Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 2 (2017), pp. 453-462

http://www.ripublication.com

Solving Fuzzy Assignment Problem Using Fourier Elimination Method

Dr. S. Muruganandam[1] **and K. Hema**[2]

[1] *Professor and Head, Department of Mathematics, Imayam College of Engineering, Kannanur, Thuraiyur , Trichy-621 206, Tamilnadu, India.*

[2] *Associate Professor, Department of Mathematics, M.A.M. College of Engineering and Technology, Trichy-621 105, Tamilnadu, India.*

Abstract

Assignment problem is one of the most-studied, well known and significant in mathematical programming. It is a combinatorial optimization problem in the field of operation research .In this paper, for a given fuzzy assignment problem, the cost are triangular fuzzy numbers and the fuzzy assignment problem is converted into crisp assignment problem by graded mean integration representation method and the crisp assignment problem is converted into linear programming problem which is solved by a proposed method called Fourier Elimination method to get the optimal solution. The proposed method is illustrated with a numerical example.

Keywords: Assignment problem, Fuzzy assignment problem, Triangular fuzzy number, Graded mean Integration representation method, Linear programming problem, Fourier elimination method.

AMS Mathematics Subject Classification (2010): 03E72, 90C05

INTRODUCTION

Assignment problem is a special type of linear programming problem whose objective is to obtain the optimum assignment for number of tasks (jobs) which is equal to the number of resources (workers) at a minimum cost or maximum profit. However, considering the decisions made in the real world where the objectives, constrains or parameters are not precise. Therefore, a decision is often made on the basis of vague information. The concept of fuzzy set theory was first introduced Bellmann & Zadeh[2] and linear programming problem was introduced by Charnes and Cooper W.W, [4]. The approach in which fuzzy assignment problem is converted into multi-objective linear programming problem where the cost co-efficient are fuzzy numbers was proposed by De.P.K and Bharati Yadav[3] .Further Kanniappan.P & Thangavel.K,[6] modified the method of Fourier elimination method which was developed by Williams.H.P[9]. In this method, solving linear programming problems is done by choosing a variable for elimination. Pandian.P & Natarajan.G,[8] discussed a Fourier transportation method to find an optimal solution for transportation problems with mixed constraints. Generalization of two phase method to solve multi-objective integer programming with three objectives was proposed by Anthony Przybylski et.al[1].

Farahi et.al [5] used fuzzy set theory and fuzzy programming to convert the multi-objective linear bi-level programming problem to a linear bi-level programming problems. An approach to solve multi-objective assignment problem with interval parameters where a weighted min-max method is applied to transform the multi-objective into single objective optimization problem was formulized by Kayvan Salehi[10] .Anil D.Gotmare et.al[1]solved the fuzzy assignment using Branch and Bound technique .In addition ,to solve multi-objective assignment problem with linear and non linear membership functions an algorithm was proposed by Atul Kumar Tiwari et.al[3].

In this paper, Fourier elimination method is proposed to find the optimal solution to the fuzzy assignment problem. We organize this paper as follows: In section 1, we introduce the fuzzy assignment problem. In section 2, we construct the mathematical model for the problem. In section 3, we give the methodology for the problem. In section 4, we give a solution procedure for the proposed method to solve the problem. In section 5, a numerical example is given to show the efficiency of the proposed method and finally we give a conclusion for the problem.

MATHEMATICAL FORMULATION

The general assignment problem

Suppose there are ‘n’ people and ‘n’ jobs. Each job must be done by exactly one person; also each person can do, at most, one job. The problem is to assign jobs to the people so as to minimize the total cost of completing all of the jobs. The general assignment problem can be mathematically stated as follows:

$$\text{Minimize } Z=\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ for } i = 1,2,\ldots, n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ for } j = 1,2,\ldots, n$$

$$x_{ij} = 0 \text{ or } 1$$

Fuzzy assignment problem

$$\text{Minimize } Z=\sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij}\, x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ for } i = 1,2,\ldots, n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ for } j = 1,2,\ldots, n$$

$$x_{ij} = 0 \text{ or } 1$$

METHODOLOGY

Triangular fuzzy number

A fuzzy number A is a triangular fuzzy number denoted by (a_1, a_2, a_3) and its membership function $\mu_A(x)$is given below:

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & a_1 \le x \le a_2, \\ \dfrac{x - a_3}{a_2 - a_3} & a_2 \le x \le a_3, \\ 0 & \text{otherwise} \end{cases}$$

Graded Mean Integration Representation:

We describe graded mean integration representation as follows:

Suppose L^{-1} and R^{-1} are inverse functions of functions L and R, respectively, and the graded mean h-level value of generalised fuzzy number $A = (a_1, a_2, a_3, a_4 : w)_{LR}$ is $h[L^{-1}(h) + R^{-1}(h)] / 2$. Then the defuzzified value P(A) by graded mean integration representation of generalised fuzzy number based on the integral value of graded mean h-level is

$$P(A) = \frac{\int_0^w h\left[\frac{L^{-1}(h) + R^{-1}(h)}{2}\right] dh}{\int_0^w h\,dh}$$

Where h is between 0 and 1, $0 < w < 1$

If $A = (a_1, a_2, a_3)$ is a triangular fuzzy number. Chen and Hsieh [6] already found the general formulae of the representation of generalized triangular fuzzy number and it is as follows:

$$P(A) = \frac{1}{2}\frac{\int_0^1 h[a_1 + h(a_2 - a_1) + a_3 - h(a_3 - a_2)]dh}{\int_0^1 h\,dh}$$

$$P(A) = \frac{a_1 + 4a_2 + a_3}{6}$$

Theorem:

Consider the system

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$$

.... (1)

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m$$

If $aij \geq 0,$ for all i, then $x_j = 0$.

Proof: Now, eliminating x_j from the given system using Fourier elimination method, we have the following system

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1j-1}x_{j-1} + a_{1j+1}x_{j+1} + a_{1n}x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2j-1}x_{j-1} + a_{2j+1}x_{j+1} + a_{2n}x_n \leq b_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots \qquad (2)$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mj-1}x_{j-1} + a_{mj+1}x_{j+1} + a_{1n}x_n \leq b_m$$

$$x_1, x_2, \ldots x_{j-1}, x_{j+1\ldots,}x_n \geq 0$$

From the system (1)&(2), we observe that there is no role of the variable x_j .that is x_j is inactive. Therefore x_j=0.

Mathematical formulation of Linear Programming Problem:

If x_j (j=1,2,…n) are the n decision variables and if the system is subject to m constraints,the general mathematical model can be written in the form :

Optimize $Z = f\left(x_{1,}x_{2,}x_{3,} \ldots x_n\right)$

Sub to $g_i(\ x_{1,}x_{2,}x_{3,} \ldots x_n)\ \leq, =, \geq\ b_{i,}(i = 1,2 \ldots m)$

$$x_{1,}x_{2,}x_{3,} \ldots x_n \geq 0.$$

THE PROPOSED METHOD - FOURIER ELIMINATION METHOD

Solution procedure:

Step: 1. Convert the fuzzy numbers into crisp numbers using Graded Mean Integration Representation Method..

Step:2 Crisp numbers are into linear programming problem .

Step:3 Select and eliminate the variables from the previous step using Fourier Elimination Method

Step:3.1 Convert the minimization problem into maximization problem as

$Min(Z) = -Max(-z) = -Max(W).$

Step:3.2 Transform the objective function in the form of linear inequalities. For eliminating the variables, we use the following procedure:

Step:3.3 First write the maximization problem having an inequality in the form of '$\leq$' from the variable in an equality constraint in the given problem.

Step:3.4 Then write the equivalent pure integer problem to the modified maximization problem.

Step:3.5 Select and remove a variable from previous step by Fourier elimination method

Step:3.6 Write the set of inequalities including the objective function after deleting the true statements and redundant constraints and also using Theorem.

Step:3.7 Repeat step 3.5 -3.6 until all variables x_{ij}'s are eliminated except the objective function variable w.

Step:3.8 Find the optimum solution for w.

Step:3.9: The values of all x_{ij}'s are calculated using back substitution method and basic algebraic method.

Step:3.10: The optimal solution and the optimal allocation for the assignment problem is achieved.

NUMERICAL EXAMPLE

Three persons are available to do three different jobs. The cost (in Rupees) that each person takes to do each job is known and are represented by triangular fuzzy numbers and are shown in following Table.

$$\begin{array}{cc} & \textbf{Jobs} \\ \textbf{Persons} & \begin{pmatrix} (7,8,9) & (14,16,18) & (1516,17) \\ (16,17,18) & (18,20,22) & (23,25,27) \\ (26,27,28) & (9,10,11) & (10,12,14) \end{pmatrix} \end{array}$$

Find an optimal assignment of persons to jobs that will minimize the total cost.

Solution:

Fuzzy assignment problem is converted into crisp assignment problem using Graded mean integration representation method.

$$\begin{array}{cc} & \textbf{Jobs} \\ \textbf{Persons} & \begin{pmatrix} 8 & 16 & 16 \\ 17 & 20 & 25 \\ 27 & 10 & 12 \end{pmatrix} \end{array}$$

Linear programming model for the assignment problem is given by

Min Z = $8x_{11} + 16x_{12} + 16x_{13} + 17x_{21} + 20x_{22} + 25x_{23} + 27x_{31} + 10x_{32} + 12x_{33}$

Sub to $x_{11} + x_{12} + x_{13} = 1$

$x_{21} + x_{22} + x_{23} = 1$

$x_{31} + x_{32} + x_{33} = 1$

$x_{11} + x_{21} + x_{31} = 1$

$x_{12} + x_{22} + x_{32} = 1$

$x_{13} + x_{23} + x_{33} = 1$

$x_{11} \geq 0$, $x_{12} \geq 0, x_{13} \geq 0$,

$x_{21} \geq 0$, $x_{22} \geq 0$, $x_{231} \geq 0$,

$x_{31} \geq 0$, $x_{321} \geq 0$ $x_{33} \geq 0$.

Min Z= - Max(-Z) = - Max W

$= -8x_{11} - 16x_{12} - 16x_{13} - 17x_{21} - 20x_{22} - 25x_{23} - 27x_{31} - 10x_{32} - 12x_{33}$ --(1)

Sub to $x_{11} = x_{22} + x_{23} + x_{32} + x_{33} - 1$ -----------------------(2)

$x_{12} = 1 - x_{22} - x_{32}$ ----------------------(3)

$x_{13} = 1 - x_{23} - x_{33}$ ----------------------(4)

$x_{21} = 1 - x_{22} - x_{23}$ ----------------------(5)

$x_{31} = 1 - x_{32} - x_{33}$ ----------------------(6)

Substituting equations (2) to (6) in equation(1) and rewriting the objective as constraint, we get

$$-5x_{22} - 0x_{23} - 25x_{32} - 23x_{33} + W \leq -68 \quad \text{------------------------ (7)}$$

$$x_{22} + x_{23} \leq 1 \quad \text{----------------------- (8)}$$

$$x_{32} + x_{33} \leq 1 \quad \text{---------------------- (9)}$$

$$x_{22} + x_{32} \leq 1 \quad \text{--------------------- (10)}$$

$$x_{23} + x_{33} \leq 1 \quad \text{--------------------- (11)}$$

Eliminating x_{22} , we get

$$-25x_{32} - 23x_{33} + 5x_{23} \leq -63 \quad \text{------------(13)}$$

$$x_{23} - x_{32} \leq 0 \quad \text{-------------(14)}$$

$$x_{32} + x_{33} \leq 1 \quad \text{-------------(15)}$$

$$x_{23} + x_{33} \leq 1 \quad \text{--------------(16)}$$

Applying Fourier elimination method, we get

$$-25x_{32} - 23x_{33} \leq -63$$

$$-x_{32} \leq 0$$

$$x_{32} + x_{33} \leq 1$$

$$x_{33} \leq 1$$

Eliminating a variable from the above equations ,we get

$$2x_{33} + w \leq -38$$

$$x_{33} \leq 1$$

By back substitution we get $x_{33} = 1$, $x_{22} = 1$, $x_{11} = 1$ & $w = -40$

Min Z = -Max(w)= 40

Min Z = 40.

Optimal assignment: 1 → 1, 2 → 2, 3 → 3

Optimal assignment cost =40

CONCLUSION

Fuzzy numbers are converted into crisp values using Graded mean integration representation method. Assignment problem is written in terms of pure linear programming problem by considering sum of each constraint equal to 1. Fourier elimination method is a novel method used to eliminate the variables to get the optimal allocation of workers into jobs and to get the optimal solution for the given fuzzy assignment problem.

REFERENCES

[1] Anil, D.Gotnare., Khot, P.G, 2016,"Solution of fuzzy assignment problem by using Branch and Bound Technique with application of linguistic variable," International Journal of Computers and Technology.,15(4),pp.6649-6653.

[2] Anthony Przybylski, Xavier Gandibleuxa, Matthias Ehrgott, 2010," A two phase method for multi-objective integer programming and its application to the assignment problem with three objectives"**,** Discrete Optimization (7)pp. 149-165.

[3] Atul Kumar et.al, 2016,"Proposal of an algorithm for solving multi-objective assignment problem (MOAP) with linear and non linear membership functions", International Journal of Industrial Engineering and Technology,(6),pp.1-12

[4] Bellman, R.E.,Zadeh, L.A., 1970,"Decision Making in a Fuzzy Environment." Management Science, 17, (4), B:141: B:164.

[5] Charnes,A.,Cooper,W.W.,Henderson,A.,1953 "An Introduction to Linear Programming problem". John Wiley & Sons, New York.

[6] Chen,S.H., and Hsieh,C.H., 1999,"Graded mean integration representation of generalized fuzzy numbers", Journal of Chinese fuzzy systems,(5).pp.1-7.

[7] De,P.K.,Bharati Yadav.,2012,"A General approach for solving assignment problems involving with fuzzy cost coefficients" ,Modern Applied Science, 6(3),pp.2-10

[8] Farahi M.H,, Ansari.E,2010, "A new approach to solve Multi-objective linear bi-level programming problems", TJMCS e (4),pp. 313-320.

[9] Kanniappan, P.,Thangavel,K., 1998,"Modified Fourier method for solving linear programming problems"OPSEARCH,35,pp.45-56.

[10] Kayvan Salehi., 1998,"An approach for solving multi-objective assignment problem with interval parameters", Management Science Letters, 4 ,pp.2155-2160.

[11] Pandian.P., Natarajan,G.,2010,"Fourier method for solving Transportation problem with mixed constraints", Int. J. Contemp. Math. Sciences, 5(28),pp.1385-1395.

[12] Williams,H.P.,1986,"Fourier's method of linear programming and its Dual", American Mathematical Monthly,93,pp.681-695.