\, X \in \mathcal{G}_{e_k}(P_{n\text{-}2}^*, i-2)\right\}$$
$$\cup \left\{\{2n-1, 2n-2\} \cup X \,/\, X \in \mathcal{G}_{e_k}(P_{n\text{-}1}^* - \{2n-2\}, i-2)\right\}$$

Proof. Proof is similar to theorem 2.4.

Theorem 4.5. Let $e_k = \{1,2\}$ be a pendant edge of the centipede ${}^{*}_{Q}$3 ≥ 2 Q For every $n \geq 5$, $g_{e_k}(P_n^* - \{2n\}, i) = g_{e_k}(P_{n\text{-}2}^*, i-2) + g_{e_k}(P_{n\text{-}1}^* - \{2n-2\}, i-2)$

Proof. Proof is similar to theorem 2.5.

Lemma 4.6. Let $e_k = \{1,2\}$ be a pendant edge of the centipede ${}^{*}_{Q}$3 ≥ 2 Q If $\mathcal{G}_{e_k}(P_{n\text{-}1}^*, i-1) = \phi$ and $\mathcal{G}_{e_k}(P_n^* - \{2n\}, i-1) = \phi$ then $\mathcal{G}_{e_k}(P_n^*, i) = \phi$.

Proof. Proof is similar to Lemma 3.1.

Lemma 4.7. Let $e_k = \{1,2\}$ be a pendant edge of the centipede $^*_{Q}$3 ≥ 2.Q Suppose $\mathcal{G}_{e_k}(P_n^*, i) \neq \phi$ then

(i) $\mathcal{G}_{e_k}(P_{n\text{-}1}^*, i-1) \neq \phi$ and $\mathcal{G}_{e_k}(P_n^* - \{2n\}, i-1) = \phi$ iff $i = n$

(ii) $\mathcal{G}_{e_k}(P_{n\text{-}1}^*, i-1) = \phi$ and $\mathcal{G}_{e_k}(P_n^* - \{2n\}, i-1) \neq \phi$ iff $i = 2n-2$

(iii) $\mathcal{G}_{e_k}(P_{n\text{-}1}^*, i-1) \neq \phi$ and $\mathcal{G}_{e_k}(\mathrm{P}_n^* - \{2n\}, i-1) \neq \phi$ iff $n+1 \leq i \leq 2n-3$

Proof. Proof is similar to lemma 3.2.

Theorem 4.8. Let $e_k = \{1,2\}$ be a pendant edge of the centipede $^*_{Q}$3 ≥ 2.Q

(i) If $\mathcal{G}_{e_k}(P_{n\text{-}1}^*, i-1) \neq \phi$ and $\mathcal{G}_{e_k}(P_n^* - \{2n\}, i-1) = \phi$ then

$$\mathcal{G}_{e_k}(P_n^*, i) = \left\{X \cup \{2n\} / X \in \mathcal{G}_{e_k}(P_{n\text{-}1}^*, i-1)\right\}$$

(ii) If $\mathcal{G}_{e_k}(P_{n\text{-}1}^*, i-1) = \phi$ and $\mathcal{G}_{e_k}(P_n^* - \{2n\}, i-1) \neq \phi$ then

$$\mathcal{G}_{e_k}(P_n^*, i) = \left\{X \cup \{2n\} / X \in \mathcal{G}_{e_k}(P_n^* - \{2n\}, i-1)\right\}$$

(iii) If $\mathcal{G}_{e_k}(P_{n\text{-}1}^*, i-1) \neq \phi$ and $\mathcal{G}_{e_k}(P_n^* - \{2n\}, i-1) \neq \phi$ then

$$\mathcal{G}_{e_k}(P_n^*, i) = \left\{X \cup \{2n\} / X \in \mathcal{G}_{e_k}(P_{n\text{-}1}^*, i-1)\right\} \cup \left\{X \cup \{2n\} / X \in \mathcal{G}_{e_k}(P_n^* - \{2n\}, i-1)\right\}$$

Proof. Proof is similar to theorem 3.3.

Theorem 4.9. Let $e_k = \{1,2\}$ be a pendant edge of the centipede $^*_{Q}$3 ≥ 2.QFor every $n \geq 4$, $g_{e_k}(P_n^*, i) = g_{e_k}(P_{n\text{-}1}^*, i-1) + g_{e_k}(P_n^* - \{2n\}, i-1)$.

Proof. Proof is similar to theorem 3.5.

Table 2 $g_{e_k}\left(P_n^*, i\right)$ *and* $g_{e_k}\left(P_n^* - \{2n\}, i\right)$

J	1	2	3	4	5	6	7	8	9	10	11	12	13	14
P_2^*	1	1												
$P_3^* - \{6\}$	0	1	1											
P_3^*	0	1	2	1										
$P_4^* - \{8\}$	0	0	1	2	1									
P_4^*	0	0	1	3	3	1								

$P_5^* - \{10\}$	0	0	0	1	3	3	1							
P_5^*	0	0	0	1	4	6	4	1						
$P_6^* - \{12\}$	0	0	0	0	1	4	6	4	1					
P_6^*	0	0	0	0	1	5	10	10	5	1				
$P_7^* - \{14\}$	0	0	0	0	0	1	5	10	10	5	1			
P_7^*	0	0	0	0	0	1	6	15	20	15	6	1		
$P_8^* - \{16\}$	0	0	0	0	0	0	1	6	15	20	15	6	1	
P_8^*	0	0	0	0	0	0	1	7	21	35	35	21	7	1

By similar way we can construct edge fixed geodominating sets corresponding to other pendant edges which are common to both P_n^* and $P_n^* - \{2n\}$.

Theorem 4.10. Let e_k be a pendant edge which is common to both P_n^* and $P_n^* - \{2n\}$. Then;

(i) For every $n \geq 3$, $G_{e_k}(P_n^*, x) = x[G_{e_k}(P_{n-1}^*, x) + G_{e_k}(P_n^* - \{2n\}, x)]$ with the initial values $G_{e_k}(P_2^*, x) = x + x^2$ and $G_{e_k}(P_3^* - \{6\}, x) = x^2 + x^3$.

(ii) For every $n \geq 4$, $G_{e_k}(P_n^* - \{2n\}, x) = x[G_{e_k}(P_{n-2}^*, x) + G_{e_k}(P_{n-1}^* - \{2n-2\}, x)]$ with the initial values $G_{e_k}(P_2^*, x) = x + x^2$ and $G_{e_k}(P_3^* - \{6\}, x) = x^2 + x^3$.

(iii) For every $n \geq 4$, $G_{e_k}(P_n^*, x) = x^{n-1}(1+x)^{n-1}$, $G_{e_k}(P_n^* - \{2n\}, x) = x^{n-1}(1+x)^{n-2}$.

Proof of (i). By theorem 4.9,

$$g_{e_k}(P_n^*, i) = g_{e_k}(P_{n-1}^*, i-1) + g_{e_k}(P_n^* - \{2n\}, i-1)$$

$$\sum_{i=n-1}^{2n-2} g_{e_k}(P_n^*, i)\, x^i = \sum_{i=n-1}^{2n-2} g_{e_k}(P_{n-1}^*, i-1)\, x^i + \sum_{i=n-1}^{2n-2} g_{e_k}(P_n^* - \{2n\}, i-1)\, x^i$$

$$G_{e_k}(P_n^*, x) = x\left[\sum_{i=n-1}^{2n-2} g_{e_k}(P_{n-1}^*, i-1)\, x^{i-1} + \sum_{i=n-1}^{2n-2} g_{e_k}(P_n^* - \{2n\}, i-1)\, x^{i-1}\right]$$

$$G_{e_k}(P_n^*, x) = x[G_{e_k}(P_{n-1}^*, x) + G_{e_k}(P_n^* - \{2n\}, x)]$$

Proof of (ii). By theorem 4.5,

$$g_{e_k}(P_n^* - \{2n\}, i) = g_{e_k}(P_{n-2}^*, i-2) + g_{e_k}(P_{n-1}^* - \{2n-2\}, i-2)$$

$$\sum_{i=n-1}^{2n-3} g_{e_k}(P_n^* - \{2n\}, \mathrm{i})\, x^i = \sum_{i=n-1}^{2n-3} g_{e_k}(P_{n-2}^*, i-2)\, x^i + \sum_{i=n-1}^{2n-3} g_{e_k}(P_{n-1}^* - \{2n-2\}, i-2)\, x^i$$

$$G_{e_k}(P_n^* - \{2n\}, x) = x^2 \left[\sum_{i=n-1}^{2n-3} g_{e_k}(P_{n-2}^*, i-2)\, x^{i-2} + \sum_{i=n-1}^{2n-3} g_{e_k}(P_{n-1}^* - \{2n-2\}, i-2)\, x^{i-2} \right]$$

$$= x^2[G_{e_k}(P_{n-2}^*, x) + G_{e_k}(P_{n-1}^* - \{2n-2\}, x)]$$

Proof of (iii). We shall prove both equalities together by induction on *n*. We have result for *n* = 4, because $G_{e_k}(P_4^*, x) = x^{4-1}(1+x)^{4-1} = x^3 + 3x^4 + 3x^5 + x^6$, $G_{e_k}(P_4^* - \{8\}, x) = x^3(1+x)^2 = x^3 + 2x^4 + x^5$ Assume that the result is true for all natural numbers less than *n*. we prove the result for *n*. We have $G_{e_k}(P_{n-1}^*, x) = x^{n-2}(1+x)^{n-2}$, $G_{e_k}(P_{n-1}^* - \{2n-2\}, x) = x^{n-2}(1+x)^{n-3}$.

Now by part (i) and induction hypothesis,

$$G_{e_k}(P_n^*, x) = x[G_{e_k}(P_{n-1}^*, x) + G_{e_k}(P_n^* - \{2n\}, x)]$$

$$= x[x^{n-2}(1+x)^{n-2} + x^{n-1}(1+x)^{n-2}]$$

$$= x^{n-1}(1+x)^{n-2}(1+x)$$

$$= x^{n-1}(1+x)^{n-1}$$

Now by part (i) and induction hypothesis,

$$G_{e_k}(P_n^* - \{2n\}, x) = \sum_{e_k \in E_1} x^2[G_{e_k}(P_{n-2}^*, x) + G_{e_k}(P_{n-1}^* - \{2n-2\}, x)]$$

$$= x^2[x^{n-3}(1+x)^{n-3} + x^{n-2}(1+x)^{n-3}$$

$$= x^{n-1}(1+x)^{n-2}$$

Theorem 4.11. Let $e_k = \{1,2\}$ be a pendant edge of a centipede $P_n^*, n \geq 2$. . Suppose that $n \geq 3$. Then for every $n-1 \leq i \leq 2n-2$, $g_{e_k}(P_n^*, i) = \binom{n-1}{i-n+1}$ and for every $n-1 \leq i \leq 2n-3$, $g_{e_k}(P_n^* - \{2n\}, i) = \binom{n-2}{i-n+1}$.

Proof. Proof is similar to theorem 3.7.

Corollary 4.12. Let $e_k = \{1,2\}$ be a pendant edge of the centipede $\overset{*}{Q}$,3 ≥ 2 .QThen the following properties hold for coefficients of $G_{e_k}(P_n^*, x)$ and $G_{e_k}(P_n^* - \{2n\}, x)$ for every $n \geq 3$;

(i) $g_{e_k}(P_n^*, 2n-2)=1, g_{e_k}(P_n^*, n-1)=1$

(ii) $g_{e_k}(P_n^*, 2n-3)=n-1, g_{e_k}(P_n^*, n)=n-1$

(iii) $g_{e_k}(P_n^*, 2n-4)=\frac{(n-1)(n-2)}{2}, g_{e_k}(P_n^*, n+1)=\frac{(n-1)(n-2)}{2}$

(iv) $g_{e_k}(P_n^*, 2n-5)=\frac{(n-1)(n-2)(n-3)}{6}, g_{e_k}(P_n^*, n+3)=\frac{(n-1)(n-2)(n-3)}{6}$

(v) $g_{e_k}(P_n^* - \{2n\}, 2n-3)=1, g_{e_k}(P_n^* - \{2n\}, n-1)=1$

(vi) $g_{e_k}(\mathrm{P}_\mathrm{n}^* - \{2n\}, 2n-4) = n-2, g_{e_k}(\mathrm{P}_\mathrm{n}^* - \{2n\}, n) = n-2$

(vii) $g_{e_k}(P_n^* - \{2n\}, 2n-5)=\frac{(n-1)(n-2)}{2}, g_{e_k}(P_n^* - \{2n\}, n+1)=\frac{(n-1)(n-2)}{2}$

(viii) If $S_n = \sum_{i=n-1}^{2n-2} g_{e_k}(P_n^*, i)$ then for every $n \geq 3$, $S_n = 2(S_{n-1})$ with the initial value $S_2 = 2$.

(ix) If $S_n = \sum_{i=n-1}^{2n-3} g_{e_k}(P_n^* - \{2n\}, i)$ then for every $n \geq 4$, $S_n = 2(S_{n-1})$ with the initial valueS$_2$ =2.

Proof . proof is similar to corollary 3.8.

Theorem 4.13. Let $e_k = \{2n-1, 2n\}$ be a pendant edge of P_n^* which is not an edge of $P_n^* - \{2n\}$ Then for every $n \geq 4$ and $i \geq n-1$

i) If $\mathcal{G}_{e_k}(P_n^*, i)$ is the family of edge fixed geodominating sets of P_n^* with cardinality i, then $g_{e_k}(P_n^*, i) = g_{e_k}(P_{n-1}^*, i-1) + g_{e_k}(P_{n-1}^*, i-2)$

ii) $G_{e_k}(P_n^*, x) = x^2 G_{e_k}(P_{n-1}^*, x) + G_{e_k}(P_{n-1}^*, x)$

iii) $G_{e_k}(P_n^*, x) = x^{n-1}(1+x)^{n-1}$

Proof. i) Consider the edge fixed geodominating sets of P_n^* with cardinality *i*. For the edge $e_k = \{2n-1, 2n\}$, the corresponding edge fixed geodominating sets of P_n^* do not contain the two vertices $2n-1, 2n$. By lemma, edge fixed geodominating sets of P_n^* must contain $n-1$ vertices. Now there remains $n-1$ vertices and we have to choose i vertices from these $n-1$ vertices. Therefore, there are $\binom{n-1}{i-n+1}$ edge fixed geodominating sets in P_n^* with cardinality *i*. $g_{e_k}(P_n^*, i) = \binom{n-1}{i-n+1}$. By the similar argument , we have $g_{e_k}\left(P_{n-1}^*, i-1\right) = \binom{n-2}{i-n+1}$ and $g_{e_k}\left(P_{n-1}^*, i-2\right) = \binom{n-2}{i-n}$ .But $\binom{n-1}{i-n+1} = \binom{n-2}{i-n+1} + \binom{n-2}{i-n}$.

Therefore $g_{e_k}(P_n^*, i) = g_{e_k}(P_{n-1}^*, i-1) + g_{e_k}(P_{n-1}^*, i-2)$

ii) By (i) above , we have $g_{e_k}(P_n^*, i) = g_{e_k}(P_{n-1}^*, i-1) + g_{e_k}(P_{n-1}^*, i-2)$

$$i = n-1 \Rightarrow g_{e_k}\left(P_n^*, n-1\right) = g_{e_k}\left(P_{n-1}^*, n-2\right) + g_{e_k}\left(P_{n-1}^*, n-3\right) \Rightarrow$$

$$x^{n-1} g_{e_k}\left(P_n^*, n-1\right) = x^{n-1} g_{e_k}\left(P_{n-1}^*, n-2\right) + x^{n-1} g_{e_k}\left(P_{n-1}^*, n-3\right)$$

....

$$i = 2n-2 \Rightarrow g_{e_k}\left(P_n^*, 2n-2\right) = g_{e_k}\left(P_{n-1}^*, 2n-3\right) + g_{e_k}\left(P_{n-1}^*, 2n-4\right) \Rightarrow$$

$$x^{2n-2} g_{e_k}\left(P_n^*, 2n-2\right) = x^{2n-2} g_{e_k}\left(P_{n-1}^*, 2n-3\right) + x^{2n-2} g_{e_k}\left(P_{n-1}^*, 2n-4\right)$$

Adding all these, we get

$$\sum_{i=n-1}^{2n-2} g_{e_k}\left(P_n^*, i\right) x^i = x \sum_{i=n-2}^{2n-4} g_{e_k}\left(P_{n-1}^*, i\right) x^i + x^2 \sum_{i=n-2}^{2n-4} g_{e_k}\left(P_{n-1}^*, i\right) x^i$$

Therefore, $G_{e_k}(P_n^*, x) = x G_{e_k}(P_{n-1}^*, x) + x^2 G_{e_k}(P_{n-1}^*, x)$

iii) By induction on *n*. The result is true for $n=4$, because $G_{e_k}\left(P_4^*, x\right) = x^3 + x^4$. Assume that the result is true for all natural numbers less than *n*. We prove the result for *n*. We have $G_{e_k}(P_{n-1}^*, x) = x^{n-2}(1+x)^{n-2}$.

Now $G_{e_k}(P_n^*, x) = x G_{e_k}(P_{n-1}^*, x) + x^2 G_{e_k}(P_{n-1}^*, x)$

$$= x\left[x^{n-2}(1+x)^{n-2}\right] + x^2\left[x^{n-2}(1+x)^{n-2}\right]$$

$$= x^{n-1}(1+x)^{n-1}$$

Therefore, the result is true for all *n*.

Theorem 4.14. Let $e_k = \{2n-3, 2n-1\}$ be a non-pendant edge of P_n^* which is a pendant edge of $P_n^* - \{2n\}$. Then for every $n \geq 4$ and $i \geq n$,

i) If $\mathcal{G}_{e_k}(P_n^*, i)$ is the family of edge fixed geodominating sets of P_n^* with cardinality *i*, then $g_{e_k}\left(P_{n-1}^*, i\right) = g_{e_k}\left(P_{n-1}^*, i-1\right) + g_{e_k}\left(P_{n-1}^*, i-2\right)$

ii) $G_{e_k}(P_n^*, x) = x^2 G_{e_k}(P_{n-1}^*, x) + G_{e_k}(P_{n-1}^*, x)$

iii) $G_{e_k}(P_n^*, x) = x^n (1+x)^{n-2}$

iv) $G_{e_k}(P_n^* - \{2n\}, x) = x^{n-1}(1+x)^{n-2}$

Proof of (i). Consider the edge fixed geodominating sets of P_n^* with cardinality *i*. By lemma, edge fixed geodominating sets of P_n^* must contain *n* vertices. Now there remains $n-1$ vertices and we have to choose *i* vertices from these $i-n$ vertices. Therefore, there are $\binom{n-1}{i-n}$ edge fixed geodominating sets in P_n^* with cardinality *i*. Therefore $g_{e_k}\left(P_n^*, i\right) = \binom{n-1}{i-n}$. By the similar argument, we have $g_{e_k}\left(P_{n-1}^*, i-1\right) = \binom{n-2}{i-n}$ and $g_{e_k}\left(P_n^*, i-2\right) = \binom{n-2}{i-n-1}$. But $\binom{n-1}{i-n} = \binom{n-2}{i-n} + \binom{n-2}{i-n-1}$.
Therefore, $g_{e_k}\left(P_n^*, i\right) = g_{e_k}\left(P_{n-1}^*, i-1\right) + g_{e_k}\left(P_{n-1}^*, i-2\right)$

Proof of (ii). By (i) above, we have $g_{e_k}\left(P_n^*, i\right) = g_{e_k}\left(P_{n-1}^*, i-1\right) + g_{e_k}\left(P_{n-1}^*, i-2\right)$

$$i = n \Rightarrow g_{e_k}\left(P_n^*, n\right) = g_{e_k}\left(P_{n-1}^*, n-1\right) + g_{e_k}\left(P_{n-1}^*, n-2\right) \Rightarrow$$

$$x^n g_{e_k}\left(P_n^*, n\right) = x^n g_{e_k}\left(P_{n-1}^*, n-1\right) + x^n g_{e_k}\left(P_{n-1}^*, n-2\right)$$

....

$$i = 2n-2 \Rightarrow g_{e_k}\left(P_n^*, 2n-2\right) = g_{e_k}\left(P_{n-1}^*, 2n-3\right) + g_{e_k}\left(P_{n-1}^*, 2n-4\right) \Rightarrow$$

$$x^{2n-2} g_{e_k}\left(P_n^*, 2n-2\right) = x^{2n-2} g_{e_k}\left(P_{n-1}^*, 2n-3\right) + x^{2n-2} g_{e_k}\left(P_{n-1}^*, 2n-4\right)$$

Adding all these, we get $\sum_{i=n}^{2n-2} g_{e_k}\left(P_n^*, i\right) x^i = x \sum_{i=n-1}^{2n-4} g_{e_k}\left(P_{n-1}^*, i\right) x^i + x^2 \sum_{i=n-1}^{2n-4} g_{e_k}\left(P_{n-1}^*, i\right) x^i$

Therefore, $G_{e_k}(P_n^*, x) = x^2 G_{e_k}(P_{n-1}^*, x) + G_{e_k}(P_{n-1}^*, x)$

Proof of (iii). By induction on *n*. The result is true for *n=4*, because $G_{e_k}\left(P_4^*, x\right) = x^4 + x^5$. Assume that the result is true for all natural numbers less than *n*. We prove the result for *n*. We have, $G_{e_k}(P_{n-1}^*, x) = x^n (1+x)^{n-2}$.

Now $G_{e_k}(P_n^*, x) = xG_{e_k}(P_{n-1}^*, x) + x^2 G_{e_k}(P_{n-1}^*, x)$

$$= x\left[x^{n-1}(1+x)^{n-3}\right] + x^2\left[x^{n-1}(1+x)^{n-3}\right]$$

$$= x^{n-1}(1+x)^{n-3} x\left[1+x\right]$$

$$= x^n (1+x)^{n-2}$$

Therefore, the result is true for all *n*.

Proof of (iv). Proof is similar to (iii).

Remark:
By similar way we can construct edge fixed geodominating sets for other non-pendant edges.

5. TOTAL EDGE FIXED GEODOMINATION POLYNOMIAL OF CENTIPEDES AND $P_n^* - \{2n\}$

Theorem 5.1.
(i)For every $n \geq 2$, $G_t(P_n^*, x) = x^{n-1}(1+x)^{n-2}((2n-1)x+n)$
(ii)For every $n \geq 3$, $G_t(P_n^* - \{2n\}, x) = x^{n-1}(1+x)^{n-3}((2n-2)x+n)$

Proof of (i). P_n^* has $n-2$ non-pendant edges which are common to P_n^* and $P_n^* - \{2n\}$, $n-1$ pendant edges which are common to P_n^* and $P_n^* - \{2n\}$, one pendant edge of P_n^* which is not an edge of $P_n^* - \{2n\}$ and one non-pendant edge of P_n^* which is a pendant edge of $P_n^* - \{2n\}$.

$$G_t(P_n^*, x) = (n-2)x^n(1+x)^{n-2} + (n-1)x^{n-1}(1+x)^{n-1} + x^{n-1}(1+x)^{n-1} + x^n(1+x)^{n-2}$$
$$= (1+x)^{n-2}x^{n-1}[(n-1)x + n(1+x)]$$
$$= x^{n-1}(1+x)^{n-2}[(2n-1)x+n]$$

Proof of (ii). $P_n^* - \{2n\}$ has $n-2$ non-pendant edges which are common to P_n^* and $P_n^* - \{2n\}$, $n-1$ pendant edges which are common to P_n^* and $P_n^* - \{2n\}$ and one pendant edge which is a non-pendant edge of P_n^*.

$$G_t(P_n^* - \{2n\}, x) = (n-2)x^n(1+x)^{n-3} + (n-1)x^{n-1}(1+x)^{n-2} + x^{n-1}(1+x)^{n-2}$$
$$= (1+x)^{n-3}x^{n-1}[(n-2)x + n(1+x)]$$
$$= x^{n-1}(1+x)^{n-3}[(2n-2)x+n]$$

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