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Common Fixed Point Theorems Governed By Rational Inequalities and Intimate Mappings in Multiplicative Metric Spaces

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Abstract

This paper is in the sequence of papers to prove the common fixed point result using rational contractive mapping for four maps, in which the pair of the maps are satisfying the compatible mapping of type (A), also the pairs of maps are assumed to satisfy the intimate mapping in the setup of multiplicative metric spaces.

Key words compatible Mappings of type (A), intimate mappings, complete multiplicative metric space.

AMS Subject Classification: 47H10, 54H25

INTRODUCTION AND PRELIMINARIES

Özavsar and Cevikel [1] defined multiplicative metric spaces with its topological properties governed by multiplicative contraction. The concept of Intimate mappings was introduced in 2001 by Sahu et al. [2] which was extension of the Compatible mappings of type (A) which was introduced by Kang et al. [3]. There is a difference between the type of mappings that is the mappings like Compatible mappings of type (A). Intimate mapping are not necessarily commute at a point of coincidence.
We are going to use four self-maps and the rational contractive maps to prove the unique common fixed point theorem. The pairs of these maps are chosen in such a way that each pair satisfies the intimate mapping as well as the compatible mapping of type (A). Some basic notations like letters $\mathbb{R}$, $\mathbb{R}_+$and $\mathbb{N}$ are used to represents real numbers, the set of all positive real numbers and the set of all natural numbers. Now , some important and necessary definitions and results in multiplicative metric space in sequel are as follows:

Definition 1.1.[4] *Let X be a nonempty set. Multiplicative metric is a mapping*

$d : X \times X \to \mathbb{R}_+$ *satisfying the following conditions:*

(m1) $d(x, y) \geq 1$ *for all* $x, y \in X$ *and* $d(x, y) = 1$ *if and only if* $x = y$,

(m2) $d(x, y) = d(y, x)$ *for all* $x, y \in X$,

(m3) $d(x, z) \leq d(x, y) \cdot d(y, z)$ *for all* $x, y, z \in X$ *(multiplicative triangle inequality).*

One of the example of complete multiplicative metric space is $\mathbb{R}_+$ *with respect to multiplicative metric space.*

Definition 1.2.([5]) *Let f and g be two mappings of a multiplicative metric space (X, d) into itself. Then f and g are said to be*

(1) Compatible if,

$$\lim_{n\to\infty} d(fgx_n, gfx_n) = 1$$

Whenever $\{x_n\}$ *is a sequence in X such that*

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$$

for some $t \in X$.

(2) Compatible of type (A) if

$$\lim_{n\to\infty} d(fgx_n, ggx_n) = 1 \text{ and } \lim_{n\to\infty} d(gfx_n,, ffx_n) = 1$$

Whenever $\{x_n\}$ *is a sequence in X such that*

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$$

For some $t \in X$.

Definition 1.3. ([3]) *Let f and g be two mappings of a multiplicative metric Space (X, d) into itself. Then f and g are said to be*

(1) g-intimate mappings if

$\alpha d(gfx_n, gx_n) \leq \alpha d(ffx_n, fx_n)$,

Where α = *limsup or liminf and* $\{x_n\}$ *is a sequence in X such that*

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$$

for some $t \in X$.

(2) f-intimate mappings if

$$\alpha d(fgx_n, fx_n) \leq \alpha d(ggx_n, gx_n),$$

where α = *limsup or liminf and* $\{x_n\}$ *is a sequence in X such that*

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$$

for some $t \in X$.

Proposition 1.4.([3]) *Let f and g be mappings of a multiplicative metric space (X, d) into itself. Assume that f and g are compatible of type (A). Then f and g are f-intimate and g-intimate.*

Proposition 1.5.([5]) *Let f and g be mappings of a multiplicative metric space (X, d) into itself. Assume that f and g are g-intimate and* $ft = gt = p \in X$. *Then* $d(gp, p) \leq d(fp, p)$.

Proof. *Suppose that* $x_n = t$ *for all* $n \geq 1$. *So*

$$fx_n = gx_n \to ft = gt = p.$$

Since f and g are g-intimate, we have

$$d(gft, gt) = \lim_{n\to\infty} d(gfx_n, gx_n) \leq \lim_{n\to\infty} d(ffx_n, fx_n) = d(fft, ft).$$

This implies that

$$d(gp, p) \leq d(fp, p).$$

MAIN RESULTS

Now we prove a common fixed point theorem using the above definitions and results.

Theorem 2.1. *Let A, B, S and T be mappings of a complete multiplicative metric space (X, d) into itself satisfying the conditions*

(2.1) $SX \subset BX$ *and* $TX \subset AX$

$$(2.2)\quad d(Sx,Ty) \leq \left\{ max \left\{ \begin{array}{c} \frac{d(Sx,Ty)[d(Ax,Sx)+d(Ax,By)]}{d(Sx,Ty)+d(By,Ty)}, \frac{d(Ax,By)[d(Ty,Sx)+d(Ax,By)]}{d(By,Ax)+d(Sx,Ty)}, \\ \frac{d(Ty,By)d(Ty,Sx)[d(By,Ax)+d(Sx,Ax)]}{d(Ty,Sx)+d(By,Ty)}, \\ \frac{d(Ty,By)[d(Sx,Ty)+d(Sx,Ax)]}{d(Ax,By)+d(Ty,By)} \end{array} \right\} \right\}^{\lambda}$$

for all $x, y \in X$, also $\lambda \in [0,1/2)$. Assume the following conditions are satisfied

(a) *one of the mappings A, B, S and T is continuous*
(b) *Assume that the pairs (A, S) and (B, T) are weakly commuting*
(c) *Assume that the pair (A, S) and (B, T) is compatible of type (A).*
(d) *Assume that A(X) is complete and the pairs (A, S) is A-intimate and (B, T) is B-intimate.*

Then A,B,S and T have unique common fixed point.

Proof. *Let us consider any arbitrary point x_0 of X.*

As given $SX \subset BX$. Hence there exist another point x_1 of X in such a way that,

$$Sx_0 = Bx_1 = y_0.$$

for the chosen x_1, there exist any x_2 of X, in such a way that

$$Tx_1 = Ax_2 = y_1.$$

Using the induction, there exist sequences $\{x_n\}$ and $\{y_n\}$ in X, so that

$$Sx_{2n} = Bx_{2n+1} = y_{2n},\ Tx_{2n+1} = Ax_{2n+2} = y_{2n+1}, \tag{2.3}$$

using (2.2), we have

$d(y_{2n}, y_{2n+1}) = d(Sx_{2n}, Tx_{2n+1})$

$$\leq \left\{ max \left\{ \begin{array}{c} \dfrac{d(Sx_{2n},Tx_{2n+1})[d(Ax_{2n},Sx_{2n}) + d(Ax_{2n},Bx_{2n+1})]}{d(Sx_{2n},Tx_{2n+1}) + d(Bx_{2n+1},Tx_{2n+1})}, \\ \dfrac{d(Ax_{2n},Bx_{2n+1})[d(Tx_{2n+1},Sx_{2n}) + d(Ax_{2n},Tx_{2n+1})]}{d(Bx_{2n+1},Ax_{2n}) + d(Sx_{2n},Bx_{2n+1})}, \\ \dfrac{d(Tx_{2n+1},Bx_{2n+1})d(Tx_{2n+1},Sx_{2n})[d(Bx_{2n+1},Ax_{2n}) + d(Sx_{2n},Ax_{2n})]}{d(Tx_{2n+1},Sx_{2n}) + d(Bx_{2n+1},Tx_{2n+1})}, \\ \dfrac{d(Tx_{2n+1},Bx_{2n+1})[d(Sx_{2n},Tx_{2n+1}) + d(Sx_{2n},Ax_{2n})]}{d(Ax_{2n},Bx_{2n+1}) + d(Tx_{2n+1},Bx_{2n+1})} \end{array} \right\} \right\}^{\lambda}$$

$$\leq \left\{ max \left\{ \begin{array}{c} \frac{d(y_{2n}, y_{2n+1})[d(y_{2n-1}, y_{2n}) + d(y_{2n-1}, y_{2n})]}{d(y_{2n}, y_{2n+1}) + d(y_{2n}, y_{2n+1})}, \\ \frac{d(y_{2n-1}, y_{2n})[d(y_{2n+1}, y_{2n}) + d(y_{2n-1}, y_{2n})]}{d(y_{2n}, y_{2n-1}) + d(y_{2n}, y_{2n})}, \\ \frac{d(y_{2n+1}, y_{2n})d(y_{2n+1}, y_{2n})[d(y_{2n}, y_{2n-1}) + d(y_{2n-1}, y_{2n})]}{d(y_{2n+1}, y_{2n}) + d(y_{2n}, y_{2n+1})}, \\ \frac{d(y_{2n+1}, y_{2n})[y_{2n}, y_{2n+1}) + d(y_{2n}, y_{2n-1})]}{d(y_{2n-1}, y_{2n}) + d(y_{2n+1}, y_{2n})} \end{array} \right\} \right\}^{\lambda}$$

$$\leq \left\{ max \left\{ \begin{array}{c} d(y_{2n-1}, y_{2n+1}), \\ d(y_{2n-1}, y_{2n}).\, d(y_{2n+1}, y_{2n}), \\ d(y_{2n+1}, y_{2n}).\, d(y_{2n}, y_{2n-1}), d(y_{2n+1}, y_{2n}) \end{array} \right\} \right\}^{\lambda}$$

$$d(y_{2n}, y_{2n+1}) \leq\ d^{\lambda}(y_{2n-1}, y_{2n}).\ d^{\lambda}(y_{2n+1}, y_{2n})$$

$$d(y_{2n}, y_{2n+1}) \leq\ d^{\frac{\lambda}{1-\lambda}}(y_{2n-1}, y_{2n})$$

(2.4) $\quad d(y_{2n}, y_{2n+1})\ \leq\ d^{h}(y_{2n-1}, y_{2n})$, *where* $\quad h=\frac{\lambda}{1-\lambda}$

In the same way,

$$d(y_{2n+1},y_{2n+2})\ \leq\ d(Tx_{2n+1},Sx_{2n+2})$$

$$=\ d(Sx_{2n+2},Tx_{2n+1})$$

$$\leq$$

$$\left\{ max \left\{ \begin{array}{c} \frac{d(Sx_{2n+2},Tx_{2n+1})[d(Ax_{2n+2},Sx_{2n+2})+d(Ax_{2n+2},Bx_{2n+1})]}{d(Sx_{2n+2},Tx_{2n+1})+d(Bx_{2n+1},Tx_{2n+1})}, \\ \frac{d(Ax_{2n+2},Bx_{2n+1})[d(Tx_{2n+1},Sx_{2n+2})+d(Ax_{2n+2},Bx_{2n+1})]}{d(Bx_{2n+1},Ax_{2n+2})+d(Sx_{2n+2},Tx_{2n+1})}, \\ \frac{d(Tx_{2n+1},Bx_{2n+1})d(Tx_{2n+1},Sx_{2n+2})[d(Bx_{2n+1},Ax_{2n+2})+d(Sx_{2n+2},Ax_{2n+2})]}{d(Tx_{2n+1},Sx_{2n+2})+d(Bx_{2n+1},Tx_{2n+1})}, \\ \frac{d(Tx_{2n+1},Bx_{2n+1})[d(Sx_{2n+2},Tx_{2n+1})+d(Sx_{2n+2},Ax_{2n+2})]}{d(Ax_{2n+2},Bx_{2n+1})+d(Tx_{2n+1},Bx_{2n+1})} \end{array} \right\} \right\}^{\lambda}$$

$$\leq \left\{ max \left\{ \begin{array}{c} \frac{d(y_{2n+2}, y_{2n+1})[d(y_{2n+1}, y_{2n+2}) + d(y_{2n+1}, y_{2n})]}{d(y_{2n+2}, y_{2n+1}) + d(y_{2n}, y_{2n+1})}, \\ \frac{d(y_{2n+1}, y_{2n})[d(y_{2n+1}, y_{2n+2}) + d(y_{2n+1}, y_{2n})]}{d(y_{2n}, y_{2n+1}) + d(y_{2n+2}, y_{2n+1})}, \\ \frac{d(y_{2n+1}, y_{2n})d(y_{2n+1}, y_{2n+2})[d(y_{2n}, y_{2n+1}) + d(y_{2n+2}, y_{2n+1})]}{d(y_{2n+1}, y_{2n+2}) + d(y_{2n}, y_{2n+1})}, \\ \frac{d(y_{2n+1}, y_{2n})[y_{2n+2}, y_{2n+1}) + d(y_{2n+2}, y_{2n+1})]}{d(y_{2n+1}, y_{2n}) + d(y_{2n+1}, y_{2n})} \end{array} \right\} \right\}^{\lambda}$$

$$\leq \left\{ max \left\{ \begin{array}{c} d(y_{2n+2}, y_{2n+1}), d(y_{2n+1}, y_{2n}), d(y_{2n+1}, y_{2n+2}). \\ d(y_{2n}, y_{2n+1}), d(y_{2n+2}, y_{2n+1}) \end{array} \right\} \right\}^{\lambda}$$

$$d(y_{2n+1}, y_{2n+2}) \leq d^{\frac{\lambda}{1-\lambda}}(y_{2n}, y_{2n+1})$$

(2.5) $$d(y_{2n+1}, y_{2n+2}) \leq d^{h}(y_{2n}, y_{2n+1}) \quad \textit{(since, } h = \frac{\lambda}{1-\lambda}\textit{)}$$

Using (2.4) and (2.5), we have

(2.6) $$d(y_n, y_{n+1}) \leq d^{h}(y_{n-1}, y_n) \leq d^{h^2}(y_{n-2}, y_{n-1}) \leq d^{h^3}(y_{n-3}, y_{n-2}) \leq \cdots \leq d^{h^n}(y_0, y_1)$$

Therefore, for all $m, n \in \mathbb{N}, n > m$,
$using\ the\ triangular\ inequality\ of\ multiplicative$

$metric\ space$ *we have,*

$$d(y_m, y_n) \leq d(y_m, y_{m-1}).d(y_{m-1}, y_{m-2}).d(y_{m-2}, y_{m-3}) \dots d(y_{n+1}, y_n)$$

$$d(y_m, y_n) \leq d^{h^{m-1}}(y_1, y_0).d^{h^{m-2}}(y_1, y_0).d^{h^{m-3}}(y_1, y_0) \dots . d^{h^n}(y_1, y_0)$$

[using (2.6)]

$$d(y_m, y_n) \leq d^{\frac{h^n}{1-h}}(y_1, y_0).$$

This implies that,

$$(y_m, y_n) \; coverges \; to \; 1, as \; n, m \; approaches \; to \; \infty.$$

$Therefore \; \{y_n\} \; is \; a \; multiplicative \; cauchy \; \; sequence.$ *Since, we have mentioned that X is a complete multiplicative metric space and hence the convergent point belong to X. Since, AX is complete there exists p* $\in$ *AX such that*

$$y_{2n+1} = Tx_{2n+1} = Ax_{2n+2}$$

approaches to p as n approaches to ∞*. so, there exist u* $\in$ *X such that ,*

(2.7) $$Au = p.$$

if $\{y_n\}$ *is a cauchy sequence then* $\{y_{2n+1}\}$ *and* $\{y_{2n}\}$ *i.e. all subsequences of* $\{y_n\}$ *are also convergent. Therefore, we have*

$$y_{2n} = Sx_{2n} = Bx_{2n+1}$$

approaches to p as n approaches to ∞ *i.e.*

$$\lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} Sx_{2n} = \lim_{n\to\infty} Bx_{2n+1} = p$$

Let's have an assertion, Su= p, Using the replacement as x=u and $y = x_{2n+1}$ *and using equation (2.2), We have*

$$d(Su, Tx_{2n+1}) \leq \left\{ max \left\{ \begin{array}{c} \dfrac{d(Su,Tx_{2n+1})[d(Au,Su)+d(Au,Bx_{2n+1})]}{d(Su,Tx_{2n+1})+d(Bx_{2n+1},Tx_{2n+1})}, \\ \dfrac{d(Au,Bx_{2n+1})[d(Tx_{2n+1},Su)+d(Au,Bx_{2n+1})]}{d(Bx_{2n+1},Au)+d(Su,Tx_{2n+1})}, \\ \dfrac{d(p,p)d(p,Su)[d(p,p)+d(Su,p)]}{d(p,Su)+d(p,p)}, \\ \dfrac{d(Tx_{2n+1},Bx_{2n+1})[d(Su,Tx_{2n+1})+d(Su,Au)]}{d(Au,Bx_{2n+1})+d(Tx_{2n+1},Bx_{2n+1})} \end{array} \right\} \right\}^{\lambda}$$

taking the limiting value of n as ∞, we have

$$d(Su, p) \leq \left\{ max \left\{ \begin{array}{c} \frac{d(Su,p)[d(p,Su)+d(p,p)]}{d(Su,p)+d(p,p)}, \\ \frac{d(p,p)[d(p,Su)+d(p,p)]}{d(p,p)+d(Su,p)}, \\ \frac{d(p,Su)[d(p,p)+d(Su,p)]}{d(p,Su)+d(p,p)}, \\ \frac{d(p,p)[d(Su,p)+d(Su,p)]}{d(p,p)+d(p,p)} \end{array} \right\} \right\}^{\lambda}$$

$$d(Su, p) \leq \{max\{d(Su,p), 1, d(Su,p), d(Su,p)\}\}^{\lambda} \quad [using\ (2.7)]$$

which is a contradiction, since λ ∈ [0,1/2). And hence, d(Su, p) = 1, i.e. Su = p.

(2.8) $$Su = Au = p.$$

Next we have assertion that p = Tv. Also p = Su ∈SX ⊂BX

Therefore there exist a point v ∈X, in such a way that

(2.9) $$Bv = p.$$

By the substitution as x= u and y = v in inequality (2.2) we have

$$d(p, Tv) = d(Su, Tv) \leq \left\{ max \left\{ \begin{array}{c} \frac{d(Su,Tv)[d(Au,Su)+d(Au,Bv)]}{d(Su,Tv)+d(Bv,Tv)}, \\ \frac{d(Au,Bv)[d(Tv,Su)+d(Au,Bv)]}{d(Bv,Au)+d(Su,Tv)}, \\ \frac{d(Tv,Bv)d(Tv,Su)[d(Bv,Au)+d(Su,Au)]}{d(Tv,Su)+d(Bv,Tv)},, \\ \frac{d(Tv,Bv)[d(Su,Tv)+d(Su,Au)]}{d(Au,Bv)+d(Tv,Bv)} \end{array} \right\} \right\}^{\lambda}$$

$$= \left\{ max \left\{ \begin{array}{c} \frac{d(p,Tv)[d(p,p)+d(p,p)]}{d(p,Tv)+d(p,Tv)}, \frac{d(p,p)[d(Tv,p)+d(p,p)]}{d(p,p)+d(p,Tv)}, \\ \frac{d(Tv,p)d(Tv,p)[d(p,p)+d(p,p)]}{d(Tv,p)+d(p,Tv)}, \\ \frac{d(Tv,p)[d(p,Tv)+d(p,p)]}{d(p,p)+d(Tv,p)} \end{array} \right\} \right\}^{\lambda}$$

$$= \{max\{1,1, d(Tv,p), d(Tv,p)\}\}^{\lambda}$$

$$d(p, Tv) \leq d^{\lambda}(p,Tv)$$

this is a contradiction because $\lambda \in [0,1/2)$. And hence,

(2.10) $$Tv = p$$

Using 2.4 we have Su= Au = p and , it is given that A, S is A-intimate, by proposition 1.5, we have

(2.11) $$d(Ap, p) \leq d(Sp, p).$$

let us consider that Sp ≠p and using the condition (2.2), we get

$$d(Sp,p) = d(Sp,Tv) \leq \left\{ \max \left\{ \begin{array}{c} \frac{d(Sp,Tv)[d(Ap,Sp)+d(Ap,Bv)]}{d(Sp,Tv)+d(Bv,Tv)}, \\ \frac{d(Ap,Bv)[d(Tv,Sp)+d(Ap,Bv)]}{d(Bv,Ap)+d(Sp,Tv)}, \\ \frac{d(Tv,Bv)d(Tv,Sp)[d(Bv,Ap)+d(Sp,Ap)]}{d(Tv,Sp)+d(Bv,Tv)}, \\ \frac{d(Tv,Bv)[d(Sp,Tv)+d(Sp,Ap)]}{d(Ap,Bv)+d(Tv,Bv)} \end{array} \right\} \right\}^{\lambda}$$

$$\leq \left\{ \max \left\{ \begin{array}{c} \frac{d(Sp,p)[d(Ap,p).d(p,Sp)+d(Ap,p)]}{d(Sp,p)+d(p,p)}, \\ \frac{d(Ap,p)[d(p,Sp)+d(Ap,p)]}{d(p,Ap)+d(Sp,p)}, \\ \frac{d(p,p)d(p,Sp)[d(p,Ap)+d(Sp,Ap)]}{d(p,Sp)+d(p,p)}, \\ \frac{d(p,p)[d(Sp,p)+d(Sp,p).d(p,Ap)]}{d(Ap,p)+d(p,p)} \end{array} \right\} \right\}^{\lambda}$$

$$= \left\{ \max \left\{ \begin{array}{c} \frac{d(Sp,p).d(Ap,p)[d(p,Sp)+1]}{d(Sp,p)+d(p,p)}, \\ \frac{d(Ap,p)[d(p,Sp)+d(Ap,p)]}{d(p,Ap)+d(Sp,p)}, \\ \frac{d(p,Sp)d(p,Ap)[1+d(Sp,p)]}{d(p,Sp)+d(Ap,p)}, \\ \frac{d(Sp,p)[1+d(p,Ap)]}{d(Ap,p)+1} \end{array} \right\} \right\}^{\lambda}$$

$$= \left\{ \max \left\{ \begin{array}{c} d(Sp,p).d(Sp,p), d(S\,p,p), \\ d(p,Sp), \\ d(Sp,p) \end{array} \right\} \right\}^{\lambda}$$

(2.12) $$= \left\{ \max \left\{ \begin{array}{c} d^2(p,Sp), d(Sp,p), \\ d(Sp,p), d(Sp,p) \end{array} \right\} \right\}^{\lambda}$$

Case 1

If $max\{d^2(p,Sp), d(Sp,p), d^2(Sp,p), d(Sp,p)\}=d^2(p,Sp)$

then, equation 2.12 become,

d(Sp, p) $\leq d^{2\lambda}(p,Sp)$, *which is obvious and hence nothing to prove*

Case 2

If $max\begin{Bmatrix} d^2(p,Sp), d(Sp,p), \\ d^2(Sp,p), d(Sp,p) \end{Bmatrix} = d(Sp,p)$

then, equation 2.12 become,

d(Sp, p) $\leq d^{\lambda}(p,Sp)$, *which is a contradiction because,* $\lambda \in [0,1/2)$,

and hence we have , d(Sp, p)=1, Sp=p.

This implies that

(2.13) *Sp =Ap = p.*

In similar manner, we get Bp=Tp=p.

Uniqueness

Let p and q are two common fixed point such that p$\neq q$, *using the equation (2.2)*

$$d(p,q) = d(Sp,Tq) \leq \left\{ max \begin{Bmatrix} \frac{d(Sp,Tq)[d(Ap,Sp)+d(Ap,Bq)]}{d(Sp,Tq)+d(Bq,Tq)}, \\ \frac{d(Ap,Bq)[d(Tq,Sp)+d(Ap,Bq)]}{d(Bq,Ap)+d(Sp,Tq)}, \\ \frac{d(Tq,Bq)d(Tq,Sp)[d(Bq,Ap)+d(Sp,Ap)]}{d(Tq,Sp)+d(Bq,Tq)}, \\ \frac{d(Tq,Bq)[d(Sp,Tq)+d(Sp,Ap)]}{d(Ap,Bq)+d(Tq,Bq)} \end{Bmatrix} \right\}^{\lambda}$$

$$\leq \left\{ max \begin{Bmatrix} \frac{d(p,q)[d(p,p)+d(p,q)]}{d(p,q)+d(q,q)}, \\ \frac{d(p,q)[d(q,p)+d(p,q)]}{d(q,p)+d(p,q)}, \\ \frac{d(q,q)d(q,p)[d(q,p)+d(p,p)]}{d(q,p)+d(q,q)}, \\ \frac{d(q,q)[d(p,q)+d(p,p)]}{d(p,q)+d(q,q)} \end{Bmatrix} \right\}^{\lambda}$$

$$= \left\{ max \begin{Bmatrix} d(p,q), d(p,q), \\ d(q,p), \\ 1 \end{Bmatrix} \right\}$$

$d(p,q) \leq d^{\lambda}(p,q)$, *which is a contraction. Hence, uniqueness is proved.*

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