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Mathematical Analysis of Mechanical Properties of Idol Making Stones

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Abstract

The aim of the paper is to investigate and compare the mechanical properties of various stones including idol (god) making special stones in India. The idol making stones are collected from renowned sthapathis (architects) in and around Kanchipuram district, a famous temple city in Tamilnadu state of India. The samples are investigated non-destructively by ultrasound technique. Various mechanical properties like Young's modulus, Bulk modulus, Poisson's ratio, Micro hardness etc., are computed by knowing the speed of ultrasound in the material. The mechanical properties of the samples are computed and discussed by comparing various stones.

Keywords: Fugacity, Idol Making stones, Micro Hardness, Non-destructive testing, Young's Modulus. MSC : 74-05

INTRODUCTION

Idols are made using different materials such as stone, metal, wood, Katu-sarkara, stucco, mud, etc. Granite used for making idols is known as 'Shila'. Shilas are classified as Purusha Shila (Male stone), Sthree Shila (Female stone) and Napumsaka Shila (Neutral stone). Purusha Shilas are used for making idols of Gods and Sthree Silas for Goddess. The Napumsaka Silas are used for carving 'peethas' (platforms). Silas, which spread spark when hit by chisel is considered as most suited for making idols. A shila selected for a particular deity should not be used for any other deity. Male deities are made from male stones and their bases are from female type stones. For Female deities, all parts of the idol are made from female type stones. Prabhavali (arch like appearance at the back of idol), platform, pillars, etc., are made from neutral stone (Napumsaka stone). Most of Indian temples are built thousands of years ago and still the deities are in stable condition. The stability of these stones over the years created an interest to study its mechanical properties and compare with ordinary stones. Study of wave propagation through a material will enable us to study the nature of the material at microscopic level. It has been shown by many researchers [1-8] about the advantage of using high frequency waves in determining the elastic constants of solids which helps to analyse its mechanical properties.

METHODS AND MATERIALS

Various stones including idol making stone samples are collected from various renowned sthapathis (architects) in and around Kanchipuram, a famous temple city located in Tamilnadu state of India. The samples are investigated non-destructively by ultrasound technique. Six stone samples are taken for study. These stones are grinded to 5mm thickness and their surfaces are grinded and made smooth to fix in the instrument. They are Black stone (sample 1), Green stone (sample 2), Neutral stone (sample 3&4), Sandal stone (sample 5) and Ordinary stone (sample 6) as shown in fig 1.

The mechanical properties are analysed by estimating various elastic constants of all stone samples. Digital ultrasonic pulse-echo velocity meter is a simple and unique direct reading digital system to determine the velocity of ultrasonic waves as well as to observe echo for attenuation measurements with a high level of accuracy. Its operating frequency is 2 MHz. A very high speed counter timer is used to precisely measure the transit time of Ultrasonic waves. The resolution in transit time is around $0.015\mu s^2$. The uncertainty in the instrument is ± 10 ms^{-1}. Velocity measurement for solids is based on pulse echo method using longitudinal & shear wave transducers. A 32 bit embedded controller with its peripherals work as standalone data acquisition system and control system for this equipment. A LCD display is provided for displaying all parameters. Density studies are done using Archimedes principle. The numerical estimation of elastic properties of various stones are computed using the formulae as given below.

Longitudinal velocity $V_L = (2d/t) ms^{-1}$

Shear velocity $V_S = (2d/t) ms^{-1}$

Longitudinal modulus $L = {V_L}^2 \rho$ G Pa

Shear modulus $G = {V_S}^2 \rho$ G Pa

Bulk modulus $K = L - (4/3)G$ G Pa

Young's modulus $E = (1+\sigma)2G$ G Pa

Micro hardness $H = (1-2\sigma)E/6(1+\sigma)$ G Pa

Poisson's ratio $\sigma = [L - 2G/2(L-G)]$ G Pa

Fugacity $(1/f_g) = (1-2\sigma)^2/2(1+\sigma)$

RESULTS AND DISCUSSION

Wave propagation through stones helps to analyse its mechanical properties. Wave interaction parameters act as an important tool in deciding the applications of these materials. Ultrasonic wave is preferred mainly because, it will not damage the sample even at microscopic level but helps to get all molecular level data about the sample.

(a) Density and velocity studies:

Density values are estimated using Archimedes principle. The maximum value of density is 3877 kg/m3 for black stone (sample 1) and it is very low (1767 kg/m3) for ordinary stone (sample 6). The density values have an intermediate value for other stone samples. It indicates that the molecules of black stone may be closely packed than other samples. The propagation speed of sound depends on the temperature, composition and the nature of the medium through which it propagates. In longitudinal waves, the displacement of the medium is parallel to the propagation of the wave. In air, sound waves behave like longitudinal pressure waves. Longitudinal waves propagate in all types of materials. Longitudinal velocity is the ratio of distance travelled and transient time and its value is 3863 m/s and 3715 m/s for sample 3 & 4, ie., neutral stones napumsaka stone) which are used for making platforms and pillars in the temples. For all other stones, it has a lesser value. The lowest value of longitudinal velocity is 3450 m/s for ordinary stone and black stone has a velocity value of 3588 m/s. Shear Velocity (Vs) can be used in a broad range of applications, including foundation stiffness assessment, earthquake site response and detection of cavities, tunnels and sinkholes. Shear velocity is useful as a method in fluid mechanics to compare many velocities such as the velocity of a flow in a stream to a velocity that relates shear between layers of flow. Shear velocity is defined as the ratio of distance travelled and transient time. Shear velocity is comparatively high for neutral stones (1005 m/s) which are used for making platforms and pillars of the temples. For all other stones, it has lesser value and the lowest value is for sample 1 ie., black stone (872 m/s).

(b) Determination of Elastic constants:

The measure of the ability of a material to withstand changes in length when subjected to lengthwise tension or compression is referred to as modulus of elasticity. Longitudinal modulus is the ratio between longitudinally applied stress and the longitudinal strain. It is high for Black stones (49 GPa, sample 1) and it is very low for ordinary stone (21 GPa, sample 6). For Neutral stones (sample 3&4), longitudinal modulus is having an average value between two. This means that black stones can withstand any kind of tension or compression in a better manner than other type of stones. Shear and longitudinal modulus of elasticity in structural lumber beams were analysed by Andre Luis Christoforo [9]. Shear modulus is the ratio of shearing stress to the shearing strain. It is the coefficient of elasticity for a shearing force. The Shear modulus is high for black stones (29 GPa) and it is low for ordinary stone (14 GPa).

For Neutral stone and Sandal stone, it is having an average value between these stones.

Bulk modulus is the ratio of bulk stress to the bulk strain. It is also defined as the relative change in the volume of a body produced by a unit compressive or tensile stress acting uniformly over its surface. It is a property that indicates the compressibility of a fluid. In the present case, Bulk modulus has a maximum value of 4.599 for black stone (sample 1) and a minimum value of 1.904 for ordinary stone (sample 6). Other stone samples have an intermediate value. Maximum value of bulk modulus reveals that sample 1 (Black stone) has the ability to withstand changes in volume when under compression on all sides. This type of stone will withstand the change in the volume when subjected to external pressure.

Young's modulus, which is also known as the elastic modulus, is a mechanical property of linear elastic solid materials. It defines the relationship between stress (force per unit area) and strain (proportional deformation) in a material. Higher value of Young’s modulus (0.867 GPa) for sample 1 (black stone) reveals that it has high elastic nature compared to ordinary stone whose Young’s modulus is 0.437 GPa. So black stones are flexible and can withstand more external pressure compared to other types of stones.

Poisson’s ratio is calculated as a function of longitudinal modulus and shear modulus. It is a measure of the dimensionality of the structure. It is also known as the coefficient of expansion on the transverse axial, the negative ratio of transverse to axial strain. When a material is compressed in one direction, it usually tends to expand in the other two directions perpendicular to the direction of compression. In present case, the value of Poisson’s ratio is almost equal for all types of stones and it is around 0.47. Among all stones, it is observed that black stones have higher value for Poisson’s ratio (0.468) compared to ordinary stone (0.461). This means that when stone samples are compressed in any direction, the stone shrinks in volume and expands in lateral directions (and when elongated in any direction, it increases in volume and shrinks in lateral directions).

The micro hardness of a substance is an important parameter to define the strength of its material. This property is basically related to the crystal structure of the material or in other words, the way in which the atoms are packed and the electronic factors operating to make the structure. Hardness testing provides useful information about the mechanical properties like elastic constants, yield strength etc of materials. Elastic deformation developed in a material when subjected to indentation is directly proportional to the plastic deformation. Bernhardt et al [10] suggested that a part of the energy absorbed during indentation is used in producing plastic deformation and the rest increases the surface free energy. Hardness value depends on the method of measurement that in turn determines the scale of hardness obtained. Micro hardness determines the stress required to eliminate the free volume. It means testing of hardness involving materials by applying small loads. Ordinary stone has a lower value (0.0038) for micro hardness. Higher value of micro hardness of stone 1 (0.0062) reveals that the stone is able to bear additional load comfortably without causing damage to its structure even at microscopic level.

Fugacity is a measure of the tendency of a substance, often a fluid, to move from one phase to another phase. Higher values of fugacity (0.0020 and 0.0023) are observed for sandal stone and ordinary stone (sample 4 & 5). These stone have more chance to change its phase from one form to other easily. Less value of fugacity of 0.0013 for black stone (sample 1, Idol making) reveals that these stones have very less chance to change its phase from one form to other under different thermo dynamical conditions. It may be for these reasons, black stone is preferred for idol preparation. The values of various elastic constants are listed in table 1 and shown in figure 2.

SUMMARY AND CONCLUSION

The mechanical properties of all stone samples are analysed by estimating its various elastic constants. It is seen from the above analysis that black stone (sample 1) has unique features compared to other stone samples and it may be concluded that due to all these features, stone 1 (Black stone) is widely preferred for making Idols compared to other stones.

Table 1. Elastic constants for various stone samples

Samples	Longitudinal Velocity m/s	Shear Velocity m/s	Density Kg/m^3	Longitudinal Modulus G pa	Shear Modulus G pa	Bulk Modulus G pa	Young's Modulus G pa	Poisson's Ratio	Micro Hardness G pa	Fugacity
Black Stone	3588	872	3877	4.993	0.295	4.599	0.867	0.468	0.0062	0.0013
Green Stone	3473	880	2621	3.163	0.203	2.892	0.595	0.465	0.0046	0.0016
Neutral Stone 1	3863	1005	2393	3.572	0.242	3.249	0.708	0.463	0.0059	0.0018
Neutral Stone 2	3715	905	2530	3.493	0.207	3.217	0.608	0.468	0.0044	0.0014
Sandal Stone	3550	972	2433	3.066	0.230	2.759	0.671	0.459	0.0062	0.0023
Ordinary Stone	3450	920	1767	2.103	0.149	1.904	0.437	0.461	0.0038	0.0020

Figure 1. Stone samples taken for study (sample 1 to 6 from L to R)

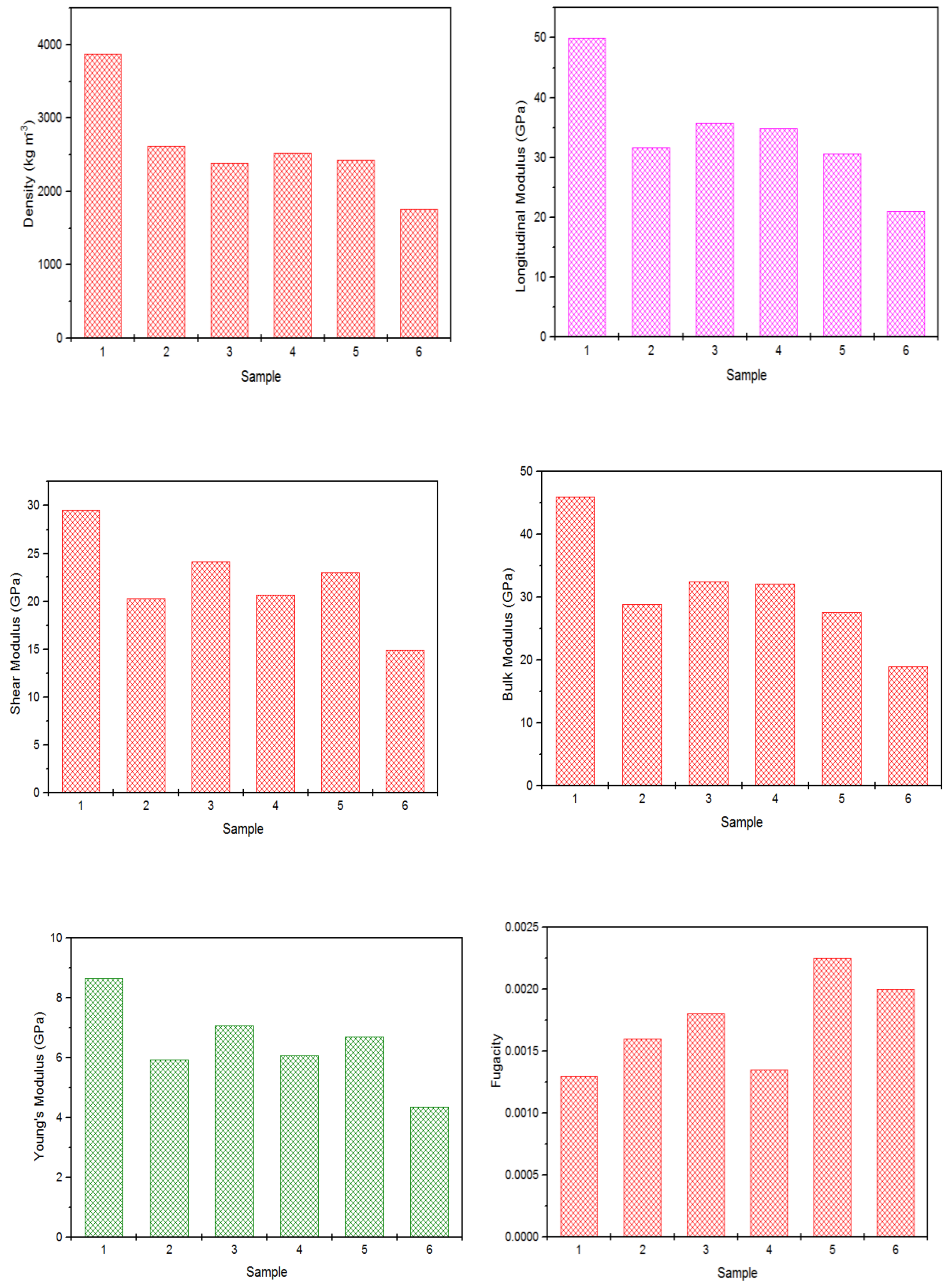

Figure 2. Various values of elastic constants for different samples

LIST OF SYMBOLS :

V_L	Longitudinal Velocity
V_S	Shear Velocity
L	Longitudinal Modulus
G	Shear Modulus
K	Bulk Modulus
E	Young's Modulus
H	Micro Hardness
σ	Poisson's Ratio
$1/f_g$	Fugacity

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